Homework #3 physics 2414

1. $A = \left(\boxed{70.0}\cos 28°, \boxed{70.0}\sin 28°\right)$

$B = \left(-40.0\cos\boxed{54.0°}, 40.0\sin\boxed{54.0°}\right)$

$C = (0, -46.8)$

a. $A + B + C = \underline{(38.29, 18.42)} = (R_x, R_y)$

b. $|R| = \sqrt{R_x^2 + R_y^2} = \underline{42.496}$

c. $\Theta = \arctan\left(\frac{R_y}{R_x}\right) = \underline{25.69°}$

2. $A = \left(\boxed{63.0}\cos 28°, \boxed{63.0}\sin 28°\right)$

$B = \left(-40.0\cos\boxed{50.5°}, 40.0\sin\boxed{50.5°}\right)$

$C = (0, -46.8)$

a. $A - B + C = (81.07, -48.09)$ Mag. $= \sqrt{81.07^2 + 48.09^2} = \underline{94.26}$

$\text{Arctan}(-48.09/81.07) = -\underline{30.68°, 329.32°}$

b. $A + B - C = (30.18, 107.24)$ Mag $= \sqrt{30.18^2 + 107.24^2} = \underline{111.41}$

$\text{Arctan}(107.24/30.18) = \underline{74.281°}$

c. $C - A - B = (-30.18, -107.242)$ Mag = " " $= \underline{111.41}$

$\text{Arctan}(-107.24/-30.18) = 180° + 74.28° = \underline{254.28°}$

(In third quad)

3. Diver starts from rest in the vertical direction:

a. $\Delta y = -\frac{1}{2}\cdot 9.80\, t^2 = -\frac{1}{2}\cdot 9.80\cdot \boxed{3.2}^2 = -50.176\,m$

Cliff height: $\underline{50.2\,m}$

b. Diver's horizontal velocity is $\boxed{1.0\,m/s}$ right:

$\Delta x = v_0 t = 1.0\,\frac{m}{s}\cdot 3.2\,s = \underline{3.2\,m}$

4. $v_x = V_0 \cos\theta$ $\quad V_y = V_0 \sin\theta$ $\quad t$ = total airborn time

a. $\Delta x = V_0 \cos\theta\, t$ $\quad \Delta y = V_0 \sin\theta\, t - \frac{1}{2} 9.8\, t^2$

$\boxed{7.8\,m} = V_0 \cos\boxed{35°}\, t$ $\quad 0 = V_0 \sin\boxed{35°}\, t - \frac{1}{2} 9.8\, t^2$

(the change in y is zero after the jump)

$t = \frac{7.8}{V_0 \cos 35°}$

Substitute t into equation for y:

$$0 = \tan 35° \cdot 7.8 - \frac{1}{2}\cdot 9.8 \cdot \frac{7.8^2}{V_0^2 (\cos 35°)^2}$$

$$V_0^2 = \frac{4.9\cdot 7.8}{\sin 35° \cos 35°} \qquad V_0 = \sqrt{\frac{4.9\cdot 7.8}{\sin 35° \cos 35°}} = \underline{9.02\,m/s}$$

b. $V_0^2 \propto \Delta x$ (other terms don't vary)

a $\boxed{3\%}$ increase in V_0 corresponds to multiplying V_0 by $\boxed{1.03}$.

$$\frac{\Delta x_0}{\Delta x_{new}} = \frac{V_0^2}{(1.03)^2 V_0^2} = \frac{7.8\,m}{\Delta x_{new}} \qquad \Delta x_{new} = 1.03^2 \Delta x_0 = 8.275\,m$$

so the distance is increased by $8.275 - 7.8 = \underline{0.475\,m}$

5. The bullet travels horizontally $\boxed{140\,m}$.

a. $\Delta x = v_0 t = \boxed{210\,m/s}\; t = \boxed{140\,m}$

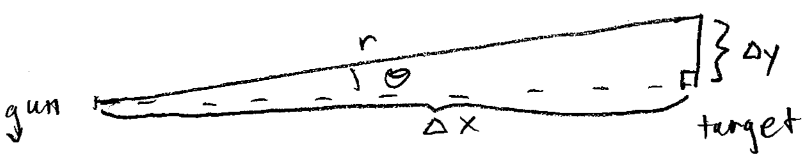
Time to target $= \dfrac{140\,m}{210\,m/s} = 0.667\,s$

During this time the bullet will fall:

$\Delta y = -\frac{1}{2}\, 9.8\, t^2 = -4.9\; 0.667^2 = -2.18\,m$

Thus it will miss the target by $\underline{2.18\,m}$

b. The change in y is small so $r \approx \Delta x$.



(not to scale)

Thus $\theta = \operatorname{Arctan}\left(\dfrac{\Delta y}{\Delta x}\right) = \operatorname{atan}\left(\dfrac{2.18}{140}\right) = \underline{0.8921^\circ}$

6. a. $y = y_0 + V_{0y}t + \frac{1}{2}at^2$ after hitting ground, projectile is at $\boxed{-305m}$ (up is positive)

$V_{0y} = \boxed{145} \sin 37°$

$-305 = 145 \sin 37° t - \frac{1}{2} 9.8 t^2$

$\downarrow$

$4.9t^2 - 145 \sin 37° t - 305$

$$t = \frac{145 \sin 37° \pm \sqrt{(145 \sin 37°)^2 + 4 \cdot 4.9 \cdot 305}}{9.8}$$

$= -2.99,\ \underline{20.8\ s}$

b. $V_x = \boxed{145} \cos 37°$

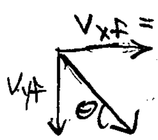
$x = x_0 + V_0 t + \frac{1}{2}at^2$

$x = 145 \cos 37° \cdot 20.8 = 2408.68m = \underline{2.41\ km}$

c. $V_{xf} = 145 \cos 37° = \underline{115.8\ m/s}$

$V_{yf} = V_{y0} + gt = 145 \sin 37° - 9.8 \times 20.8 = \underline{-116.6\ m/s}$

d. $|V| = \sqrt{V_{xf}^2 + V_{yf}^2} = (115.8^2 + 116.6^2)^{1/2} = \underline{164.3\ m/s}$

e, 

$\arctan\left(\frac{V_{yf}}{V_{xf}}\right) = \underline{45.2°}$

f. $V_{yi}^2 = 2g\Delta y_{max}$ $\Delta y_{max} = \frac{(145 \sin 37°)^2}{2 \cdot 9.8} = \underline{388.5m}$

7. a. The goods will have fallen 235m in time t:

$$235\text{m} = \frac{1}{2} \cdot 9.8\frac{\text{m}}{\text{s}^2} t^2 \qquad t = \left(\frac{2 \cdot 235}{9.8}\right)^{1/2} = 6.9253\text{s}$$

This means they'll have to be dropped Δx away:

$$\Delta x = V_0 t = \boxed{81.9\frac{\text{m}}{\text{s}}}\, 6.9253\text{s} = \underline{567.2\text{m}}$$

b. The supplies travel horizontally 425m in time t:

$$\Delta x = V_0 t = 425\text{m} = 81.9\frac{\text{m}}{\text{s}} t \qquad t = 5.1893\text{s}$$

They must fall Δy in this time:

$$\Delta y = -235\text{m} = V_y t + \frac{1}{2} g t^2 = V_y(5.2) - \frac{1}{2} 9.8 (5.2)^2$$

$$\downarrow$$

$$V_y = \frac{\frac{1}{2} 9.8 (5.2)^2 - 235}{5.2} = \underline{-19.8585\text{m/s}}$$

(down)

c. The x-component of velocity does not change:

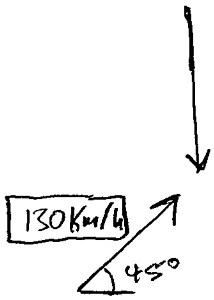
$$V_x = \boxed{81.9\text{ m/s}}$$

The y-component will experience gravity for ~5.2s

$$V_y = -19.8585 - 9.8(5.1893) = -70.7136\text{ m/s}$$

$$V = \sqrt{V_x^2 + V_y^2} = \underline{108.2\text{ m/s}}$$

8. a.



$V_1 = (0, -600)$

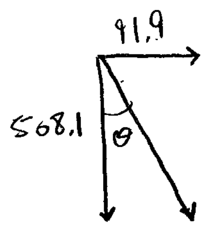
$V_2 = (130 \cos 45°, 130 \sin 45°)$

$V_1 + V_2 = (91.924 \text{ Km/hr}, -508.076 \text{ Km/hr})$

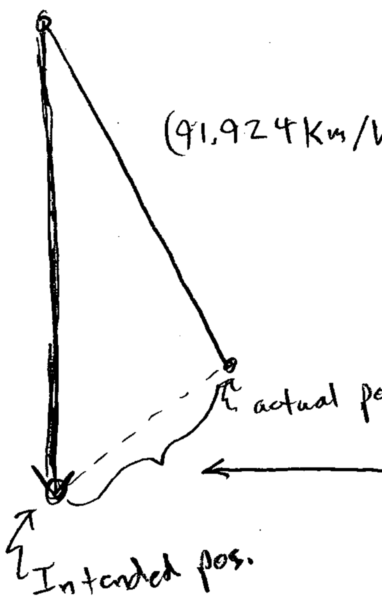
a. Magnitude $= \sqrt{91.92^2 + 508.1^2} = \underline{516.3 \text{ Km/h}}$

b. $\theta°$ east of south $= \text{Arctan}(91.924/508.076)$

$= \underline{10.2553°}$



c.



$(91.924 \text{ Km/hr}, -508.076 \text{ Km/hr}) \times 10 \text{min} \times \frac{1 \text{hr}}{60 \text{min}} = (15.3 \text{Km}, -84.7 \text{Km})$

$= \text{pos.}1$

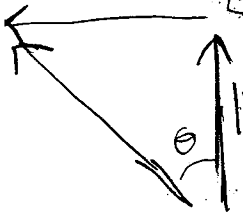
$|\text{pos.}1 - \text{pos.}2| = \sqrt{15.3^2 + (-84.7 + 100)^2}$

$= \underline{21.66 \text{ Km}}$

$(0, -600 \text{ Km/hr}) \times 10 \text{min} \times \frac{1 \text{hr}}{60 \text{min}} = (0, -100 \text{ Km}) = \text{pos.}2$

9. a. Car 2 sees:

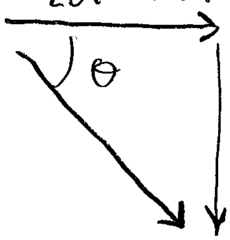
$|v_{2E}| = \boxed{51\,\text{Km/h}}$ magnitude $= \sqrt{51^2 + 27^2} = \underline{57.7\,\text{Km/h}}$



$|V_{1E}| = \boxed{27\,\text{Km/h}}$ $\quad \text{Arctan}\left(\frac{51}{27}\right) = \underline{62.1°}$

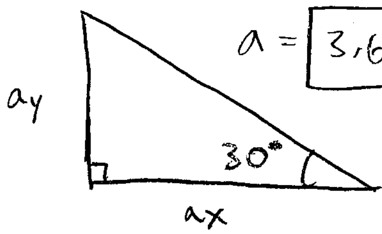
b. Car 1 sees:

$|v_{2E}| = 51\,\text{Km/h}$ $\quad$ magnitude $= \sqrt{51^2 + 27^2} = \underline{57.7\,\text{Km/h}}$



$|V_{1E}| = 27\,\text{Km/h}$ $\quad \text{Arctan}\left(\frac{27}{51}\right) = \underline{27.9°}$

10. a.



$a = \boxed{3.60\,\text{m/s}^2}$

$a_y = a \sin 30°$

$= 3.60 \sin 30° = \underline{1.80\,\text{m/s}^2}$

b. Elevation change is $\boxed{295\,\text{m}}$ with uniform y acceleration.

$\Delta y = v_0 t + \frac{1}{2} a t^2$ $\qquad -295\,\text{m} = -\frac{1}{2} \cdot 1.8\, t^2$

$t = \sqrt{\frac{2 \cdot 295}{1.8}} = \underline{18.1\,\text{s}}$

11. a. $V_{xi} = V_i \cos\theta = \boxed{19}\cos\boxed{40°} = \underline{14.55\ m/s}$

$V_{yi} = V_i \sin\theta = \boxed{19}\sin\boxed{40°} = \underline{12.213\ m/s}$

b. $55\ m = V_{xi}t = 14.55\ m/s\ t \qquad t = \frac{55}{14.55} = 3.779\ s$

During time t the equation for the y-axis is:

$y = y_0 + V_{yi}t + \frac{1}{2}gt^2 = 0 + 12.213t - \frac{1}{2}9.8t^2$

Plug in t:

$y = 12.213 \cdot 3.779 - \frac{1}{2}9.8 \cdot 3.779^2$

$= -23.818\ m$

so $\underline{23.8\ m}$ below

12. $\Delta x = V_0\cos\theta\, t \qquad \Delta y = V_0\sin\theta\, t - \frac{1}{2}|g|t^2$

What the question means to ask is what is the maximum horizontal range from the barrel of the gun. This means $\Delta y = 0$ after some time. (You'd have to know the height of the barrel from the floor to find true Δx!)

$0 = V_0\sin\theta\, t - \frac{1}{2}|g|t^2 \rightarrow t = \frac{2V_0\sin\theta}{g}$ $\xrightarrow{\text{substitute for } \Delta x}$ $\Delta x = \frac{2V_0^2}{g}\sin\theta\cos\theta$

$\rightarrow \Delta x = \frac{V_0^2}{g}\sin 2\theta$ The max θ will make $\frac{d\Delta x}{d\theta} = 0$.

$\frac{d\Delta x}{d\theta} = \frac{2V_0^2}{g}\cos 2\theta = 0$ For $\theta = 45°$ this works. The range will be defined as $\Delta x = \frac{V_0^2}{g}$ since $\sin 2 \cdot 45° = 1$

To find V_0 we use $\Delta y = V_0 \cdot \boxed{3.9} - \frac{1}{2}9.8\boxed{3.9}^2 = 0 \qquad V_0 = 19.11\ m/s$

so, $\Delta x = \frac{V_0^2}{g} = \frac{19.11^2}{9.8} = \underline{37.3\ m}$