Virial Theorem:

From Q.M 1

$$\frac{d}{dt}\langle \hat{A} \rangle = \frac{i}{\hbar} \langle [\hat{H}, \hat{A}] \rangle + \langle \frac{\partial \hat{A}}{\partial t} \rangle$$

let $\hat{A} = xp$

$$\frac{d}{dt}\langle xp \rangle = \frac{i}{\hbar} \langle [\hat{H}, xp] \rangle + 0$$

$$= \frac{i}{\hbar} \langle [H,x]p + x[H,p] \rangle$$

$[V(x), x] = 0$

$$[H,x] = [\tfrac{p^2}{2m} + V, x] = \tfrac{1}{2m}[p^2, x] + [V,x]$$

$$= \tfrac{1}{2m}\{p[p,x] + [p,x]p\} = \tfrac{1}{2m}\{p\, i\hbar + i\hbar p\} = i\hbar\, p/m$$

$$[H,x] = i\hbar\, p/m$$

$$[H,p] = [\tfrac{p^2}{2m} + V, p] = \overbrace{[\tfrac{p^2}{2m}, p]}^{0} + [V,p] = [V,p]$$

$$[V,p] =$$

write $V(x)$ as a series in x $\quad V(x) = \sum_{n=0}^{\infty} a_n x^n$

$$\sum_{n=0}^{\infty} a_n [x^n, p]$$

what is $[x^n, p]$

$[x,p] = i\hbar$

$[x^2,p] = x[x,p] + [x,p]x = 2i\hbar x$

$[x^3,p] = x[x^2,p] + [x,p]x^2 = 3i\hbar x^2$

$[x^n,p] = i\hbar n x^{n-1}$

$$\sum_{n=0}^{\infty} a_n \cdot i\hbar n x^{n-1} = i\hbar \sum_{n=0}^{\infty} a_n n x^{n-1} = \underline{i\hbar \frac{dV}{dx}}$$

so we get

$$\frac{d}{dt}\langle xp\rangle = \frac{i}{\hbar}\left[\langle \frac{i\hbar}{m} p\cdot p\rangle + i\hbar\langle x\frac{dV}{dx}\rangle\right]$$

$$= 2\langle \frac{p^2}{2m}\rangle - \langle x\frac{dV}{dx}\rangle$$

$$= 2\langle T\rangle - \langle x\frac{dV}{dx}\rangle$$

in a stationary state, the expectation value is time-independent) (for operators which do not explicitly depend on time)

$$\Rightarrow \frac{d}{dt}\langle xp\rangle = 0$$

so $2\langle T\rangle = \langle x\frac{dV}{dx}\rangle$ <u>Virial theorem</u>