Physics 2414, Spring 2008
Group Exercise 8, Mar 27, 2008

Name 1: __________ OUID 1: __________

Name 2: __________ OUID 2: __________

Name 3: Solutions OUID 3: Solutions

Name 4: __________ OUID 4: __________

Section Number: ____

Conservation of Energy

A mass m moves from point 'i' to point 'f' under the action of one or more forces. The change in kinetic energy of the mass equals the sum of the work done by the individual forces,

$$K_f - K_i = \Delta K = \Sigma\, W. \tag{1}$$

The work done by conservative forces (eg. gravity, spring force, etc.) are denoted by W_c, and work done by non-conservative forces (eg. friction) are denoted by W_{nc}. Thus we can re-write eqn. (1) as

$$\Delta K - \Sigma\, W_c = \Sigma\, W_{nc}. \tag{2}$$

Negative of the work done by all the conservatice forces is defined to be the change in **potential energy**, $\Delta U = -\Sigma\, W_c$. Thus we can write eqn. (2) as

$$\Delta K + \Delta U = \Sigma\, W_{nc}. \tag{3}$$

This is the statement of conservation of energy.

In particular, the contribution to ΔU due to gravity is given by

$$\Delta U_g = -W_g = mgh_f - mgh_i \tag{4}$$

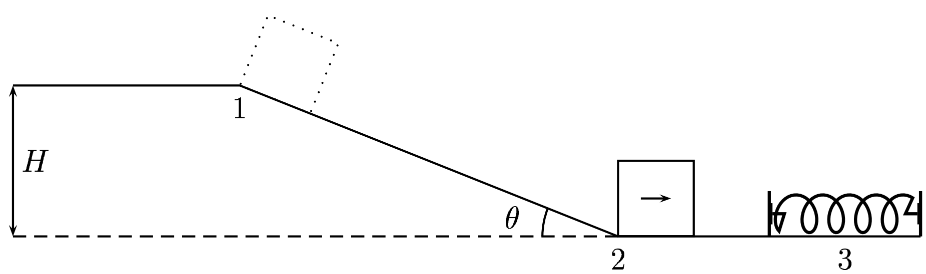
where h is the positive height above the ground. The contribution to ΔU due to the spring force is given by

$$\Delta U_s = -W_s = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 \tag{5}$$

where k is the spring constant of the spring, and x_i (x_f) is the initial (final) change in the length of the spring from its length in its relaxed state. *Note that x_i (x_f) are not the lengths of the spring itself.*

Problems

A mass $M = 100$ kg starts from rest from the highest point on an incline which has friction. The incline makes an angle $\theta = 45^o$ with the horizontal, and vertical height of the incline is $H = 20$ meters.



1. *Mass sliding down from Point 1 to point 2:*

The mass starts from rest ($v_1 = 0$) at point '1' and reaches point '2' with velocity v_2. The goal of this section will be to evaluate v_2.

(a) What is the expression for the change in kinetic energy in going from point '1' to point '2'? (The superscript '12' denotes the end points '1' and '2'.)

$$\Delta K^{12} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2} \times 100 \times v_2^2 - 0 = 50v_2^2 \qquad (6)$$

(b) What is the change in gravitational potential energy in going from point '1' to point '2'?

$$\Delta U_g^{12} = 0 - mgH = 0 - 100 \times 9.8 \times 20 = -19600 \text{ Joules} \qquad (7)$$

(c) Are there any non-conservative forces acting on the mass between point '1' to point '2'?

Yes, the frictional force.

(d) If the frictional force acting on the mass while sliding down the incline is $|\vec{\mathbf{F}}_f| = 173.2$ Newtons, calculate the work done by the frictional force in going from point '1' to point '2'. Thus calculate

W_{nc}^{12}.

$$W_{nc}^{12} = W_f = |\vec{\mathbf{F}}_f| d \cos 180 \qquad (d = \frac{H}{\sin\theta}) \tag{8}$$

$$= 173.2 \times \frac{20}{0.7} \times (-1) = -4949 \text{ Joules} \tag{9}$$

(e) Using the answers in (a) to (d) above in eqn. (3) write the expression for the conservation of energy statement between point '1' and point '2'.

$$\Delta K + \Delta U = \Sigma\, W_{nc}.$$

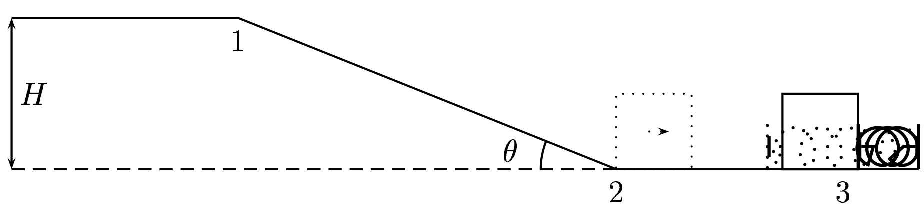
$$50 v_2^2 - 19600 \text{ Joules} = -4949 \text{ Joules} \tag{10}$$

(f) Evaluate the velocity of the mass v_2 at point '2'.

$$v_2 = \sqrt{\frac{19600 - 4949}{50}} = 17.12 \text{ m/s} \tag{11}$$

2. *Compression of the spring:*

The mass keeps moving on the horizontal *frictionless* surface untill it hits the spring with spring constant $k = 10^4$ N/m and comes to rest after compressing the spring by a length x. Note that in the relaxed state the compression in the spring is zero. The goal of this section will be to evaluate x.



(a) What is the change in kinetic energy in going from point '2' to point '3'?

$$\Delta K^{23} = 0 - \frac{1}{2} m v_2^2 = \frac{1}{2} \times 100 \times 17.12^2 = -14655 \text{ Joules} \tag{12}$$

(b) What is the change in gravitational potential energy in going from point '2' to point '3'?

$$\Delta U_g^{23} = 0 \qquad \text{(since they are at the same height.)} \tag{13}$$

(c) What is the expression for the change in spring potential energy in going from point '2' to point '3'?

$$\Delta U_s^{23} = \frac{1}{2}kx^2 - 0 = \frac{1}{2} \times 10^4 \times x^2 - 0 = 5000x^2 \tag{14}$$

(d) Is there any non-conservative force acting on the mass between point '2' and point '3'? Determine W_{nc}^{23}.

$$W_{nc}^{23} = 0 \qquad \text{(No non-conservative force acting.)} \tag{15}$$

(e) Using the results from (a) to (d) above in eqn. (3) write the expression for the conservation of energy statement between point '2' and point '3'.

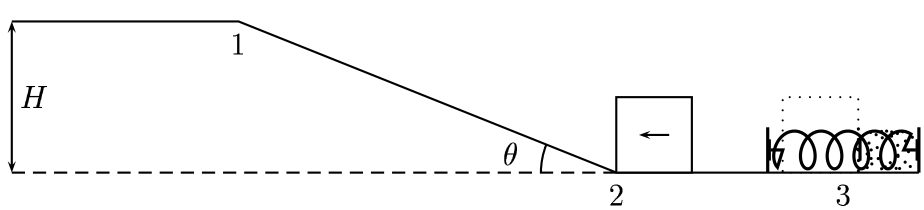
$$\Delta K + \Delta U = \Sigma\, W_{nc}.$$
$$-14655 \text{ Joules} + 5000x^2 = 0 \tag{16}$$

(f) Evaluate the compression x in the spring.

$$x = \sqrt{\frac{14655}{5000}} = 1.7 \text{ meters} \tag{17}$$

3. *Recoil of the spring:*

The compressed spring pushes the mass back. The mass reaches point '2' again with velocity v_2'.



(a) What can you conclude about v_2'? Pick the correct answer.

(i) $v_2' > v_2$

(ii) $v_2' = v_2$ ✓

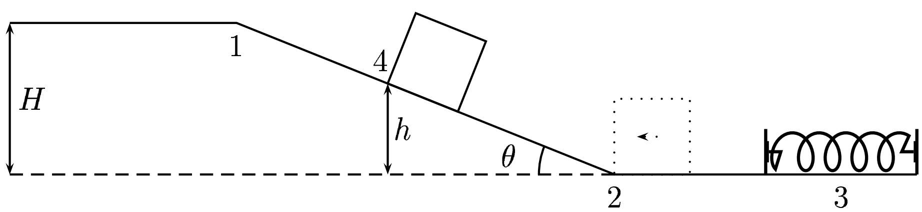
(iii) $v_2' < v_2$

(b) How will your answer to (a) change if the surface of the horizontal floor had friction?

$$v_2' < v_2 \tag{18}$$

4. *Climbing the incline from point '2' to point '4':*

The mass starts climbing up the incline and eventually comes to a stop at a height h from the ground denoted as point '4'. The goal of this section is to evaluate h.



(a) What is the change in kinetic energy in going from point '2' to point '4'?

$$\Delta K^{24} = 0 - \frac{1}{2} m v_2^2 = 0 - 14655 \text{ Joules} \tag{19}$$

(b) What is the expression for the change in gravitational potential energy in going from point '2' to point '4'?

$$\Delta U_g^{24} = mgh - 0 = 100 \times 9.8 \times h - 0 = 980h \text{ Joules} \tag{20}$$

(c) Is there any non-conservative force acting on the mass between point '2' and point '4'?

Yes, the frictional force.

(d) If the frictional force acting on the mass while climbing up the incline is $|\vec{\mathbf{F}}_f| = 173.2$ Newtons, calculate the work done by the frictional force in going from point '2' to point '4'. Thus determine the expression for W_{nc}^{24}.

$$W_{nc}^{24} = W_f = |\vec{\mathbf{F}}_f| d \cos 180 \qquad (d = \frac{h}{\sin\theta}) \tag{21}$$

$$= 173.2 \times \frac{h}{0.7} \times (-1) = -247.4h \tag{22}$$

(e) Using the results from (a) to (d) above in eqn. (3) write the conservation of energy statement between point '2' and point '4'.

$$\Delta K + \Delta U = \Sigma\, W_{nc}.$$

$$-14655 \text{ Joules} + 980h = -247.4h \tag{23}$$

(f) Evaluate the height h.

$$h = \frac{14655}{980 + 247.4} = 11.9 \text{ meters} \tag{24}$$

5. *Analysis:*

(a) How will the result for h change if we decrease the frictional force acting on the mass while on the incline?

(i) increase ✓

(ii) decrease

(iii) remain the same

(b) How will the result for h change if we decrease the spring constant k of the spring?

(i) increase

(ii) decrease

(iii) remain the same ✓