

Physics 2414, Spring 2008

Group Exercise 7, Mar 6, 2008

Name 1: _____ OUID 1: _____
Name 2: _____ OUID 2: _____
Name 3: _____ OUID 3: _____
Name 4: _____ OUID 4: _____

Section Number: _____

Work and Energy

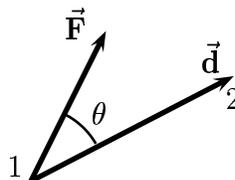
A mass m moves from point '1' to point '2' under the action of the force \vec{F} . Kinetic energy of the mass is given by the expression

$$K = \frac{1}{2}mv^2 \quad (1)$$

where v is the velocity of the particle. The change in kinetic energy of the mass is

$$\Delta K = K_f - K_i \quad (2)$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad (3)$$



Work done *by* the force \vec{F} *on* the mass m is given by the expression

$$W = |\vec{F}|d \cos \theta \quad (4)$$

where, \vec{d} is the displacement of the mass m , and θ is the angle between the force vector and the displacement vector (see figure).

The work done on the mass m equals the change in kinetic energy. The expression relating this is given by

$$\Delta K = W. \quad (5)$$

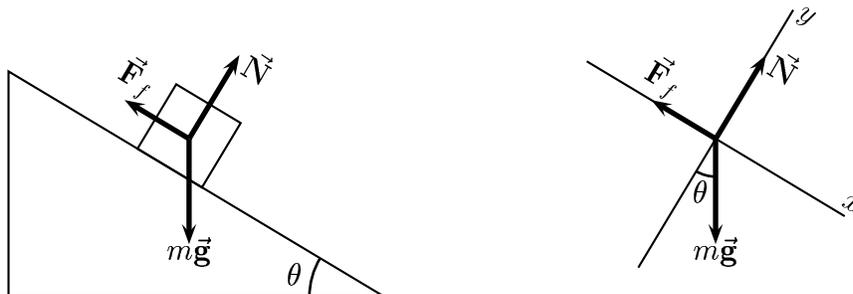
If there is more than one force acting on the mass, the change in kinetic energy equals the total work done by all the forces, and is

given by the expression

$$\Delta K = \sum_i W_i. \quad (6)$$

Problems

A block of mass $M = 100$ kg slides on a frictional incline plane under gravity. The incline makes an angle $\theta = 30^\circ$ with the horizontal. The coefficient of kinetic friction between the mass and the surface of the incline is $\mu_k = 0.25$. The mass starts from the highest point on the incline plane and reaches the lowest point on the plane. The free body force diagram is provided to you.

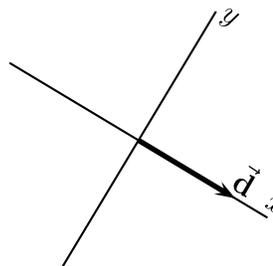


1. *Displacement vector:*

(a) If the base of the incline plane measures 5 meters, what is the magnitude of the displacement of the block.

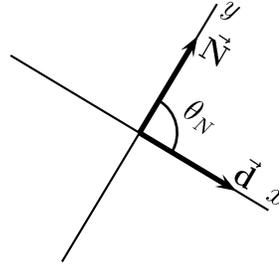
$$|\vec{d}| = \frac{5}{\cos \theta} = \frac{5}{\cos 30} = 5.75\text{m} \quad (7)$$

(b) Show the displacement vector \vec{d} on the graph.



2. *Work done by the normal force \vec{N} :*

(a) Show the normal force \vec{N} and the displacement \vec{d} on the graph.



(b) What is the angle between the Normal force \vec{N} and the displacement vector \vec{d} ?

$$\theta_N = 90^\circ \quad (8)$$

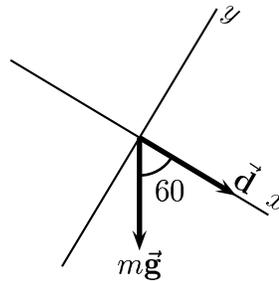
(c) Calculate the work done by the Normal force on the mass.

$$W_N = |\vec{N}|d \cos \theta_N \quad (9)$$

$$= 0 \quad (\cos 90 = 0) \quad (10)$$

3. *Work done by gravity $m\vec{g}$:*

(a) Show the force $m\vec{g}$ and the displacement \vec{d} on the graph.



(b) What is the angle between the force $m\vec{g}$ and the displacement vector \vec{d} ?

$$\theta_{mg} = 90 - \theta = 60 \quad (11)$$

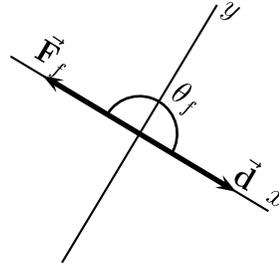
(c) Calculate the work done by the force $m\vec{g}$ on the mass.

$$W_{mg} = mgd \cos(90 - \theta) \quad (12)$$

$$= 100 \times 9.8 \times 5.75 \times \cos 60 = 2818\text{J} \quad (13)$$

4. *Work done by the frictional force \vec{F}_f :*

- (a) Show the frictional force $\vec{\mathbf{F}}_f$ and the displacement $\vec{\mathbf{d}}$ on the graph.



- (b) What is the angle between the frictional force $\vec{\mathbf{F}}_f$ and the displacement vector $\vec{\mathbf{d}}$?

$$\theta_f = 180^\circ \quad (14)$$

- (c) Calculate the work done by the frictional force on the mass. (Hint: $|\vec{\mathbf{F}}_f| = \mu_k |\vec{\mathbf{N}}| = \mu_k mg \cos \theta$)

$$W_f = \mu_k mg \cos \theta d \cos \theta_f \quad (15)$$

$$= 0.25 \times 100 \times 9.8 \times \cos 30 \times 5.75 \times \cos 180 \quad (16)$$

$$= 1225\text{J} \quad (17)$$

- (d) What is the work done by static frictional force?

Since static frictional forces do not move the mass, the work done by static frictional force is zero.

5. *Total work done by the forces on the mass:*

The total work done by the three forces on the mass is equal to the sum of the individual work done.

$$W_{\text{tot}} = W_N + W_{mg} + W_f \quad (18)$$

- (a) Determine the total work done by the three forces on the mass.

$$W_{\text{tot}} = 0 + 2818\text{J} - 1225\text{J} = +1593\text{J} \quad (19)$$

6. *Change in kinetic energy:*

- (a) What is the kinetic energy of the mass just before it starts to

slide.

$$K_i = \frac{1}{2}mv_i^2 = 0 \quad (v_i = 0) \quad (20)$$

(b) The acceleration of the mass is determined by the expression

$$a = g \sin \theta - \mu_k g \cos \theta. \quad (21)$$

Evaluate a .

$$a = 9.8 \times \sin 30 - 0.25 \times 9.8 \times \cos 30 = 2.77 \text{m/s}^2 \quad (22)$$

(c) Determine the velocity of the mass just before it reaches the ground. (Hint: $v_f^2 - v_i^2 = 2ad$.)

$$v_f = \sqrt{v_i^2 + 2ad} \quad (23)$$

$$= \sqrt{0 + 2 \times 2.77 \times 5.75} = 5.64 \text{m/s} \quad (24)$$

(d) Determine the kinetic energy of the block just before it reaches the ground.

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2} \times 100 \times 5.64^2 = 1590 \text{J} \quad (25)$$

(e) Determine the change in the kinetic energy of the mass.

$$\Delta K = 1.6 \times 10^3 \text{J} \quad (26)$$

7. *Total work done by the forces on the mass equals the change in kinetic energy of the mass:*

(a) Verify that the total work done by the forces on the mass equals the change in the kinetic energy of the mass.

$$W_{\text{tot}} = 1593 \text{J} = 1.6 \times 10^3 \text{J} \quad (27)$$

$$\Delta K = 1590 \text{J} = 1.6 \times 10^3 \text{J} \quad (28)$$