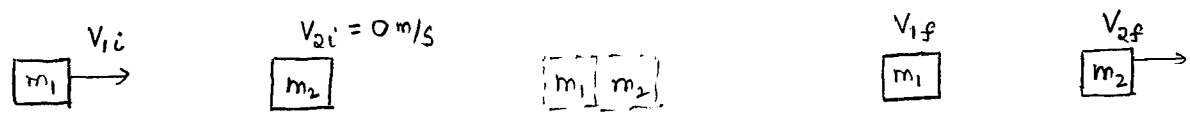
PHYS 2414 HW#8 SOLUTIONS

1. Giancoli6 7.P.022. [355796] **0/4 points** Show Details

A ball of mass 0.340 kg moving east (+*x* direction) with a speed of 3.20 m/s collides head-on with a 0.200 kg ball at rest. If the collision is perfectly elastic, what will be the speed and direction of each ball after the collision?

ball originally at rest	4.03	m/s	---Select---	east
ball originally moving east	0.83	m/s	---Select---	east

Sol: Before collision   Collision   After collision



Since collision is perfectly elastic both momentum and kinetic energy are conserved. i.e

$$(\vec{P}_{tot})_{initial} = (\vec{P}_{tot})_{final}.$$

$$K_{initial} = K_{final}.$$

$\vec{P} = m\vec{V}$ $(kg\,m/s)$ depends on direction (vector)

$K = \frac{1}{2}mV^2$ (J) does not depend on direction of motion. (scalar)

Therefore,

$$m_1V_{1i} + m_2V_{2i} = m_1V_{1f} + m_2V_{2f} \quad — (1)$$

$$\frac{1}{2}m_1V_{1i}^2 + \frac{1}{2}m_2V_{2i}^2 = \frac{1}{2}m_1V_{1f}^2 + \frac{1}{2}m_2V_{2f}^2 \quad — (2)$$

{ Note: We can right away put $V_{2i} = 0$, which will simplify calculation. But we will do this algebra once keeping everything so that we can use it in other problems. }

collect terms with mass m_1 on one side and terms with mass m_2 on other side in both equations.

$$m_1(V_{1i} - V_{1f}) = m_2(V_{2f} - V_{2i}) \quad \text{— (3)}$$

$$m_1(V_{1i}^2 - V_{1f}^2) = m_2(V_{2f}^2 - V_{2i}^2) \quad \text{— (4)}$$

divide (4) by (3) [This can be done excluding a rare case of $V_{1i} = V_{1f}$ and $V_{2i} = V_{2f}$.]

$$\frac{\cancel{m_1}(V_{1i}^2 - V_{1f}^2)}{\cancel{m_1}(V_{1i} - V_{1f})} = \frac{\cancel{m_2}(V_{2f}^2 - V_{2i}^2)}{\cancel{m_2}(V_{2f} - V_{2i})}$$

$$\Rightarrow \frac{\cancel{(V_{1i} - V_{1f})}(V_{1i} + V_{1f})}{\cancel{(V_{1i} - V_{1f})}} = \frac{\cancel{(V_{2f} - V_{2i})}(V_{2f} + V_{2i})}{\cancel{(V_{2f} - V_{2i})}}$$

using $a^2 - b^2 = (a+b)(a-b)$

$$\Rightarrow V_{1i} + V_{1f} = V_{2f} + V_{2i} \quad \text{— (5)}$$

Solve for V_{1f}

$$\boxed{V_{1f} = V_{2f} + V_{2i} - V_{1i}} \quad \text{— (A)}$$

Plug in V_{1f} in eq (1)

$$m_1 V_{1i} + m_2 V_{2i} = m_1(V_{2f} + V_{2i} - V_{1i}) + m_2 V_{2f}$$

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{2f} + m_1 V_{2i} - m_1 V_{1i} + m_2 V_{2f}$$

solving for V_{2f},

$$m_1 V_{2f} + m_2 V_{2f} = m_1 V_{1i} + m_2 V_{2i} - m_1 V_{2i} + m_1 V_{1i}$$

$$V_{2f}(m_1 + m_2) = 2 m_1 V_{1i} + (m_2 - m_1) V_{2i}$$

$$\boxed{V_{2f} = \frac{2 m_1 V_{1i} + (m_2 - m_1) V_{2i}}{(m_1 + m_2)}} \quad \text{— (B)}$$

Once we have number for V_{2f} we can plug-it back in (A) to get V_{1f}.

For problem 1, given

$m_1 = \boxed{0.340}$ kg $\qquad V_{1i} = \boxed{3.20}$ m/s

$m_2 = \boxed{0.200}$ kg $\qquad V_{2i} = 0$ m/s.

using (B) $\qquad \because V_{2i} = 0$

$$V_{2f} = \frac{2 m_1 V_{1i} + \cancel{(m_2 - m_1) V_{2i}}^{\,0}}{m_1 + m_2}$$

$$V_{2f} = \frac{2\left(\boxed{0.340}\,\cancel{kg}\right)\left(\boxed{3.20}\text{ m/s}\right)}{\left(\boxed{0.340} + \boxed{0.200}\right)\cancel{kg}} = \boxed{4.03}\text{ m/s}$$

This is +, so ball originally at rest is moving east after collision.

Plug this in eq. (A)

$$V_{1f} = V_{2f} + \cancel{V_{2i}}^{\,0} - V_{1i} = \left(\boxed{4.03} - \boxed{3.20}\right)\text{ m/s} = \boxed{0.83}\text{ m/s.}$$

which is again +, so ball originally moving in east direction, will keep moving in east direction.

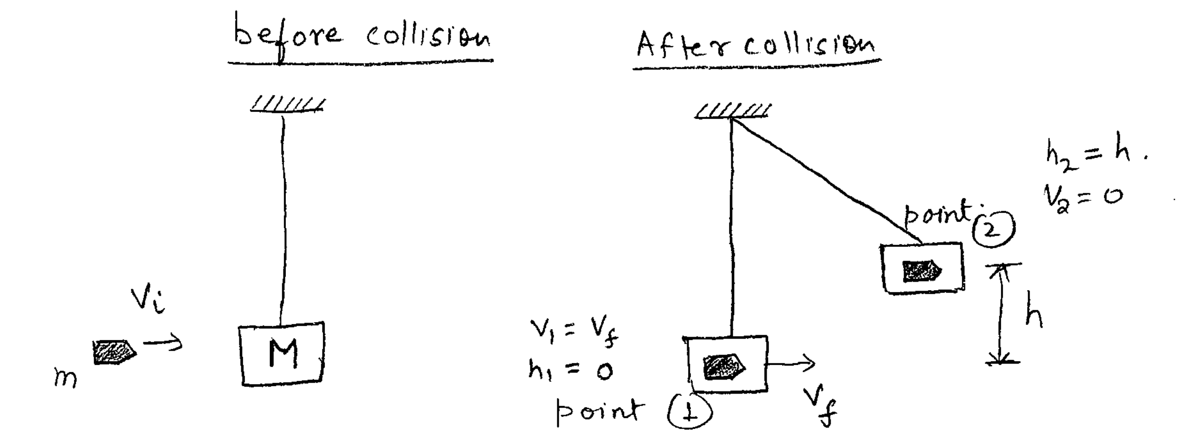
{Note: mass m_1 can also bounce back in certain cases depending on value of two masses and initial velocities.}

2. Giancoli6 7.P.031. [352911] **0/1 points** Show Details

In a ballistic pendulum experiment, projectile 1 results in a maximum height *h* of the pendulum equal to 2.6 cm. A second projectile causes the the pendulum to swing higher, to h_2 = 5.5 cm. The second projectile was how many times faster than the first?

✗ 1.45 x

Sol: Ballistic pendulum experiment.- A bullet moving with initial velocity V_i, hits a pendulum and embeds in it. (This is case of perfectly inelastic collision.) After collision (bullet + pendulum) starts moving with final velocity V_f and rises to a maximum height 'h'



Momentum is conserved in inelastic collision.

$$m V_i = (m+M) V_f \qquad — \; (1)$$

Since there is no non-conservative force acting after collision, so mechanical energy is conserved.

$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}(m+M)\cancel{V_2^2}^{\,0} - \frac{1}{2}(m+M)V_1^2 + (m+M) g h_2 - (m+M)\cancel{g h_1}^{\,0} = 0$$

$$-\frac{1}{2}\cancel{(m+M)} V_f^2 + \cancel{(m+M)} g h = 0$$

$$V_f = \sqrt{2gh} \qquad — \; (2)$$

Plug-in V_f in eq 1

$$m v_i = (m+M)\sqrt{2gh}$$

$$V_i = \frac{(m+M)}{m}\sqrt{2gh} \qquad - \quad (3)$$

Now, we have two projectiles

<u>Case 1</u> $\quad V_i = V_{1i} \qquad h = \boxed{2.6}\text{ cm}$

<u>case 2</u> $\quad V_i = V_{2i} \qquad h = \boxed{5.5}\text{ cm}$

$$V_{1i} = \frac{m+M}{m}\sqrt{2g(\boxed{2.6}\text{ cm})}$$

$$V_{2i} = \frac{m+M}{m}\sqrt{2g(\boxed{5.5}\text{ cm})}$$

we are interested in ratio V_{2i}/V_{1i}

$$\frac{V_{2i}}{V_{1i}} = \frac{\cancel{\frac{m+M}{m}}}{\cancel{\frac{m+M}{m}}}\frac{\sqrt{2g(\boxed{5.5}\text{ cm})}}{\sqrt{2g(\boxed{2.6}\text{ cm})}} = \sqrt{\frac{\cancel{2g}(\boxed{5.5}\text{ cm})}{\cancel{2g}(\boxed{2.6}\text{ cm})}}$$

$$\frac{V_{2i}}{V_{1i}} = \boxed{1.45} \checkmark$$

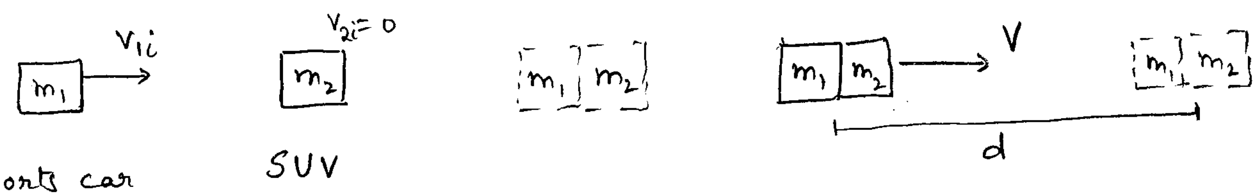
{ <u>Note</u>: This approach will work only if mass of pendulum and projectile remains same in both cases. }

3. Giancoli6 7.P.035. [355790] **0/1 points** Show Details

A 920 kg sports car collides into the rear end of a 2300 kg SUV stopped at a red light. The bumpers lock, the brakes are locked, and the two cars skid forward $\boxed{3.9}$ m before stopping. The police officer, knowing that the coefficient of kinetic friction between tires and road is $\boxed{0.30,}$ calculates the speed of the sports car at impact. What was that speed?

✗ $\boxed{16.8}$ m/s

Sol. Before collision — After collision



given: $m_1 = 920$ kg $\quad V_{1i} = ?$

$m_2 = 2300$ kg $\quad V_{2i} = 0$

$d = \boxed{3.9}$ m

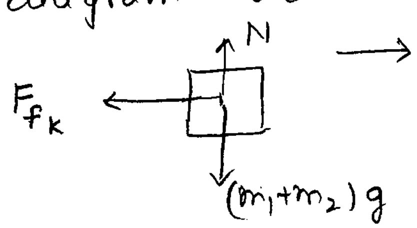
$\mu_k = \boxed{0.30}$ coefficient of kinetic friction.

Since, two cars stick together, this is perfectly inelastic collision. Therefore only momentum is conserved since there is no net force on the system. We have assumed that immediately before and after collision cars were moving with constant velocity.

$$m_1 V_{1i} + \cancel{m_2 V_{2i}}^{\,0} = (m_1 + m_2)\, V \qquad —(1)$$

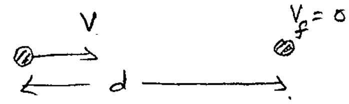
$$\Rightarrow \quad V_{1i} = \frac{(m_1 + m_2)}{m_1}\, V$$

After collision two cars skid as single body. Free body diagram for them is



x dir: $\Sigma F_x = m a_x$

y dir: $\Sigma F_y = m a_y$



$\cancel{v_f^2}^{\,0} = v^2 + 2a_x d$

$a_x = \frac{-v^2}{2d}$

$a_y = 0$ No motion in y-direction.

$\Rightarrow$ x dir: $\cancel{-} F_{fk} = (m_1+m_2)\, a_x = \cancel{-}(m_1+m_2)\frac{v^2}{2d}$

$F_{fk} = \mu_k N = \mu_k (m_1+m_2)\, g$.

$\Rightarrow \mu_k \cancel{(m_1+m_2)}\, g = \cancel{(m_1+m_2)} \frac{v^2}{2d}$

$v = \sqrt{2\mu_k g d}$ — (2)

Plug-in eq. (1). So speed of sports car at impact is

$v_{1i} = \frac{(m_1+m_2)}{m_1}\sqrt{2\mu_k g d}$

$v_{1i} = \left(\frac{920\ kg + 2300\ kg}{920\ kg}\right)\sqrt{2(\boxed{0.30})(9.8\ \frac{m}{s^2})(\boxed{3.9}\ m)}$

$v_{1i} = \boxed{16.76}$ m/s

$v_{1i} = \boxed{16.8}$ m/s ✓

This problem combines kinematics, Newton's Law and momentum conservation concepts.

4. Giancoli6 7.P.039. [355793] **0/11 points** Show Details

A $\boxed{5.5}$ kg object moving in the $+x$ direction at 5.5 m/s collides head-on with a $\boxed{2.6}$ kg object moving in the $-x$ direction at 4.0 m/s. Find the final velocity of each mass for each of the following situations. (Take the positive direction to be $+x$.)

(a) The bodies stick together.

$\boxed{2.45}$ m/s (5.5 kg mass)

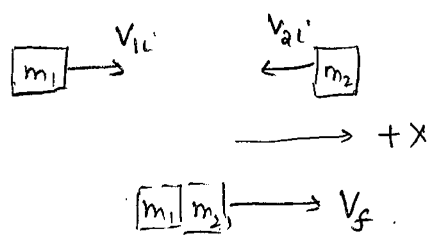
$\boxed{2.45}$ m/s (2.6 kg mass)

$m_1 = \boxed{5.5}$ kg $\quad V_{1i} = 5.5$ m/s

$m_2 = \boxed{2.6}$ kg $\quad V_{2i} = -4.0$ m/s

Sol: (a) Since bodies stick together, this is the case of perfectly inelastic collision. The direction in which they move after collision depends on their individual initial momentum. Whichever body has higher initial momentum will take other body along.

Only momentum is conserved in this case ($F_{net} = 0$)



$$m_1 V_{1i} + m_2 V_{2i} = (m_1 + m_2) V_f$$

$$V_f = \frac{m_1 V_{1i} + m_2 V_{2i}}{m_1 + m_2}$$

$$V_f = \frac{(\boxed{5.5}\text{ kg})(5.5\text{ m/s}) + (\boxed{2.6}\text{ kg})(-4.0\text{ m/s})}{(\boxed{5.5} + \boxed{2.6})\text{ kg}}$$

$$V_f = \boxed{2.451}\text{ m/s}$$

Both masses move with same velocity in $+X$ direction.

4 (b) The collision is elastic.

✗ $\boxed{-0.599}$ m/s (5.5 kg mass)

✗ $\boxed{8.9}$ m/s (2.6 kg mass)

Before collision   After collision

final direction of motion depends on initial momentum of each body.

This is a case of perfectly elastic collision, so both momentum and KE are conserved.

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$$

and

$$\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2.$$

* We solved this in problem 1. Refer eq (A) and (B) on page 2

eq (B) $$V_{2f} = \frac{2 m_1 V_{1i} + (m_2 - m_1) V_{2i}}{(m_1 + m_2)}$$

$$V_{2f} = \frac{2 (\boxed{5.5}\,kg)(5.5\ m/s) + (\boxed{2.6} - \boxed{5.5})\,kg\,(-4.0\ m/s)}{(\boxed{5.5} + \boxed{2.6})\ kg}$$

$$V_{2f} = \boxed{8.9012}\ m/s = \boxed{8.90}\ m/s.$$

eq (A) $$V_{1f} = V_{2f} + V_{2i} - V_{1i}$$

$$V_{1f} = (\boxed{8.9012}\ m/s) + (-4.0\ m/s) - (5.5\ m/s)$$

$$V_{1f} = \boxed{-0.599}\ m/s$$

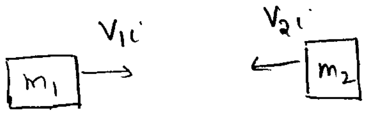
Bounces back.

4. (c) The $\boxed{5.5}$ kg body is at rest after the collision.

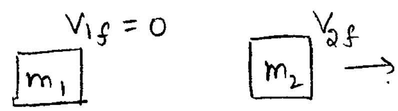
$\boxed{0}$ m/s (5.5 kg mass)

$\boxed{7.63}$ m/s (2.6 kg mass)

Sol: Before collision



After collision



Using conservation of momentum. -

$$m_1 V_{1i} + m_2 V_{2i} = \cancel{m_1 V_{1f}}^{\,0} + m_2 V_{2f}$$

$$V_{2f} = \frac{m_1 V_{1i} + m_2 V_{2i}}{m_2} = \frac{(\boxed{5.5}\,kg)(5.5\,m/s) + (\boxed{2.6}\,kg)(-4.0\,m/s)}{(\boxed{2.6}\,kg)}$$

$$V_{2f} = \boxed{7.635}\ m/s = \boxed{7.63}\ m/s.$$

$$V_{1f} = 0\ m/s.$$

4(d) The $\boxed{2.6}$ kg body is at rest after collision.

Again using conservation of momentum -

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + \cancel{m_2 V_{2f}}^{\,0}$$

$$V_{1f} = \frac{m_1 V_{1i} + m_2 V_{2i}}{m_1} = \frac{(\boxed{5.5}\,kg)(5.5\,m/s) + (\boxed{2.6}\,kg)(-4.0\,m/s)}{(\boxed{5.5}\,kg)}$$

$$V_{1f} = \boxed{3.61}\ m/s$$

This result does not make sense since object 1 has to keep moving in +x direction after collision to obey conservation of momentum. This amounts to saying that it has to go through object 2, since object is at rest after collision !!

4. (e) The 5.5 kg body has a velocity of 4.0 m/s in the -*x* direction after the collision.

-4 m/s (5.5 kg mass)

16.1 m/s(2.6 kg mass)

Sol: Given $V_{1f} = -4.0\,m/s$.

Using conservation of momentum.

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$$

$$\Rightarrow \quad V_{2f} = \frac{m_1 V_{1i} + m_2 V_{2i} - m_1 V_{1f}}{m_2}$$

$$V_{2f} = \frac{(\boxed{5.5}\,kg)(5.5\,m/s) + (\boxed{2.6}\,kg)(-4.0\,m/s) - (\boxed{5.5}\,kg)(-4.0\,m/s)}{(\boxed{2.6}\,kg)}$$

$$V_{2f} = \boxed{16.1}\,m/s.$$

This is a case of elastic collision, therefore kinetic energy should be conserved.

$$\text{Initial KE} = \frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2$$

$$= \frac{1}{2}\left[(\boxed{5.5}\,kg)(5.5\,m/s)^2 + (\boxed{2.6}\,kg)(-4.0\,m/s)^2\right]$$

$$= \boxed{103.99}\ \text{Joules.} = \boxed{104}\ J.$$

$$\text{Final KE} = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

$$= \frac{1}{2}\left[(\boxed{5.5}\,kg)(-4.0\,m/s)^2 + (\boxed{2.6}\,kg)(16.1\,m/s)^2\right]$$

$$= \boxed{380.97} = \boxed{381}\ J$$

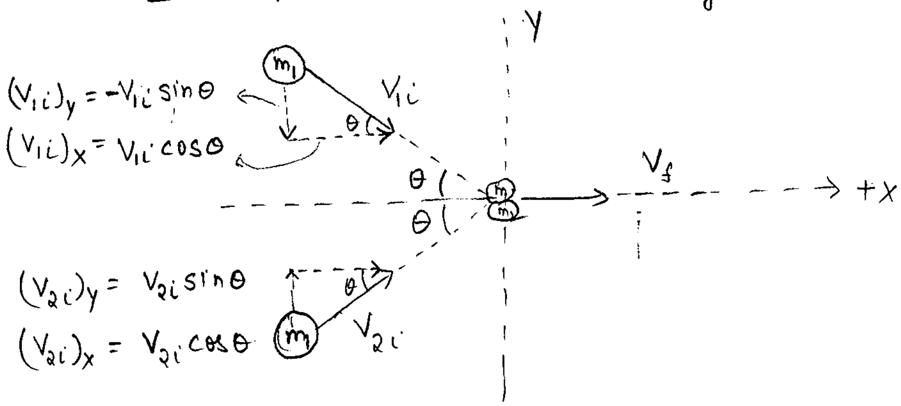
Kinetic energy after collision has increased, which is not possible unless we provide energy to bodies from outside. Therefore, these numbers are unreasonable.

5. Giancoli6 7.P.043. [352940] **0/1 points** Show Details

After a completely inelastic collision between two objects of equal mass, each having initial speed *v*, the two move off together with speed *v*/2. What was the angle between their initial directions?

✖ 120°

Sol: Assume that finally objects move in +x direction.



Also, Let both objects are coming at an angle θ with horizontal. Therefore, total angle between their initial directions is 2θ.

Since, this is perfectly inelastic collision and $F_{net} = 0$ only momentum is conserved.

$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = (m_1 + m_2)\vec{V}_f$$

This is a two-dimensional problem and momentum in both x and y directions are separately conserved.

x dir: $m_1 (V_{1i})_x + m_2 (V_{2i})_x = (m_1 + m_2) V_{fx}$

y dir: $m_1 (V_{1i})_y + m_2 (V_{2i})_y = (m_1 + m_2) V_{fy}$

Given: $V_{1i} = V \Rightarrow (V_{1i})_x = V\cos\theta \quad (V_{1i})_y = -V\sin\theta$.

$V_{2i} = V \Rightarrow (V_{2i})_x = V\cos\theta \quad (V_{2i})_y = V\sin\theta$.

$V_{fx} = \frac{V}{\boxed{2}}$, $V_{fy} = 0$

$m_1 = m_2$

$$\Rightarrow mv\cos\theta + mv\cos\theta = (m+m)\frac{V}{\boxed{2}} = 2m\frac{V}{\boxed{2}}$$

$$\cancel{2}\,\cancel{m}\cancel{v}\cos\theta = \cancel{2}\,\cancel{m}\frac{V}{\boxed{2}} \Rightarrow \cos\theta = \frac{1}{\boxed{2}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\boxed{2}}\right)$$

$$\theta = \boxed{60^\circ}$$

$\Rightarrow$ Angle between two objects.

$$2\theta = 2\,\boxed{60^\circ} = \boxed{120^\circ}$$ ✓✓

6. Giancoli6 7.P.046. [352931] **0/1 points** Show Details

Find the center of mass of the three mass system shown in Figure 7-38 given that m_1 = $\boxed{1.60}$ kg and m_2 = $\boxed{1.30}$ kg. Specify relative to the center of the left hand (1.00 kg) mass.

✖ $\boxed{0.455}$ m

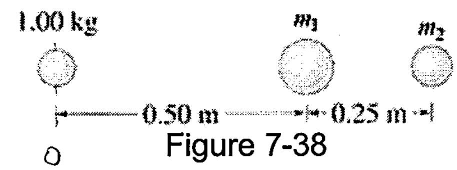


Figure 7-38

Sol: We are setting origin at 1.00 kg mass. Center of mass is defined as.

$$X_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{M} \quad \text{where} \quad M = \sum_{i=1}^{n} m_i$$

This is a three body problem.

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

Given: $m_1 = \boxed{1.60}$ kg $\quad x_1 = 0.50$ m

$m_2 = \boxed{1.30}$ kg $\quad x_2 = (0.50 + 0.25) = 0.75$ m.

$m_3 = 1.00$ kg $\quad x_3 = 0$

$$X_{cm} = \frac{(\boxed{1.60}\,kg)(0.50m) + (\boxed{1.30}\,kg)(0.75m) + (1.00\,kg)(0)}{(\boxed{1.60} + \boxed{1.30} + 1.00)\,kg}$$

$$X_{cm} = \boxed{0.455}\,m$$

7. Giancoli6 7.P.058. [525552] **0/3 points** Show Details

A $\boxed{50}$ kg woman and an $\boxed{85}$ kg man stand $\boxed{10.0}$ m apart on frictionless ice.

(a) How far from the woman is their CM?

✖ $\boxed{6.3}$ m

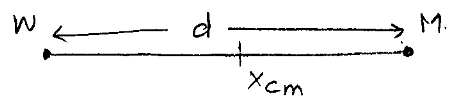
(b) If each holds one end of a rope, and the man pulls on the rope so that he moves $\boxed{1.5}$ m, how far from the woman will he be now?

✖ $\boxed{5.95}$ m

(c) How far will the man have moved when he collides with the woman?

✖ $\boxed{3.7}$ m

Sol: Set origin at position of women as we are interested in distance of CM (center of mass) from woman.



(a) Given

mass of Woman $m_1 = \boxed{50}$ kg.

position of woman $x_1 = 0$ m.

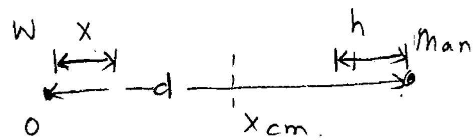
mass of man $m_2 = \boxed{85}$ kg, position of man $x_2 = \boxed{10.0}$ m

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad \text{(two body problem)}$$

$$X_{cm} = \frac{(\boxed{50}\,kg)(0) + (\boxed{85}\,kg)(\boxed{10.0}\,m)}{(\boxed{50} + \boxed{85})\,kg} = \boxed{6.296}\,m = \boxed{6.3}\,m.$$

(b) Man moved a distance.

$h = \boxed{1.5}$ m $\Rightarrow x_2 = (\boxed{10.0} - \boxed{1.5})$ m.



woman moved say 'x' distance.

$$X_{cm} = \boxed{6.296}\,m = \frac{(\boxed{50}\,kg)(x) + (\boxed{85}\,kg)(\boxed{10.0} - \boxed{1.5})\,m}{(\boxed{50} + \boxed{85})\,kg}$$

$$x = \frac{(\boxed{6.296}\,m)(\boxed{135}\,kg) - (\boxed{85}\,kg)(\boxed{8.5}\,m)}{(\boxed{50}\,kg)} = \boxed{2.55}\,m$$

(b) contd...

(b) contd..

Distance between man and woman now is

$$= \left(\boxed{10.0} - \boxed{1.5} - \boxed{2.55}\right) m = \boxed{5.95}\ m$$

(c) They will collide at center of mass. position.

$$x_{cm} = \boxed{6.296}\ m.$$

Man would have moved. $= \left(\boxed{10.0} - \boxed{6.296}\right) m$

$$= \boxed{3.704}\ m$$

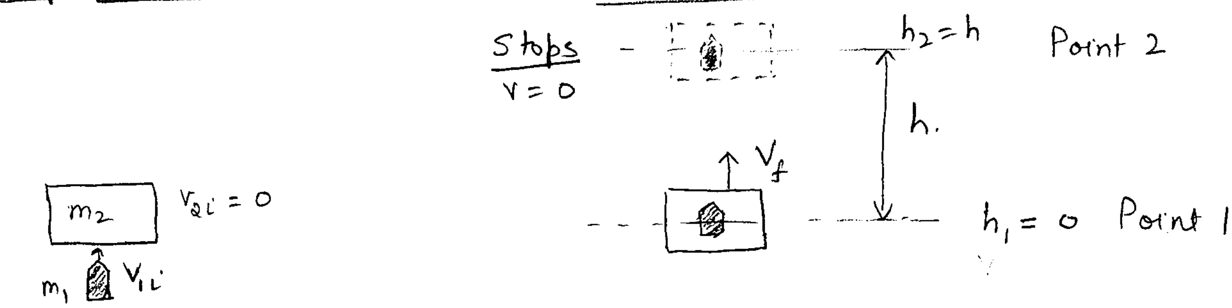
$$= \boxed{3.7}\ m.$$

8. Giancoli6 7.P.070. [352961] **0/1 points** Show Details

A bullet is fired vertically into a $\boxed{1.20}$ kg block of wood at rest directly above it. If the bullet has a mass of 29.0 g and a speed of $\boxed{550}$ m/s, how high will the block rise after the bullet becomes embedded in it?

$\boxed{8.59}$ m

Sol: Before Collision — After Collision



There is gravitational force acting throughout but it's effect is small, so we can assume that immediately before and after collision bodies are moving at constant velocity for a short time and therefore do not experience acceleration ($F_{net} = 0$). Therefore we can treat first part of problem as inelastic collision problem in which momentum is conserved.

$$m_1 V_{1i} + m_2 \cancel{V_{2i}}^{\,0} = (m_1 + m_2) V_f$$

$$V_f = \frac{m_1 V_{1i}}{m_1 + m_2} = \frac{(0.029\ kg)(\boxed{550}\ m/s)}{(\boxed{1.20} + 0.029)\ kg} = \boxed{12.978}\ m/s.$$

Second part when (bullet + block) rises, total energy is conserved.

$$\Delta K^{12} + \Delta U^{12} = 0.$$

$$\frac{1}{2} m \cancel{V_2}^{\,0\,2} - \frac{1}{2} m V_f^2 + mg h_2 - \cancel{mg h_1}^{\,0} = 0.$$

$$h_2 = h = \frac{V_f^2}{2g} = \frac{(\boxed{12.978}\ m/s)^2}{2\,(9.8\ m/s^2)} = \boxed{8.59}\ m.$$

9. Giancoli6 7.P.076. [352925] **0/5 points** Show Details

Two balls, of masses $m_A = \boxed{20}$ g and $m_B = \boxed{68}$ g are suspended as shown in Figure 7-44. The lighter ball is pulled away to a 60° angle with the vertical and released.

Sol: This problem can be divided in three parts - (1) Before collision when total energy is conserved as there is no non-conservative forces acting.

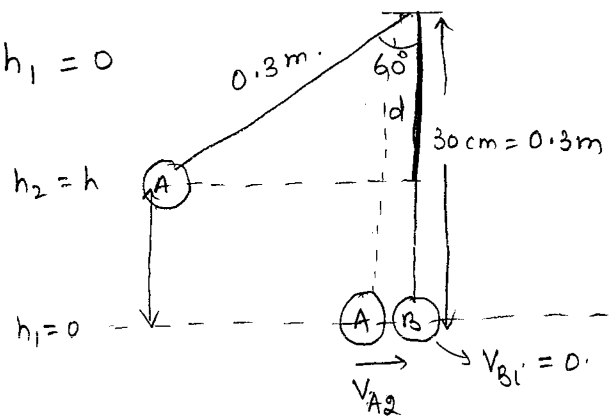
(2) Collision which is perfectly elastic.

(3) After collision - again energy (total) is conserved)

(a) Consider energy change from point 1 to point 2

$$\Delta K^{12} + \Delta U^{12} = \cancel{W_{nc}} \to 0$$

$$\frac{1}{2} m_A V_{A2}^2 - \cancel{\frac{1}{2} m_A V_{A1}^2}^{\,0} + \cancel{m_A g h_2}^{\,0} - m_A g h_1 = 0$$



$$V_{A2} = \sqrt{2gh}$$

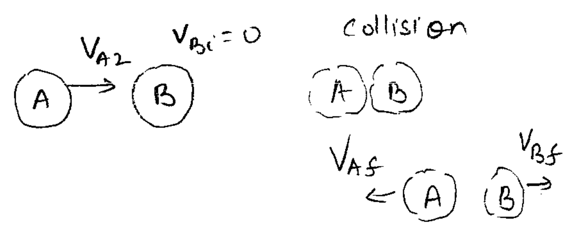
$$h = (0.3\,m) - (0.3\,m)\cos 60^\circ$$

$$h = 0.15 \ m.$$

$$\Rightarrow \quad V_{A2} = \sqrt{2\,(9.8\,m/s)\,(0.15\,m)}$$

$$V_{A2} = 1.715 \ m/s \ \checkmark$$

(b) This is perfectly elastic collision, so both momentum and kinetic energy are conserved -



$$m_A V_{A2} + m_B V_{Bi} = m_A V_{Af} + m_B V_{Bf}$$

and

$$\frac{1}{2} m_A V_{A2}^2 + \frac{1}{2} m_B V_{Bi}^2 = \frac{1}{2} m_A V_{Af}^2 + \frac{1}{2} m_B V_{Bf}^2$$

(b) contd...

(b) contd....

We solved this case in problem 1. Refer eq. (A) and (B) on page 2

eq (B)

$$V_{Bf} = \frac{2 m_A V_{A2} + (m_B - m_A)\cancel{V_{Bi}}^{\,0}}{(m_A + m_B)} = \frac{2(\boxed{0.02}\,kg)(1.715\ m/s)}{(\boxed{0.02} + \boxed{0.068})\,kg}$$

$$V_{Bf} = \boxed{0.779}\ m/s$$

eq (A)

$$V_{Af} = V_{Bf} + \cancel{V_{Bi}}^{\,0} - V_{Ai} = (\boxed{0.779}\ m/s) - (1.715\ m/s)$$

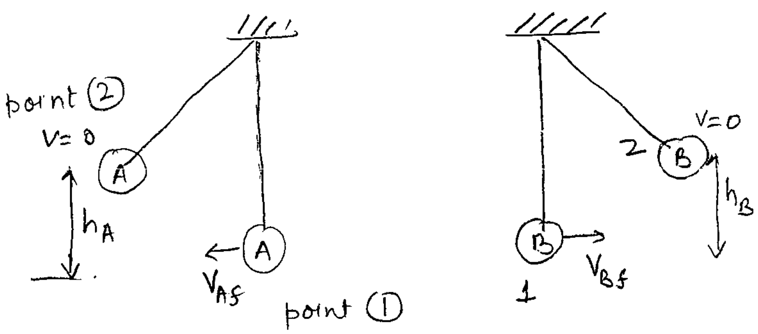
$$V_{Af} = -\boxed{0.935}\ m/s$$

Thus object A bounces backward.

(c) After collision both object A and B starts with V_{Af} and V_{Bf} velocities, rises to a maximum height say h_A and h_B respectively before stopping. Since there are no non-conservative forces acting total energy is conserved. $\Delta K + \Delta U = 0$

Object (A)

$$\frac{1}{2}\cancel{m_A V^2}^{\,0} - \frac{1}{2} m_A V_{Af}^2 + m_A g h_A - \cancel{m_A g h}^{\,0} = 0$$



$$\Rightarrow \quad h_A = \frac{V_{Af}^2}{2g} = \frac{(-\boxed{0.935}\ m/s)^2}{2(9.8)\ m/s^2}$$

$$h_A = \boxed{0.0446}\ m$$

Object (B)

$$h_B = \frac{V_{Bf}^2}{2g} = \frac{(\boxed{0.779}\ m/s)^2}{2 \times 9.8\ m/s^2} = \boxed{0.0309}\ m = \boxed{0.031}\ m$$

10. Giancoli6 7.P.079. [352921] **0/3 points** Show Details

A block of mass m = $\boxed{2.50}$ kg slides down a 30.0° incline which is 3.60 m high. At the bottom, it strikes a block of mass M = $\boxed{6.80}$ kg which is at rest on a horizontal surface, Fig. 7-46. (Assume a smooth transition at the bottom of the incline, an elastic collision, and ignore friction.)

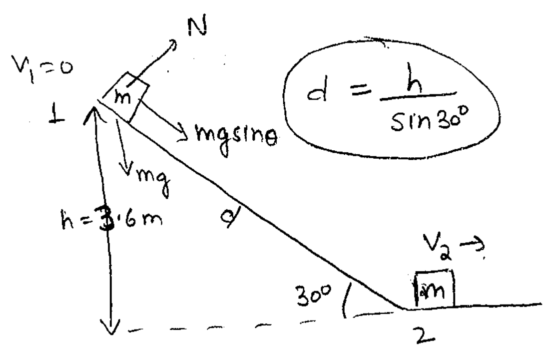
Sol (a) This problem in approach
is similar to problem 9.
we can divide it is
same three parts

① Before collision -
Since no non-conservative
force is acting total mechanical
energy is conserved.

② During collision — This is elastic collision. Both momentum
and kinetic energy are conserved.

③ After collision - Again total energy is conserved.

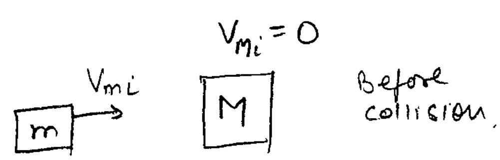
(a) We are interested in final velocities after collision.
For this we need to determine the initial velocities
of the objects.



Block of mass 'M' is at rest.
Block of mass 'm' starts at height
h = 3.6m down the incline.

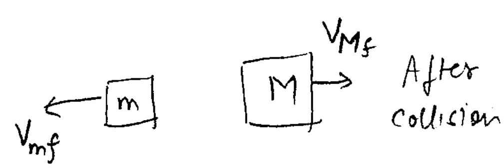
$$\cancel{m} g h = \frac{1}{2} \cancel{m} v_2^2$$

$$v_2 = \sqrt{2gh} = \sqrt{2 \times (9.8\ m/s^2)(3.6\ m)} = 8.4\ m/s$$



This is perfectly elastic collision

$$V_{mi} = V_2 = 8.4\ m/s.$$



contd..

10 (a) contd...

momentum conservation:

$$m\,V_{mi} + \cancel{M V_{Mi}}^{\,0} = m\,V_{mf} + M\,V_{Mf} \qquad -(1)$$

Energy conservation.

$$\frac{1}{2} m\, V_{mi}^2 + \frac{1}{2}\cancel{M V_{Mi}^2}^{\,0} = \frac{1}{2} m\, V_{mf}^2 + \frac{1}{2} M\, V_{Mf}^2 \qquad -(2)$$

from (1)

$$m\,(V_{mi} - V_{mf}) = M\,V_{Mf} \qquad -(3)$$

from (2)

$$m\,(V_{mi}^2 - V_{mf}^2) = M\,V_{Mf}^2 \qquad -(4)$$

divide (4) by (3) and use $a^2 - b^2 = (a+b)(a-b)$

$$\frac{\cancel{M} V_{Mf}^{\cancel{2}}}{\cancel{M}\,\cancel{V_{Mf}}} = \frac{\cancel{m}\,\cancel{(V_{mi} - V_{mf})}(V_{mi} + V_{mf})}{\cancel{m}\,\cancel{(V_{mi} - V_{mf})}}$$

$$V_{Mf} = V_{mi} + V_{mf} \qquad -(5)$$

Plug this in eq. (1).

$$m\,V_{mi} = m\,V_{mf} + M\,(V_{mi} + V_{mf})$$

Solve for final Velocity of little block V_{mf}

$$V_{mf} = \frac{m V_{mi} - M V_{mi}}{m+M} = \frac{(m-M)\,V_{mi}}{m+M}$$

$$V_{mf} = \frac{(\boxed{2.50}\,\text{kg} - \boxed{6.80}\,\text{kg})(8.4\,\text{m/s})}{(\boxed{2.50} + \boxed{6.80})\,\text{kg}} = -\boxed{3.88}\,\text{m/s} \qquad -(6)$$

speed of lighter block $= |V_{mf}| = \boxed{3.88}$ m/s ✓

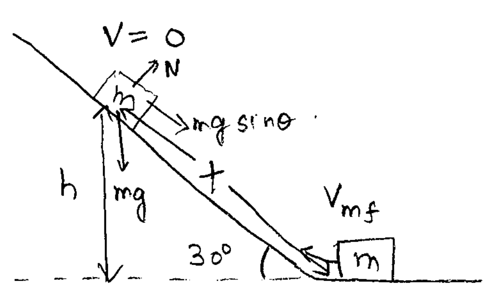
Plug V_{mf} from (6) in eq. (5) to calculate velocity of heavier block.

$$V_{Mf} = V_{mi} + V_{mf} = (8.4\ m/s) + (-\boxed{3.88}\ m/s) = \boxed{4.52}\ m/s.$$

speed of heavier block.

$$|V_{Mf}| = \boxed{4.52}\ m/s$$

(b) Since, lighter block is moving with some velocity after collision V_{mf}, it will rise up the incline till force due to gravity stops it completely. We are interested in finding distance 'x' up the incline it will rise,



Total mechanical Energy is conserved. i.e $\Delta K + \Delta U = 0$.

$$\Rightarrow \frac{1}{2} \cancel{m} V_{mf}^2 = \cancel{m} g h \Rightarrow h = \frac{V_{mf}^2}{2g}$$

then $\frac{h}{x} = \sin 30^\circ \Rightarrow x = \frac{V_{mf}^2}{2g \sin\theta}$

$$x = \frac{(\boxed{-3.88}\ m/s)^2}{2(9.8\ m/s)\ \sin 30^\circ} = \boxed{1.536}\ m = \boxed{1.54}\ m$$