Chapter 6

Energy and Oscillations

Conservation of Energy

In this chapter we will discuss one of the most important and fundamental principles in the universe. Energy is conserved. This means that in any process, the total amount of energy after the process is the same as the total amount of energy before the process. Energy can be transformed from one form to another but it can not be created or destroyed.

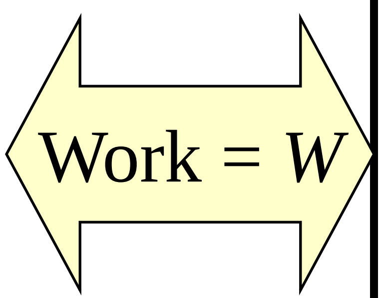
If energy is always conserved, what do we mean when we say that you should turn the lights off or drive a fuel efficient vehicle to “conserve” energy?

Energy and Systems

Although the total energy in the universe cannot change, we usually don't do problems involving the entire universe. So we define a part of the universe we are interested in as the "system" and the rest of the universe as the "environment."

Energy transfer

Environment



System

- Motional energy
  = kinetic energy = KE
- Stored energy
  = potential energy = PE
- Other kinds of energy we won't discuss

Energy Transfer and Energy Conservation

If there is no energy transfer (work done) on the system then,

$$KE_i + PE_i = KE_f + PE_f \qquad \text{(i=“initial”, f=final)}$$

which can be written as

$$0 = KE_f + PE_f - KE_i - PE_i$$

If energy is transferred into or out of the system using work, then

$$W = KE_f + PE_f - KE_i - PE_i$$

But now we’re getting way ahead of ourselves. This is the most important principle in the chapter, and where we want to end up, but first we need to define what all these terms mean. So let’s look at work, kinetic energy, and potential energy, and then come back to this principle.

Work

In physics, work has a very specific definition. It involves a force that acts on an object while the object is moving.

$$W = F_{\|}d$$

W is the work done by the force F acting on an object, when the object moves a distance d.

The only part of the force that can do work is the part of the force parallel or antiparallel ($\|$) to the motion.

When the force is parallel to the motion, the work done is positive (adding energy to the system). When the force is antiparallel to the motion, the work done is negative (removing energy from the system).

The SI unit of work is N·m which is a Joule: 1 N·m = 1 J

Problem: You push a 28 kg crate 2.3 m across the floor with a horizontal force of 52 N. How much work did you do on the crate?

Problem: You pull a crate for a distance of 6.2 m. The force you exert has a horizontal component of 58 N and a vertical component of 28 N?

(a) How much work does your horizontal force do on the crate?
(b) How much work does your vertical force do on the crate?
(c) How much total work do you do on the crate?

How much work is done by …

…the team on the right if the rope doesn’t move?

…the groom as he carries his bride over the threshold?

…the person as he pushes the rock if it doesn’t move.

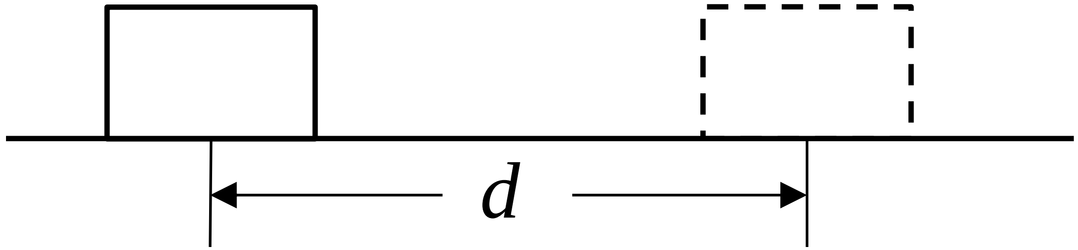
Interactive Question

You push a crate across the floor by applying a constant horizontal force. Four forces can be identified on the crate: 1) the horizontal push 2) the gravitational force, 3) the normal force, and 4) the frictional force. Which forces do NO work on the crate?

A) 2 only
B) 1 and 4 only
C) 2 and 3 only
D) 2, 3, and 4
E) 1, 2, and 3

Interactive Question

If there is a frictional force opposing the motion of a block as it slide across the ground to the right does this frictional force do work on the block?



A) No, the frictional force does no work.
B) Sort of, only part of the frictional force does work.
C) Yes, the frictional force does positive work.
D) Yes, the frictional force does negative work.

Interactive Question

You raise a 10 N physics book up in the air a distance of 1 meter at a constant velocity of 0.5 m/s. The work done by gravity is

A) +10 J
B) –10 J
C) +5 J
D) –5 J
E) zero

Interactive Question

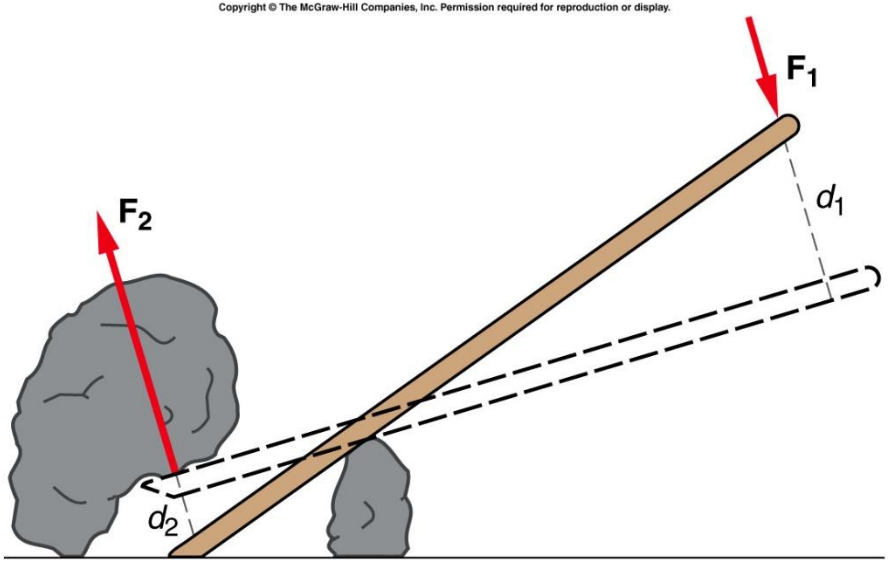
You raise a 10 N physics book up in the air a distance of 1 meter at a constant velocity of 0.5 m/s. The work done by you is

A) +10 J
B) –10 J
C) +5 J
D) –5 J
E) zero

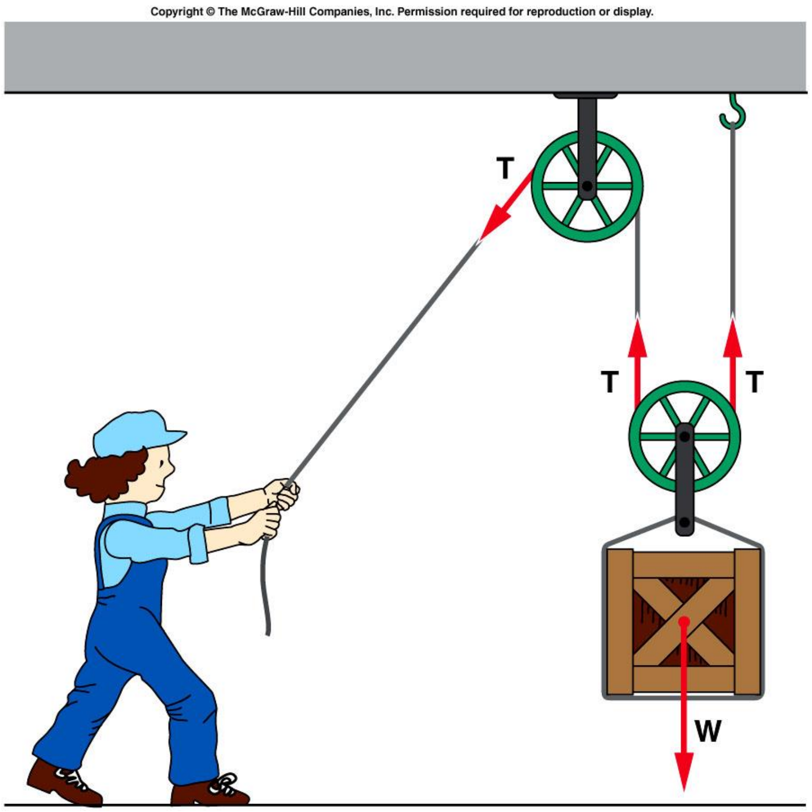
Simple Machines

A simple machine multiplies the effect of an applied force but requires the same work. $W = F_{\|}d$

1. A lever
    - A small force applied to one end delivers a large force to the rock.
    - The small force acting through a large distance moves the rock a small distance.



2. A pulley
   - A small tension applied to one end delivers twice as much tension to lift the box.
   - The person exerts a small tension acting through a large distance which moves the box a small distance.



The mechanical advantage of a simple machine is the ratio of the output force to the input force.

- For this pulley the mechanical advantage is 2
- The work done by the person, though is identical to the work done on the crate.

Kinetic Energy and the Work-Energy Theorem

Something that is moving has energy due to its motion. We call this kinetic energy (KE). We can derive an expression for kinetic energy.

$$\mathrm{KE} = (1/2)mv^2$$

where *m* is the mass of an object moving with speed *v*.

When work is done on an object, the object's kinetic energy will change. We can derive an expression between the total work done on an object and the change in KE.

$$W_{\mathrm{net}} = \mathrm{KE}_{\mathrm{f}} - \mathrm{KE}_{\mathrm{i}}$$
$$W_{\mathrm{net}} = (1/2)mv_{\mathrm{f}}^2 - (1/2)mv_{\mathrm{i}}^2$$

And for some problems: $F_{||\mathrm{net}}d = (1/2)mv_{\mathrm{f}}^2 - (1/2)mv_{\mathrm{i}}^2$

Interactive Question

In which of the following situations will there be an increase in kinetic energy?

A) A projectile approaches its maximum height
B) A box is pulled across a floor at a constant speed.
C) A child is pushing a merry-go-round causing it to rotate faster.
D) A satellite travels in a circular orbit around a planet at a fixed altitude.
E) A stone at the end of a string is whirled in a horizontal circle at a constant speed.

Interactive Question

Two marbles, one twice as heavy as the other, are dropped to the ground from the roof of a building. Just before hitting the ground, the heavier marble has

A) as much kinetic energy as the lighter one.
B) twice as much kinetic energy as the lighter one.
C) half as much kinetic energy as the lighter one.
D) four times as much kinetic energy as the lighter one.
E) impossible to tell.

Interactive Question

Compare the kinetic energy of two objects: The first has a mass of *m* and a speed of 2*v*. The second has a mass of 2*m* and a speed of *v*.

A) $KE_1 = KE_2$
B) $2KE_1 = KE_2$
C) $KE_1 = 2KE_2$
D) $KE_1 = 4KE_2$
E) $4KE_1 = KE_2$

Problem: A 1500 kg car moving at a speed of 35 m/s comes to a stop over a distance of 82 meters.

a) What is the car's initial kinetic energy?
b) What is the car's final kinetic energy?
c) How much work was done to stop this car?
d) What was the net average force stopping the car?

Potential Energy

- Potential energy (PE) can be defined as the potential ability to do work or gain kinetic energy.
- Potential energy is dependent on the relative position of the two objects.
- We will define two types of potential energy:

1. Gravitational potential energy: PE = mgh

   m is the mass of the object, g is the acceleration due to gravity, and h is the height above some reference point.

2. Elastic potential energy: PE = $(1/2)kx^2$

   x is the distance a spring has been compressed or stretched, and k is a characteristic of the stiffness of the spring, called the spring constant.

Interactive Question

In which system is there a decrease in potential energy?

A) A boy stretches a spring.
B) A child slides down a sliding board.
C) A crate rests at the bottom of an inclined plane.
D) A car ascends up a steep hill.
E) More than one of the above

Interactive Question

If the height of an object above a reference level is quadrupled its gravitational potential energy is multiplied by

A) 16
B) 4
C) 2
D) 1/4
E) 1/16

Interactive Question

If the distance a spring is stretched from its equilibrium position is halved the potential energy of the spring is multiplied by

A) 4
B) 2
C) 1
D) 1/2
E) 1/4

Conservation of Energy

When we add potential energy, then our conservation of energy equation becomes:

$$W = \mathrm{KE_f} + \mathrm{PE_f} - \mathrm{KE_i} - \mathrm{PE_i}$$

$$W = \tfrac{1}{2}mv_f^2 + mgh_f + \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}mv_i^2 - mgh_i - \tfrac{1}{2}kx_i^2$$

This is the conservation of energy.

When W=0, which means there is no friction, air resistance, or other forces changing the speed, (i.e. only gravity and/or springs are affecting the speed), then

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}$$

$$\tfrac{1}{2}mv_i^2 + mgh_i + \tfrac{1}{2}kx_i^2 = \tfrac{1}{2}mv_f^2 + mgh_f + \tfrac{1}{2}kx_f^2$$

This is called the conservation of *mechanical* energy

Solving Problems with Conservation of *Mechanical* Energy

1. Use this principle when the only forces affecting the speed are from gravity or springs.
   - Friction and other dissipative forces are non-existent.
2. Find initial and final velocities (for kinetic energy), height (for gravitational potential energy), and spring compression/stretching distances (for elastic potential energy)
3. Set the initial mechanical energy equal to the final mechanical energy.

$$KE_i + PE_i = KE_f + PE_f$$

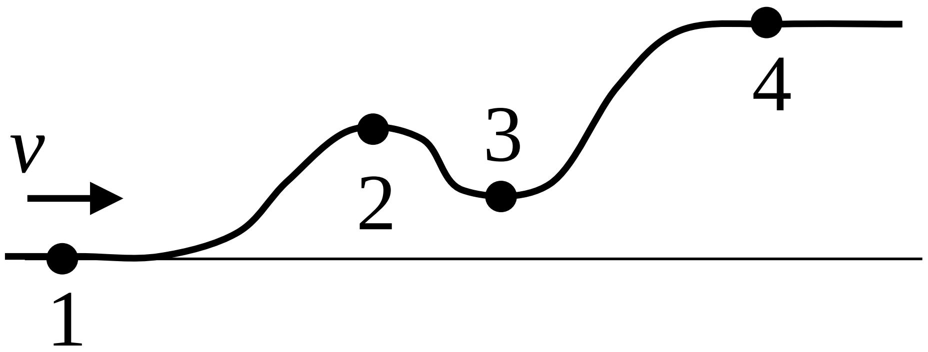
$$½mv_i^2 + mgh_i + ½kx_i^2 = ½mv_f^2 + mgh_f + ½kx_f^2$$

Problem: A child and sled with mass of 50 kg start from rest at the top of a 1.8 m high frictionless hill.

a) What is the potential energy at the top of the hill?
b) What is the kinetic energy at the bottom of the hill?
c) How fast is the child and sled going at the bottom?

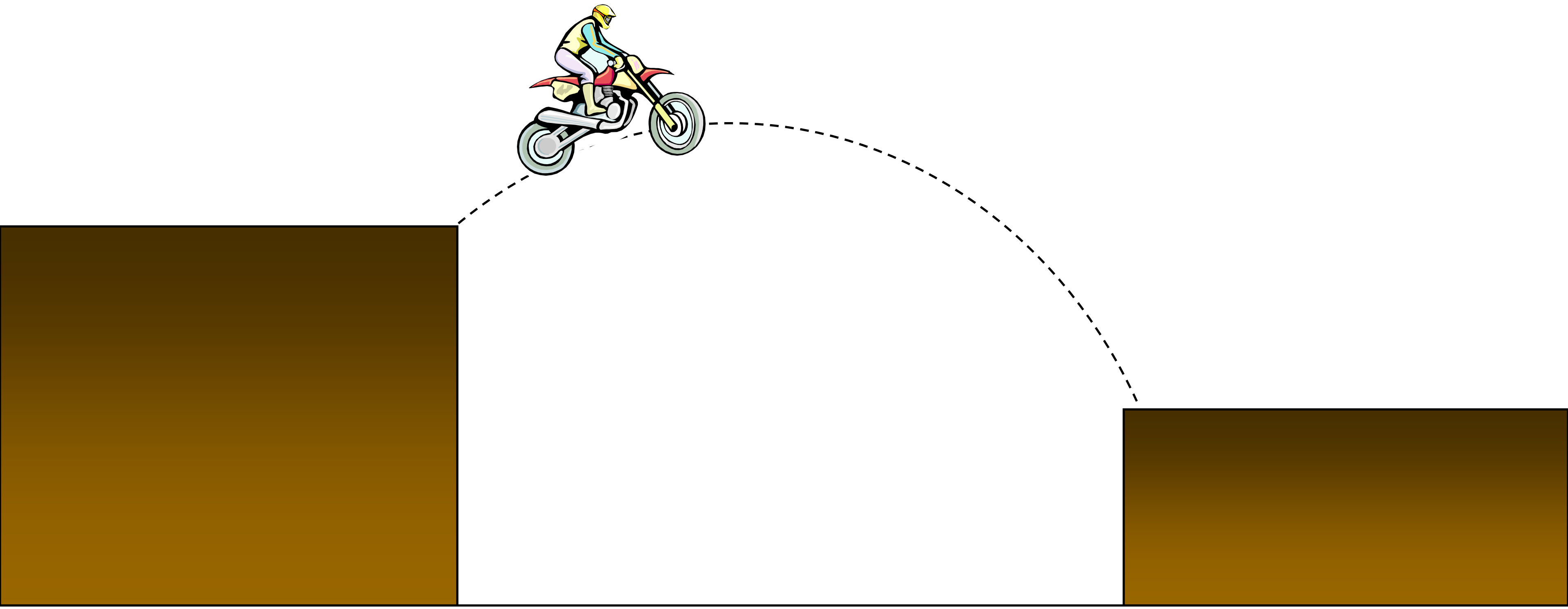
Interactive Question

A cart starts with velocity *v* and rolls up the frictionless path shown. Which of the following is *not* true?



A) The potential energy at point 3 is greater than the potential energy at point 2
B) The potential energy is greatest at point 4.
C) The kinetic energy is greatest at point 1.
D) The mechanical energy does not change.
E) The cart has kinetic and potential energy at point 2.

Problem: A motorcycle rider leaps across a canyon with an initial speed of 38.0 m/s He lands at a height of 35.0 m with a speed of 46 m/s. How high above the canyon was his starting point?



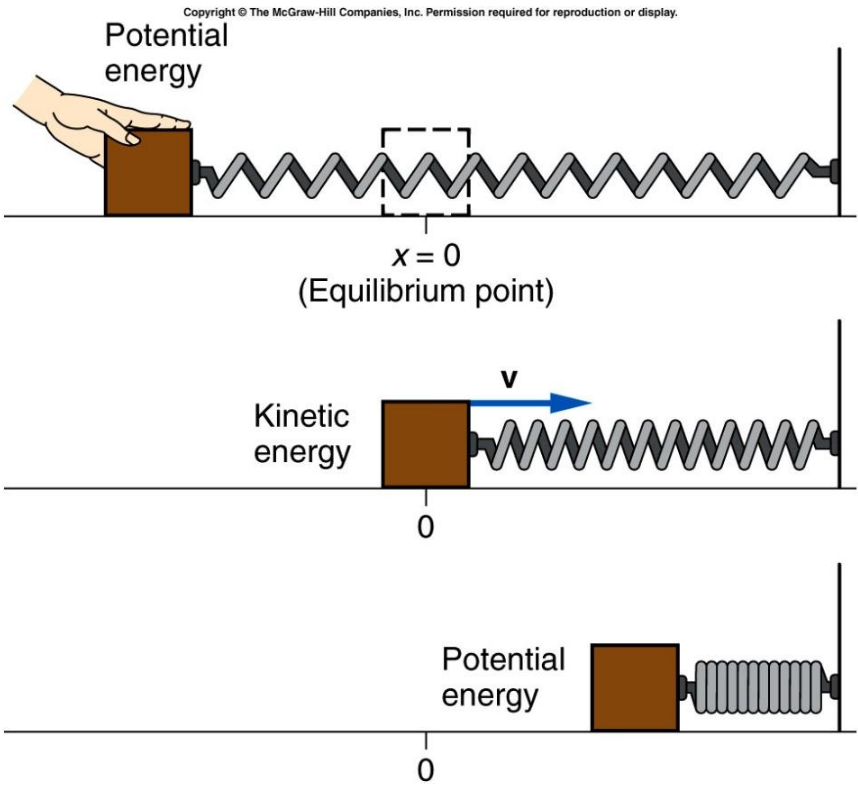
Interactive Question

Suppose you wanted to ride your mountain bike down a steep hill. Two paths lead from the top to the base, one twice as long as the other. Neglect friction and air resistance. Compared to the maximum speed you would reach if you took the short path, the maximum speed you will reach along the longer path is

A) one quarter as fast.
B) twice as fast.
C) the same speed.
D) half as fast.
E) four times as fast.

Springs and Simple Harmonic Motion

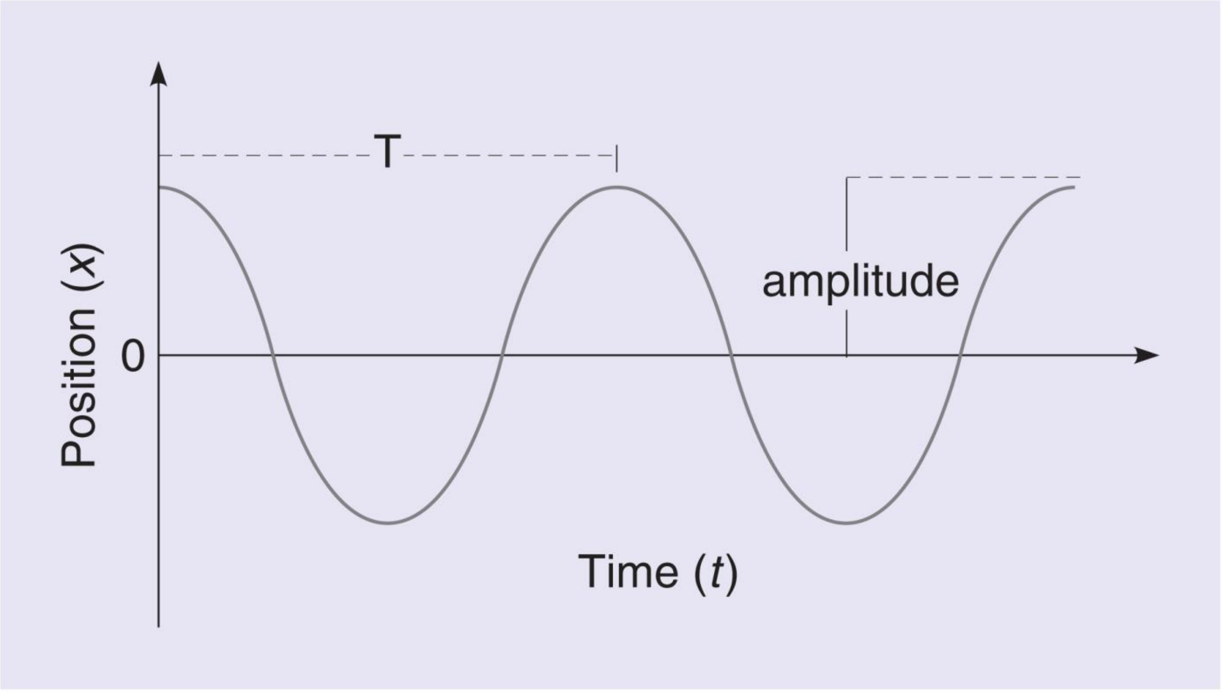
- Simple harmonic motion occurs when the energy of a system repeatedly changes from potential energy to kinetic energy and back again.
- Energy added by doing work to stretch the spring is transformed back and forth between potential energy and kinetic energy.



- At the equilibrium point the kinetic energy is greatest and the potential energy is zero. At the end points the potential energy is greatest and the kinetic energy is zero.
- With no friction, for any two points: $KE_1 + PE_1 = KE_2 + PE_2$

Period and Frequency

- The time it takes to complete one oscillation is called the period, T, and the number of oscillations in a given time is the frequency, f. $f = 1/T$
- Frequency has units of s^{-1} or Hertz (Hz). Something that oscillates 5 times a second has a frequency of 5 Hz.
- The amplitude is the maximum distance from the equilibrium position.



Problem: A .24-kg mass is attached to a horizontal spring that has a spring constant of 86 N/m. The spring is initially stretched to 0.23 m. If there is no friction so that the spring oscillates with SHM how much energy is kinetic and potential when the spring is at

(a) $x = 0.23$ m

(b) $x = 0.0$ m

(c) $x = 0.1$ m

Problem: A spring with spring constant of 24 N/m is attached to a 0.38 kg block and stretched a distance of 18 cm from its equilibrium position and released.

A) How fast is it going when it reaches the equilibrium position?

B) How fast is it going when it is 18 cm away from equilibrium on the other side?

C) How fast is it going when it is 9 cm from the equilibrium position?

Problem: A spring loaded toy dart gun shoots a dart straight up in the air and the dart reaches a maximum height of 24 m. The same dart is shot straight up a second time but this time the spring is compressed only half as far. How far up does the dart go this time neglecting friction?

Work from friction and other nonconservative forces

When we have forces other than gravity or springs pushing or pulling parallel to the direction of motion, then those forces do work and we must use the total conservation of energy equation. Often those forces will be dissipative forces like friction or air resistance.

$$W = \mathrm{KE_f} + \mathrm{PE_f} - \mathrm{KE_i} - \mathrm{PE_i}$$

$$W = \tfrac{1}{2}mv_\mathrm{f}^2 + mgh_\mathrm{f} + \tfrac{1}{2}kx_\mathrm{f}^2 - \tfrac{1}{2}mv_\mathrm{i}^2 - mgh_\mathrm{i} - \tfrac{1}{2}kx_\mathrm{i}^2$$

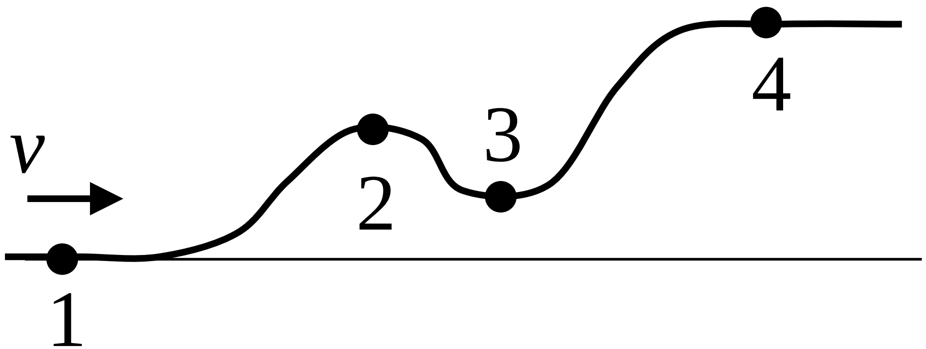
Problem: A truck with 3.0×10^6 J of kinetic energy at the bottom of a hill, coasts up the hill and comes to rest after gaining 1.2×10^6 J of potential energy. How much work was done on the truck in bringing it to rest?

Problem: A child and sled with mass of 50 kg slide down a hill with a height of 1.8 m.

a) If the sled starts from rest and has a speed of 4.6 m/s at the bottom, how much thermal energy is lost due to friction. (This will be the work done.)
b) If the hill were 15 meters long, what was the average force of friction.

Interactive Question

A cart starts with velocity *v* and rolls up the path shown. Do not neglect friction. Which of the following is *not* true?



A) The potential energy at point 2 is greater than the potential energy at point 3.
B) The potential energy is greatest at point 4.
C) The kinetic energy is greatest at point 1.
D) The mechanical energy at point 4 is equal to the mechanical energy at point 1.
E) The cart has kinetic and potential energy at point 2.

<u>Problem:</u> In a circus a spring loaded cannon is used to shoot a human cannonball (with a mass of 68 kg) over a net that is 7.5 m high. The spring in the cannon has a spring constant of 780 N/m and is compressed 4.5 m. As the human cannonball just clears the net, he is moving at a speed of 3.3 m/s. How much energy was dissipated by air resistance as the person just cleared the net?

Interactive Question

A ball is dropped from a certain height and after it has fallen some distance it has gained 30 J of kinetic energy. Do not neglect air resistance. The ball must have started out with

A) more than 30 J of potential energy
B) exactly 30 J of potential energy
C) less than 30 J of potential energy

Interactive Question Answer

A ball is dropped from a certain height and after it has fallen some distance it has gained 30 J of kinetic energy. Do not neglect air resistance. The ball must have started out with

If mechanical energy were conserved, (that is there is no friction and only springs or gravity do work on the ball), then the ball would have to lose 30 J of potential energy to gain 30 J of kinetic energy. Since some of the energy is lost to friction, it takes more than 30 J of potential energy to give the ball 30 J of kinetic energy.

A) more than 30 J of potential energy

Power

Power is defined as the rate at which energy is transferred or transformed. Energy transfer into the system is done by external work, so power is often defined as the rate that work is done.

$$P = W/t$$

The SI unit of power is watts (W): 1 J/s = 1 W

Sometimes we measure power in horsepower (hp)

1 hp = 550 ft·lb/s = 746 W

Problem: A 50 kg woman runs up a flight of stairs in 5.0 s. Her net upward displacement is 5.0 m.

a) How much work did she do?

b) What power did the woman exert while she was running?

Interactive Question

If an equivalent amount of work is done in a shorter period of time

A) more power is required

B) the same amount of power is required

C) less power is required

Elastic (Spring) Force

We have discussed the potential energy that is stored in a spring and found that it is

$$\text{PE} = (1/2)kx^2$$

where k is the spring constant and x is the displacement from the equilibrium position.

A spring also exerts a force when it is stretched or compressed. The magnitude of that force, for an ideal spring, is

$$F = kx$$

- Only objects with this relationship between force and displacement will exhibit simple harmonic motion.
- This is a force that can be used in Newton's 2nd Law.

<u>Problem</u>: A 2.5 kg fish is weighed by hanging the fish on a spring scale. When the fish hangs on the scale, the spring is stretched by 3.2 cm. What is the spring constant for this spring?