Physics 2414, Spring 2008
Group Exercise 7, Mar 6, 2008

Name 1: _____   OUID 1: _____

Name 2: _____   OUID 2: _____

Name 3: _____   OUID 3: _____

Name 4: _____   OUID 4: _____

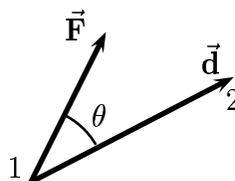Section Number: ____

Work and Energy

A mass m moves from point '1' to point '2' under the action of the force $\vec{\mathbf{F}}$. Kinetic energy of the mass is given by the expression

$$K = \frac{1}{2}mv^2 \tag{1}$$

where v is the velocity of the particle. The change in kinetic energy of the mass is

$$\Delta K = K_f - K_i \tag{2}$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \tag{3}$$

Work done *by* the force $\vec{\mathbf{F}}$ *on* the mass m is given by the expression

$$W = |\vec{\mathbf{F}}|d\cos\theta \tag{4}$$

where, $d = |\vec{\mathbf{d}}|$ is the magnitude of displacement of the mass m, and θ is the angle between the force vector and the displacement vector (see figure).

The work done on the mass m equals the change in kinetic energy. The expression relating this is given by

$$\Delta K = W. \tag{5}$$

If there is more than one force acting on the mass, the change in kinetic energy equals the total work done by all the forces, and is

given by the expression

$$\Delta K = \sum_i W_i. \qquad (6)$$

Problems

A block of mass $M = 100$ kg slides on a frictional incline plane under gravity. The incline makes an angle $\theta = 30^o$ with the horizontal. The coefficient of kinetic friction between the mass and the surface of the incline is $\mu_k = 0.25$. The mass starts from the highest point on the incline plane and reaches the lowest point on the plane. The free body force diagram is provided to you.
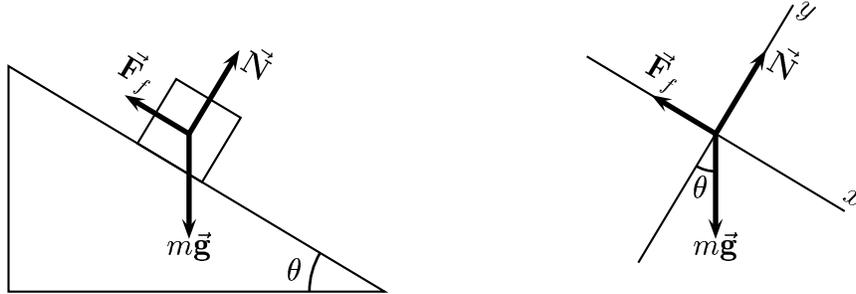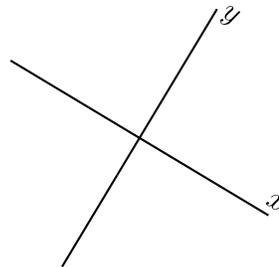

1. *Displacement vector:*

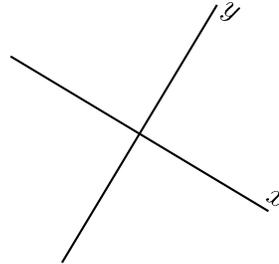(a) If the base of the incline plane measures 5 meters, what is the magnitude of the displacement of the block.

$$d = |\vec{d}| = \qquad (7)$$

(b) Show the displacement vector $\vec{d}$ on the graph.

2. *Work done by the normal force $\vec{N}$:*

(a) Show the normal force $\vec{\mathbf{N}}$ and the displacement $\vec{\mathbf{d}}$ on the graph.

(b) What is the angle between the Normal force $\vec{\mathbf{N}}$ and the displacement vector $\vec{\mathbf{d}}$?
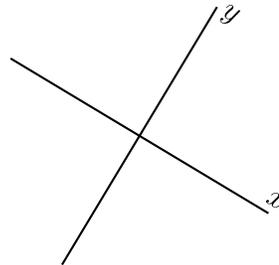
$$\theta_N = \tag{8}$$

(c) Calculate the work done by the Normal force on the mass.

$$W_N = |\vec{\mathbf{N}}|d\cos\theta_N \tag{9}$$
$$= \tag{10}$$

3. *Work done by gravity $m\vec{\mathbf{g}}$:*

(a) Show the force $m\vec{\mathbf{g}}$ and the displacement $\vec{\mathbf{d}}$ on the graph.

(b) What is the angle between the force $m\vec{\mathbf{g}}$ and the displacement vector $\vec{\mathbf{d}}$?
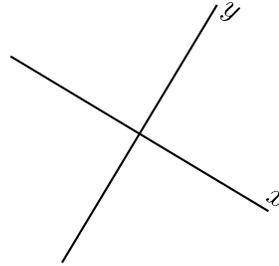
$$\theta_{mg} = \tag{11}$$

(c) Calculate the work done by the force $m\vec{\mathbf{g}}$ on the mass.

$$W_{mg} = \tag{12}$$

4. *Work done by the frictional force $\vec{\mathbf{F}}_f$:*

(a) Show the frictional force $\vec{\mathbf{F}}_f$ and the displacement $\vec{\mathbf{d}}$ on the graph.

(b) What is the angle between the frictional force $\vec{\mathbf{F}}_f$ and the displacement vector $\vec{\mathbf{d}}$?

$$\theta_f = \tag{13}$$

(c) Calculate the work done by the frictional force on the mass. (Hint: $|\vec{\mathbf{F}}_f| = \mu_k |\vec{\mathbf{N}}| = \mu_k mg \cos\theta$)

$$W_f = \tag{14}$$

(d) What is the work done by static frictional force?

5. *Total work done by the forces on the mass:*

The total work done by the three forces on the mass is equal to the sum of the individual work done.

$$W_{\text{tot}} = W_N + W_{mg} + W_f \tag{15}$$

(a) Determine the total work done by the three forces on the mass.

$$W_{\text{tot}} = \tag{16}$$

6. *Change in kinetic energy:*

(a) What is the kinetic energy of the mass just before it starts to slide.

$$K_i = \tag{17}$$

(b) The acceleration of the mass is determined by the expression

$$a = g \sin\theta - \mu_k g \cos\theta. \tag{18}$$

4

Evaluate a.

(c) Determine the velocity of the mass just before it reaches the ground. (Hint: $v_f^2 - v_i^2 = 2ad$.)

$$v_f = \hspace{6cm} (19)$$

(d) Determine the kinetic energy of the block just before it reaches the ground.

$$K_f = \hspace{6cm} (20)$$

(e) Determine the change in the kinetic energy of the mass.

$$\Delta K = \hspace{6cm} (21)$$

7. *Total work done by the forces on the mass equals the change in kinetic energy of the mass:*

(a) Verify that the total work done by the forces on the mass equals the change in the kinetic energy of the mass.

$$W_{\text{tot}} = \hspace{6cm} (22)$$

$$\Delta K = \hspace{6cm} (23)$$