Physics 3803
Homework Assignment 2
Due Jan 28 at 5:00

1) In classical mechanics, it is possible, in principle, to drop a point object so precisely that it lands exactly on a point target directly below it. In quantum mechanics, because of the uncertainty principle, one cannot do this.
(a) Assume that one drops an object from a height $L$. Assume also that there is both an initial horizontal $x_0$ and $v_0$. Determine the distance $\delta x$, by which one misses the target in terms of $x_0$ and $v_0$.
(b) Use $\Delta x \Delta p \approx \hbar$ to eliminate one variable and find the minimum horizontal "error" $\delta x$ in terms of $L$, $g$, $m$, and $\hbar$. What is the typical error for a macroscopic object dropped from 1 m?

2) Consider the wave function $\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t}$, where $A$, $\lambda$, and $\omega$ are positive real constants.
(a) Normalize $\Psi$.
(b) Determine the expectation values of $x$ and $x^2$.
(c) Find the standard deviation of $x$. Sketch the graph of $|\Psi|^2$ as a function of $x$, and mark the points ($< x > + \sigma$) and ($< x > - \sigma$) to illustrate the sense in which $\sigma$ represents the "spread" in $x$. What is the probability that the particle would be found outside this range?

3) Show that $d < p > /dt = < -\partial V/\partial x >$. This is known as Ehrenfest's theorem; it tells us that the expectation values obey Newton's second law.

4) What is the expectation of momentum $< p >$ for a particle in the state $\Psi(x,t) = Ae^{-\lambda(x/a)^2}e^{-i\omega t} \sin kx$?

5) A particle of mass $m$ is in the state $\Psi(x,t) = Ae^{-a[(mx^2|/\hbar)+id]}$
(a) For what potential energy function $V(x)$ does $\Psi$ satisfy the Schrodinger equation?
(b) Find $\sigma_x$ and $\sigma_p$. Is their product consistent with the uncertainty principle?

6) The correspondence principle states: "Quantum theory must agree with classical theory in the limit of large quantum numbers". Using the Bohr model of the atom, determine the frequency of radiation emitted both classically and quantum mechanically and show that they are identical for large quantum numbers.