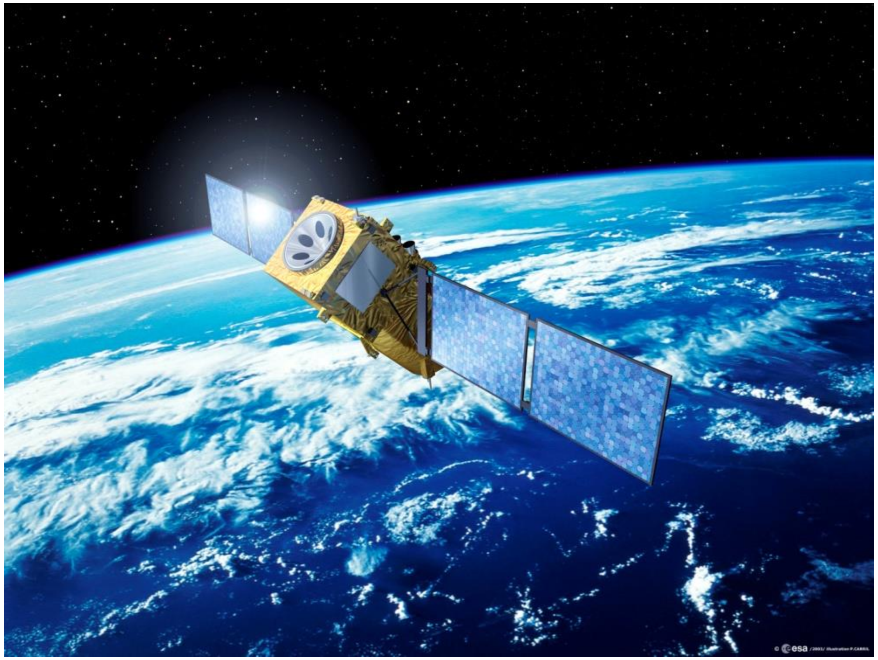
Chapter 5

Circular Motion, the Planets, and Gravity

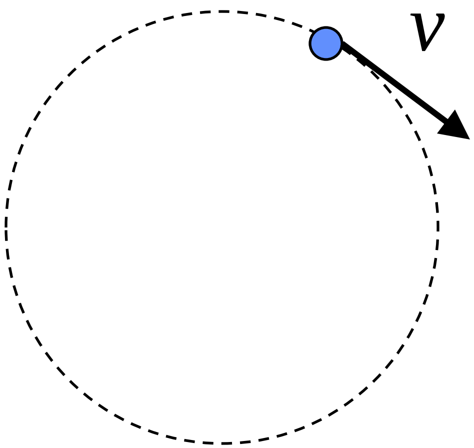


Uniform Circular Motion

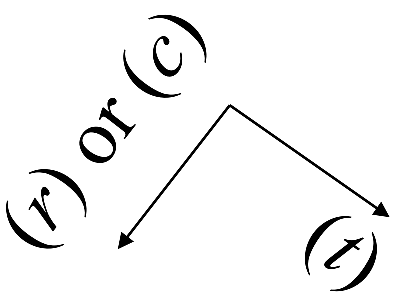
Uniform circular motion is the motion of an object traveling at a constant (uniform) speed in a circular path.

- Because velocity is a vector, with a magnitude and direction, an object that is turning at a constant speed has a change in velocity. The object is accelerating.
  - If the object was not accelerating, it would be traveling at a constant speed *in a straight line*.
  - We will determine the magnitude and direction of the acceleration for an object moving at a constant speed in a circle

Direction of Acceleration for Uniform Circular Motion.



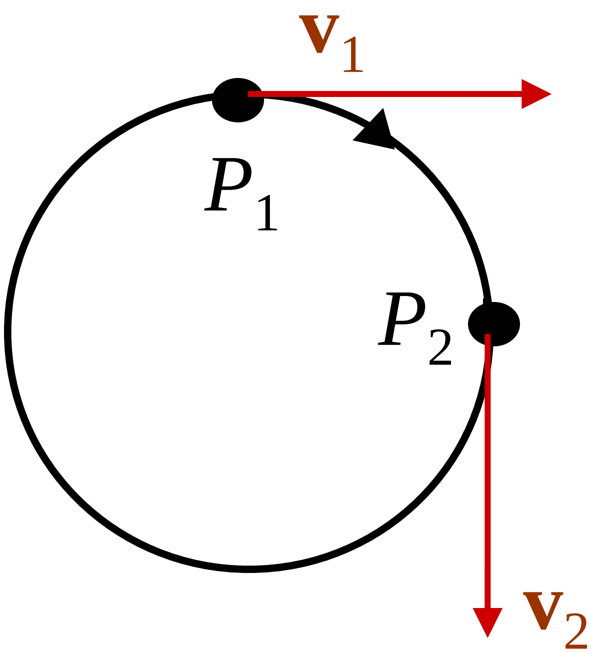
Define an axis that is always tangential to the velocity (*t*), and an axis that is always perpendicular to that, radial (*r*), or centripetal (*c*).



What would an acceleration in the tangential direction do?

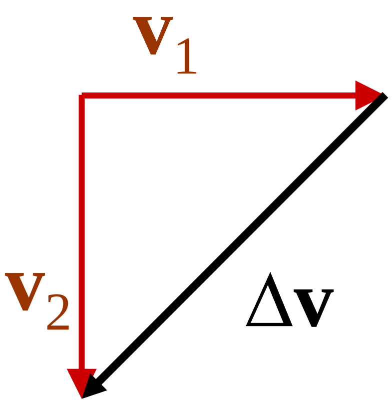
It must change the speed of the object. But for uniform circular motion, the speed doesn't change, so the acceleration can not be tangential and must be radial.

Consider a object that is twirled on a string in a circle at a constant speed in a clockwise direction, as shown. Let's find the direction of the average acceleration between points P_1 and P_2?



$\mathbf{v}_2 - \mathbf{v}_1 = \Delta\mathbf{v}$

$\mathbf{v}_1 + \Delta\mathbf{v} = \mathbf{v}_2$

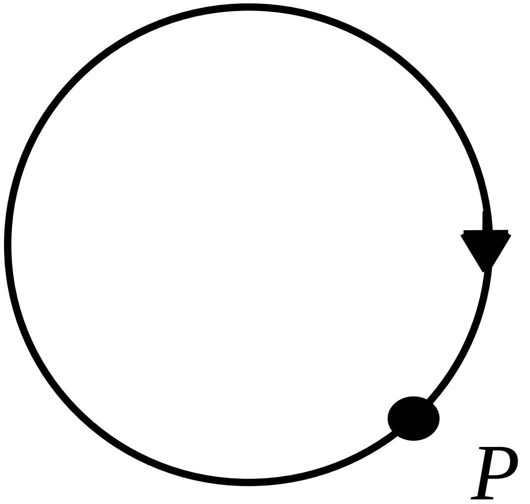


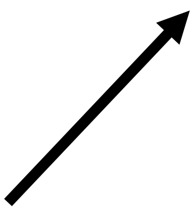
This direction of $\Delta\mathbf{v}$ is the direction of the acceleration. It is directly toward the center of the circle at the point midway between P_1 and P_2.

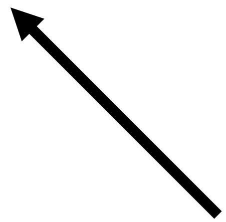
The direction of acceleration is toward the center of the circle. It is called *centripetal* (center-seeking) acceleration.

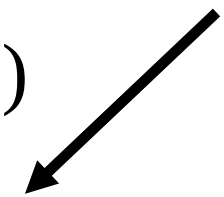
Interactive Question

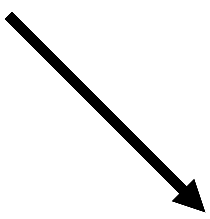
A rock is twirled on a string at a constant speed in a clockwise direction as shown. The direction of its acceleration at point *P* is

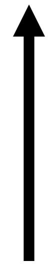


A) 

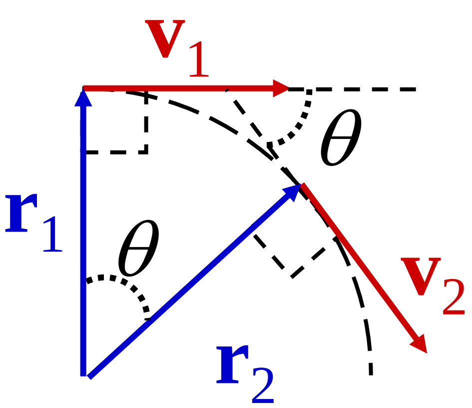
B) 

C) 

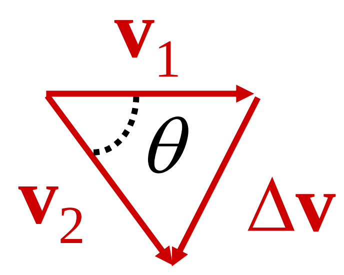
D) 

E) 

Magnitude of Acceleration for Uniform Circular Motion.

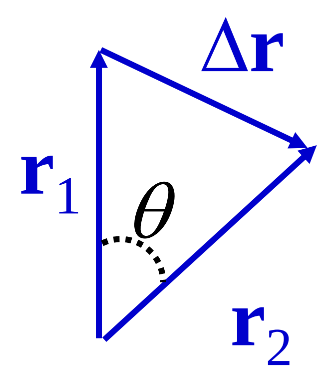


$$v_1 = v_2 = v,$$
$$r_1 = r_2 = r$$
$$\theta = \theta$$



$$\mathbf{v}_1 + \Delta\mathbf{v} = \mathbf{v}_2$$

$$\Delta v/v = \Delta r/r$$
$$\Delta v = v\Delta r/r$$
$$\Delta v/\Delta t = (v/r)\Delta r/\Delta t = (v/r)v = v^2/r$$
$$a_c = v^2/r$$



$$\mathbf{r}_1 + \Delta\mathbf{r} = \mathbf{r}_2$$

Interactive Question

A 1500 kg car travels at a constant speed of 22 m/s around a circular track which has a radius of 80 m. Which statement is true concerning this car?

A) The velocity of the car is changing.
B) The car is characterized by constant velocity.
C) The car is characterized by constant acceleration.
D) The car has a velocity vector that points along the radius of the circle.
E) More than one of the above is true.

Problem: In an Olympic bobsled race, the sled is traveling at a speed of 35 m/s when it moves through a banked curve. The bobsledders feel a centripetal acceleration of 2.2 “g’s.” What is the radius of the curve?

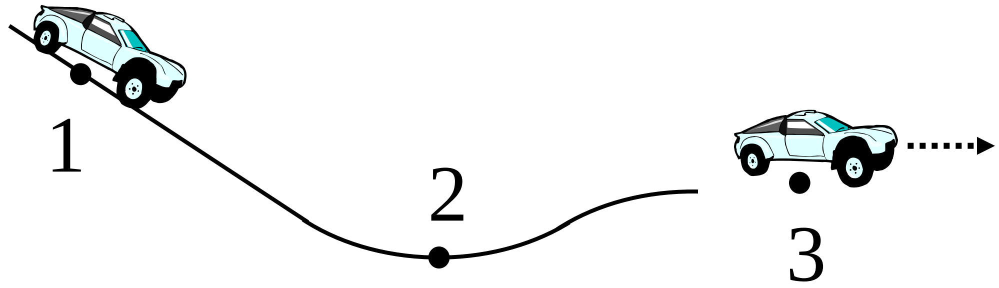
Interactive Question

You are riding on a merry-go-round. While sitting on a horse, the merry-go-round slows down so that it is rotating one-half as fast. By what factor does your centripetal acceleration change?

A) It is one-fourth as much
B) It is one-half as much
C) It is the same
D) It is twice as much
E) It is four times as much

Interactive Question

A toy car rolls down a track and flies off the end. What direction is the instantaneous acceleration at points 1, 2 and 3?



	(1)	(2)	(3)
A)	↓	←	→
B)	↓	←	↓
C)	↓	↑	→
D)	↘	←	↓
E)	↘	↑	↓

Uniform Circular Motion and Force

For an object undergoing uniform circular motion, consider the forces and acceleration in the radial direction

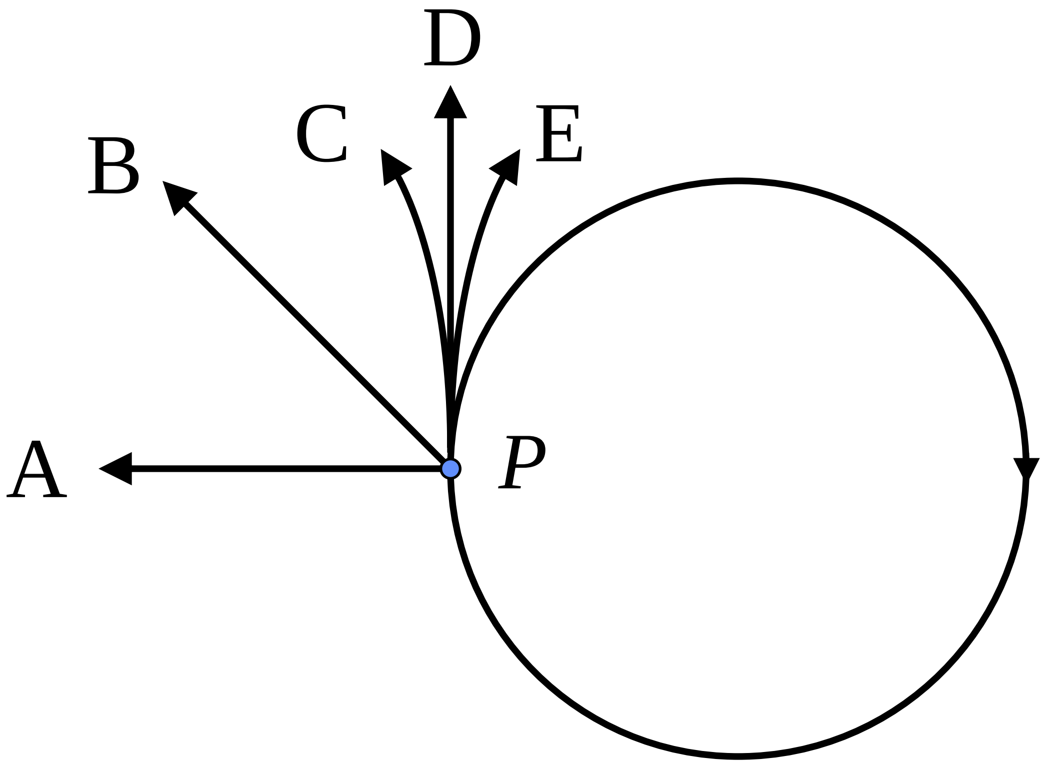
$F_{\text{net-radial}} = ma_c = mv^2/r$

This "net radial force" is sometimes called the centripetal, or "center-seeking" force.

It is simply the sum of all the forces acting on an object, like tension, gravity, friction, the normal force, etc. When the sum of all of the forces has a net direction toward the center of a circle, it is a "net radial force," or "centripetal" force. *It is not a new force!*

Interactive Question

A boy attaches a rock to a string which he then swings clockwise in a horizontal circle. The string breaks at point *P* on the sketch which shows a view from above. What path will the rock follow?

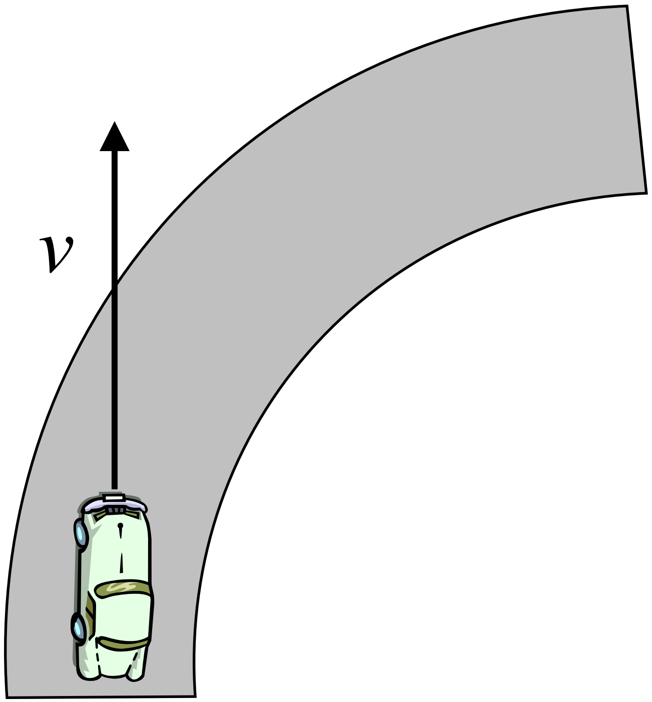


Whenever an object is turning, some force or combination of forces must provide the needed centripetal acceleration.

The friction between the tires and road produces the centripetal acceleration on a level curve.

Centripetal Force and "Centrifugal" Force

Centripetal: Center Seeking
Centrifugal: Center Fleeing – no such thing really.

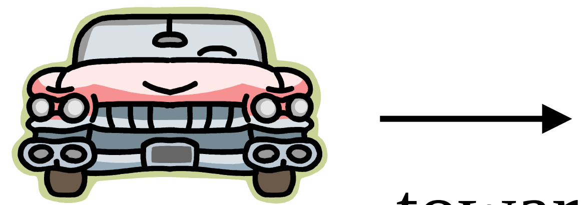


While driving the car around the corner, you think you are trying to "fly outward," but you are really just trying to go straight by Newton's 1st law, while the car is turning.

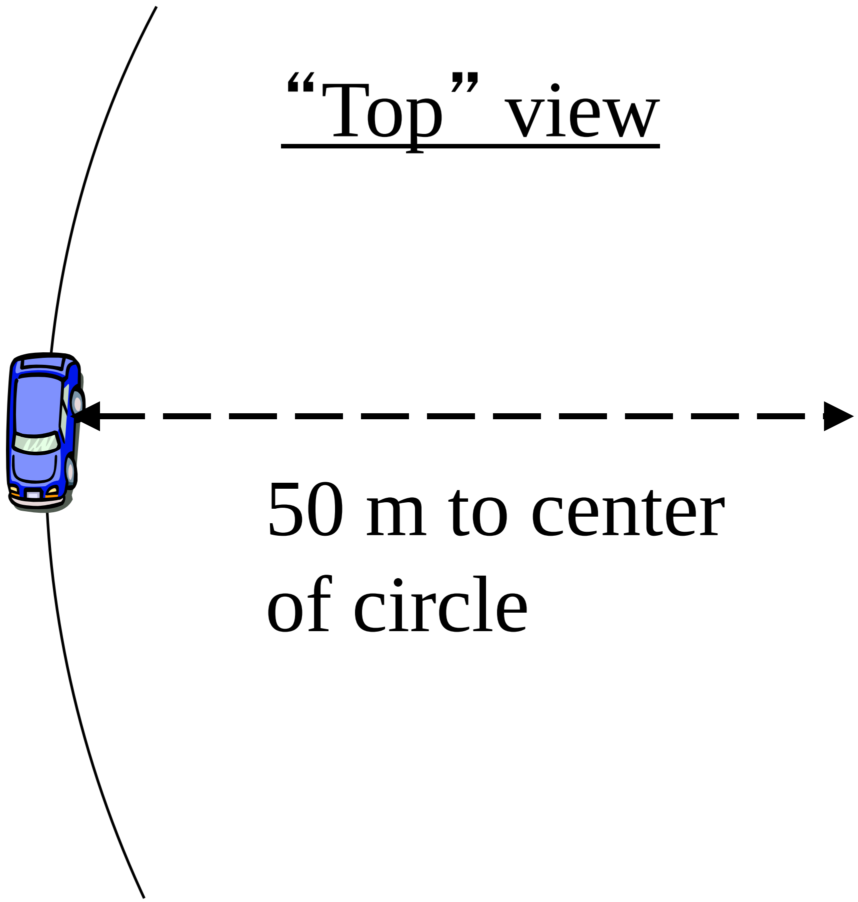
We "invent" a pretend force that we think is making us fly outward and call it "centrifugal" force, but it is a pseudo-force that exists only because we are in a "non-inertial" reference frame, (i.e. an accelerating reference frame).

Problem: A 1900 kg car turns a corner on a flat road of radius 50.0 m. If the maximum force the road can exert on the tires is 12,000 N, what is the maximum speed the car can negotiate the turn without sliding?

“Front” view



“Top” view



Interactive Question

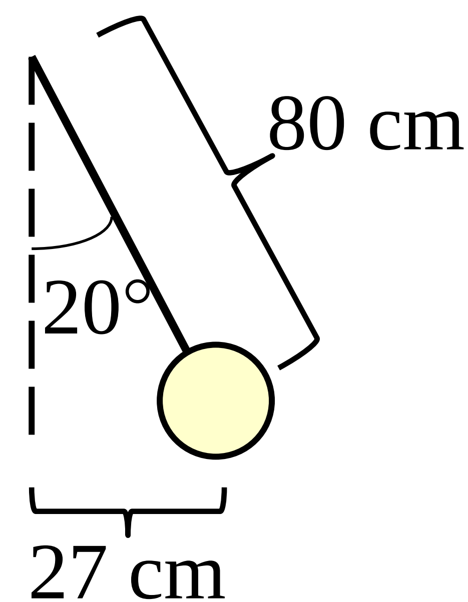
A boy is whirling a stone around his head by means of a string. Assume the string is exactly horizontal. The string makes one revolution every second and the tension in the string is *T*. The boy then doubles the speed of the stone, keeping the radius of the circle unchanged so that the string makes two complete revolutions every second. What happens to the tension in the string? (You can neglect the effects of gravity).

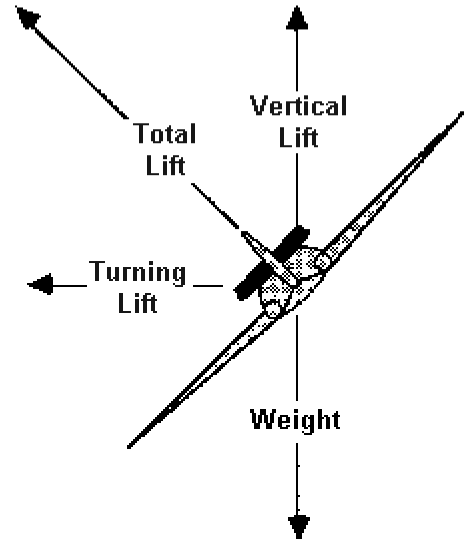
A) It remains unchanged
B) It is reduced to half of its original value.
C) It is increased to twice its original value.
D) It is reduced to one-fourth of its original value.
E) It is increased to four times its original value.

Problem: You tie a ball to a 0.80 m long string and rotate the ball in a horizontal circle that makes an angle of 20° with the vertical direction. The distance from the ball to the "center of the circle" is 27 cm and the ball takes 1.7 seconds to complete one circle.

(a) What is the centripetal acceleration?

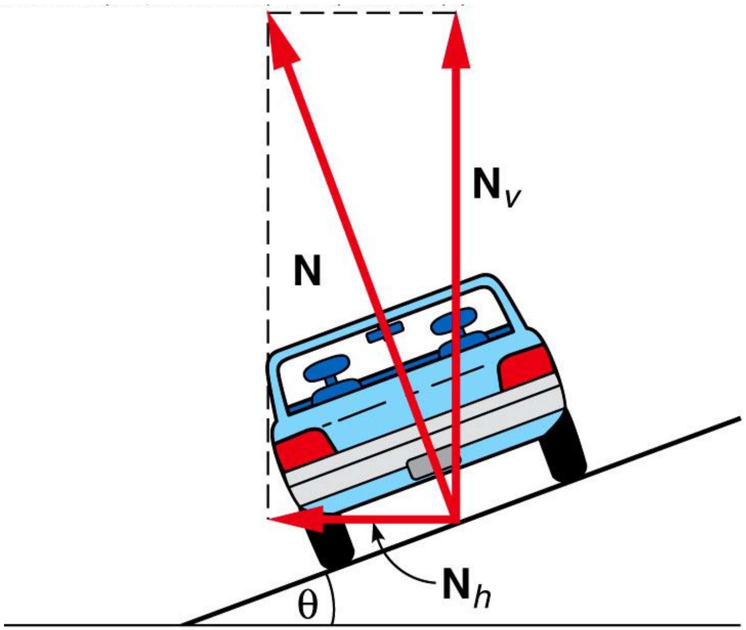
(b) Why is the string at an angle?



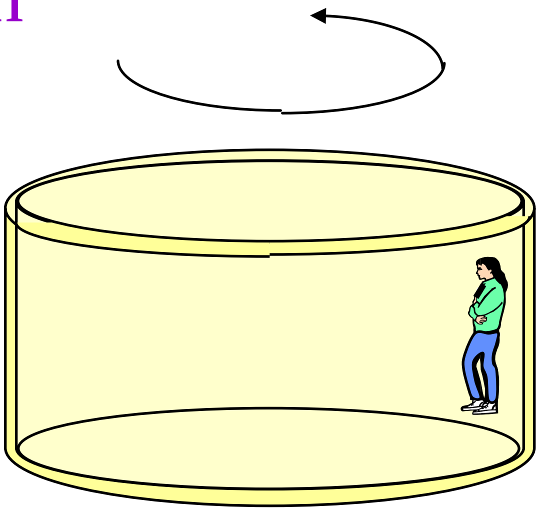

Total
Lift
Vertical
Lift
Turning
Lift
Weight


W
N
F

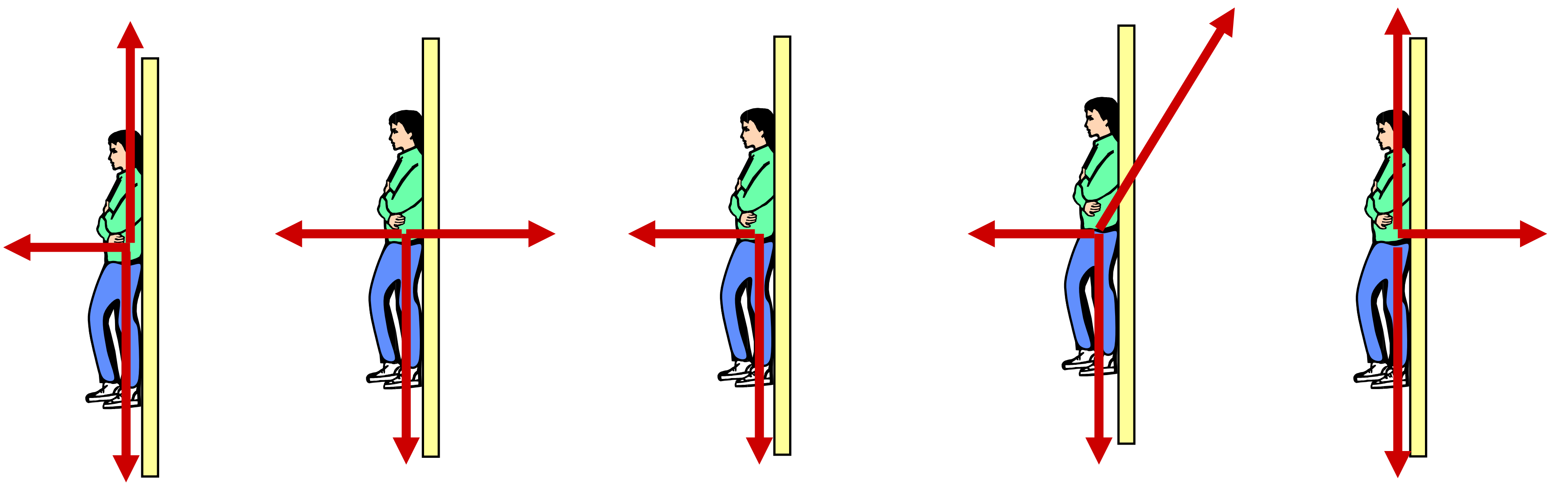
On a banked curve, the horizontal component of the normal force contributes to the centripetal acceleration in addition to the friction of the tires.



Interactive Question

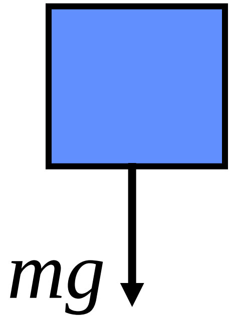


A rider in an amusement park ride, the "barrel of fun" finds herself stuck with her back to the wall. Which diagram correctly shows the forces acting on her?

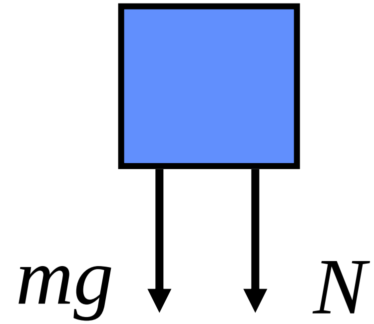


A) B) C) D) E)

Problem: What is the minimum velocity you can spin a can filled with water attached to a 1.0 m long string in a vertical direction without the water falling out?



Water falling out



Water not falling out

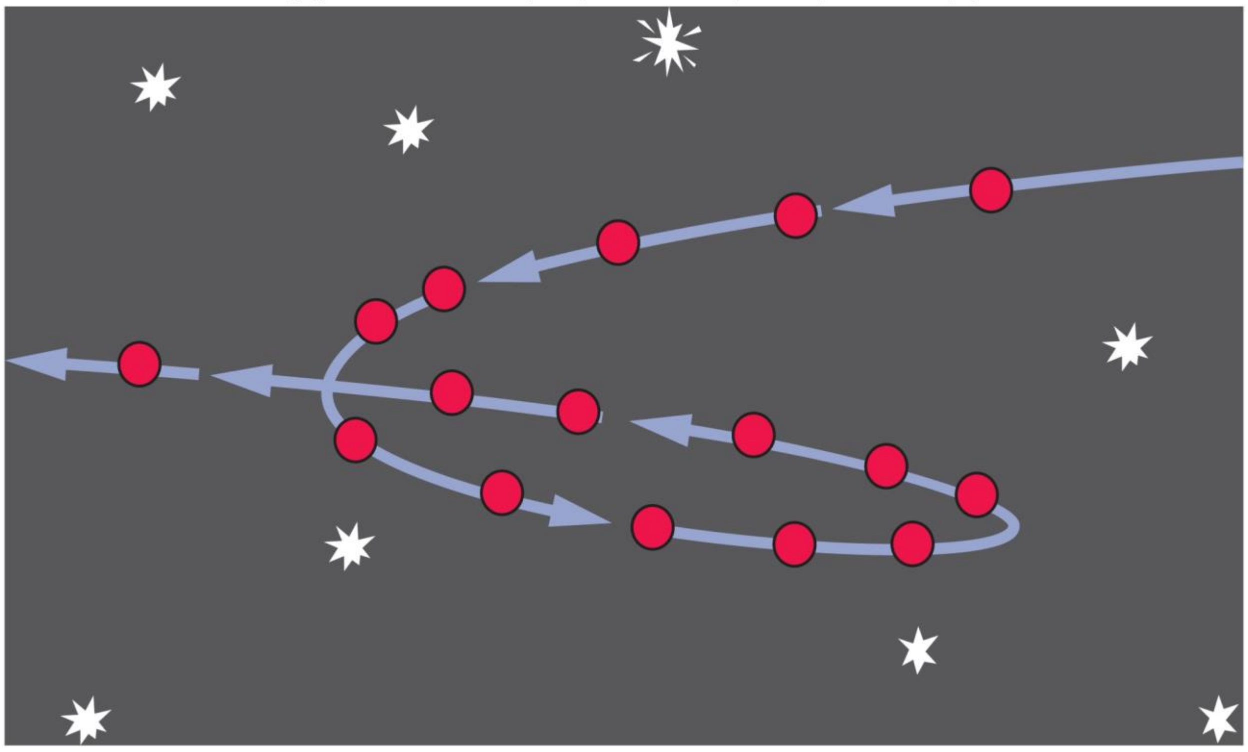
Why is there a greater downward force when the water is not falling out of the can?

The only way the water can not fall out of the can is if it actually accelerates downward faster than it would if only the force of gravity acted on it.

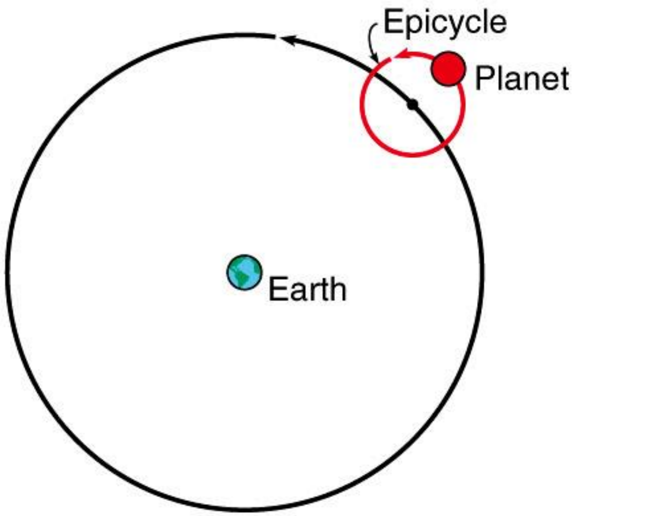
<u>Problem:</u> If you are spinning a can as in the previous problem, what is the normal force at the bottom of the circle?

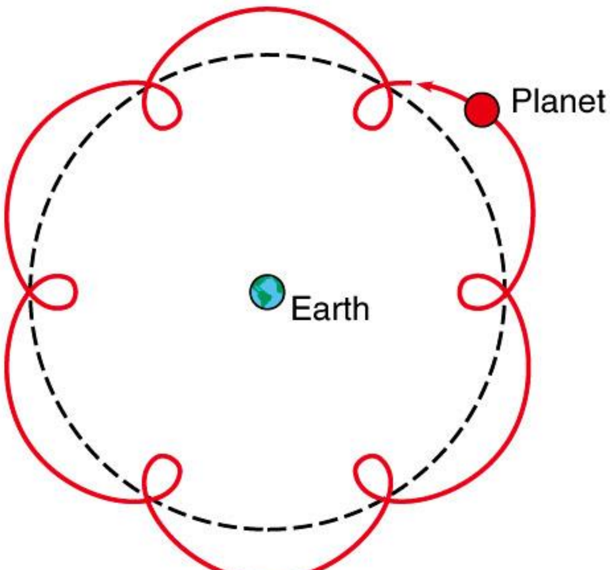
Planetary Motion and Centripetal Force

The planets seem to wander around compared with the stars which are much farther away. This is called "**retrograde motion**." Over the course of a few months, if you chart the position of a planet compared with the stars, this is what you'll see.



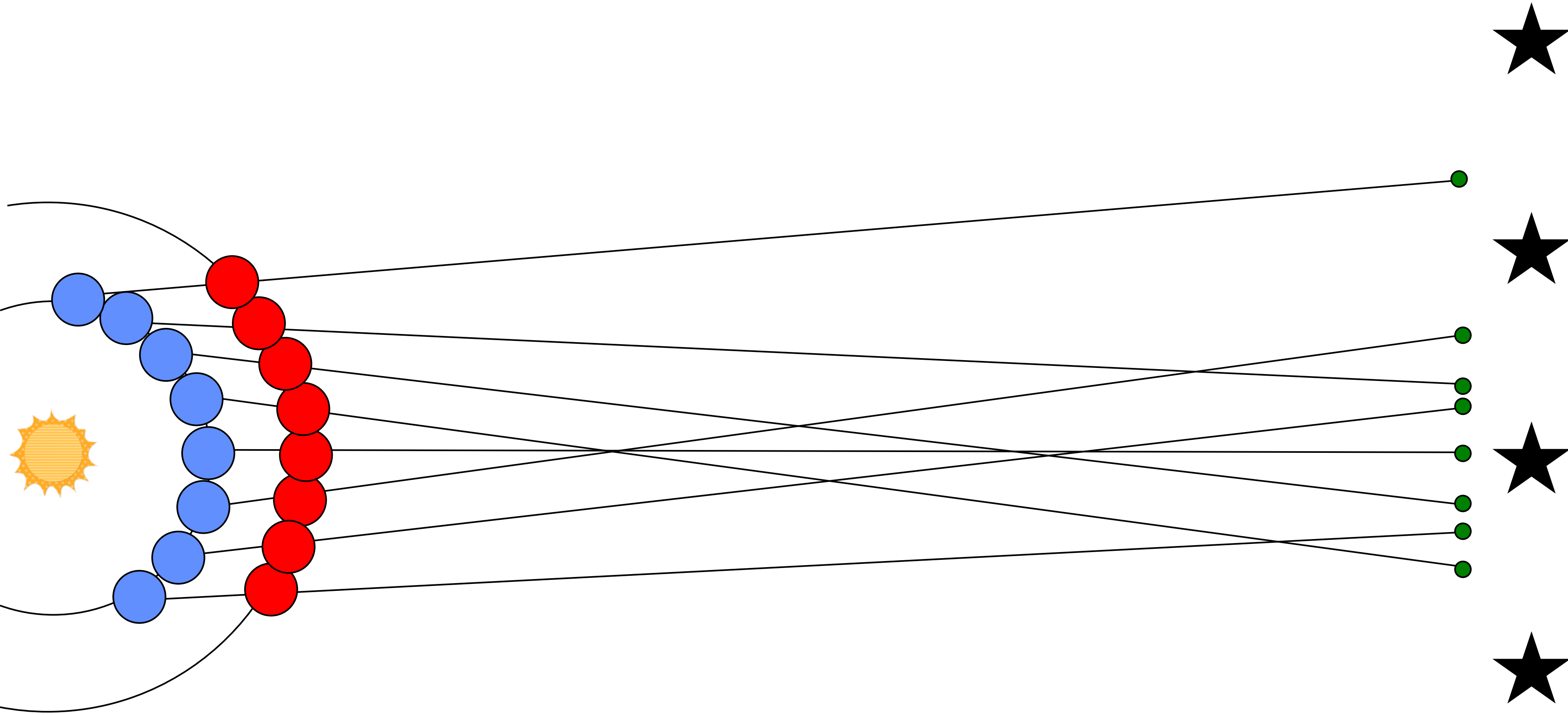
To explain the apparent retrograde motion of the planets, with the Earth at the center of the solar system. Ptolemy invented the idea of epicycles.





- ***Epicycles*** are imaginary circles the planets supposedly travel while also traveling along their main (larger) orbits around the Earth.
- This would explain the occasional “backward motion” the planets seemed to follow.
- But is there a better way to explain retrograde motion

Another explanation of retrograde motion.
Put the sun at the center of the solar system



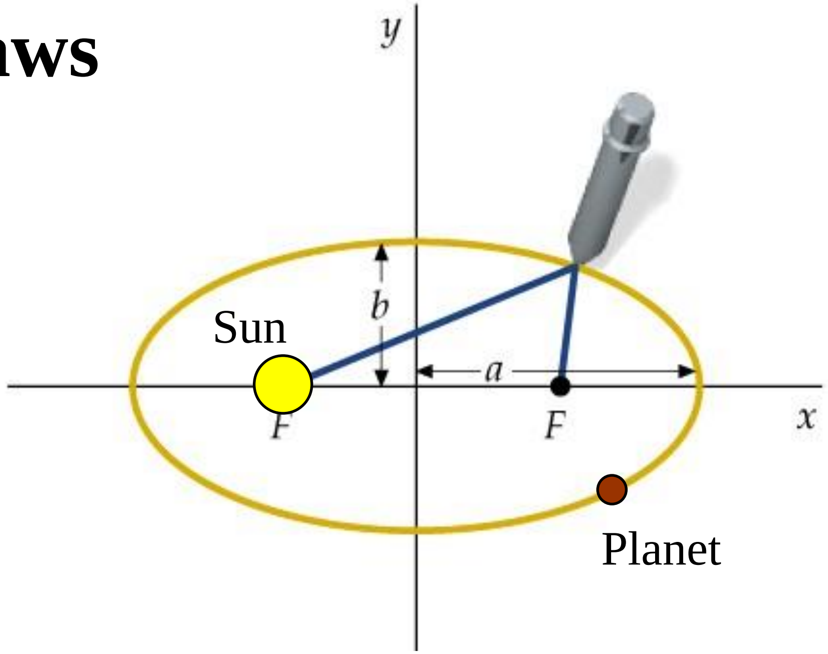
Both explanations work. But neither perfectly explained the motion of the stars and planets.

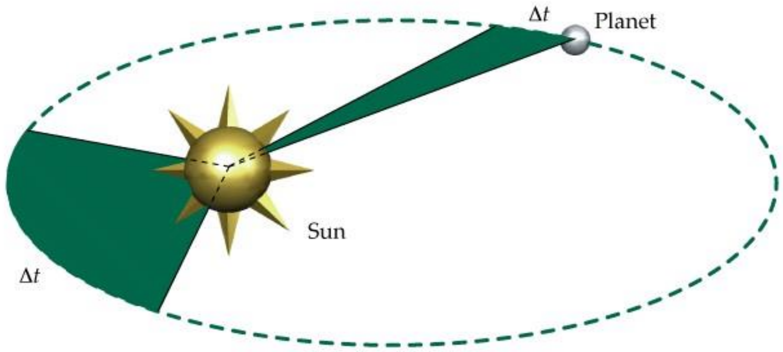
Kepler's Laws

1. All planets move in elliptical orbits with the Sun at one focal point.
2. A line drawn from the Sun to any planet sweeps out equal areas in equal time intervals.
3. The square of the orbital period of any planet (T) is proportional to the cube of the average distance from the planet to the Sun (r).

$T^2 \propto r^3$

$T_1^2/T_2^2 = r_1^3/r_2^3$



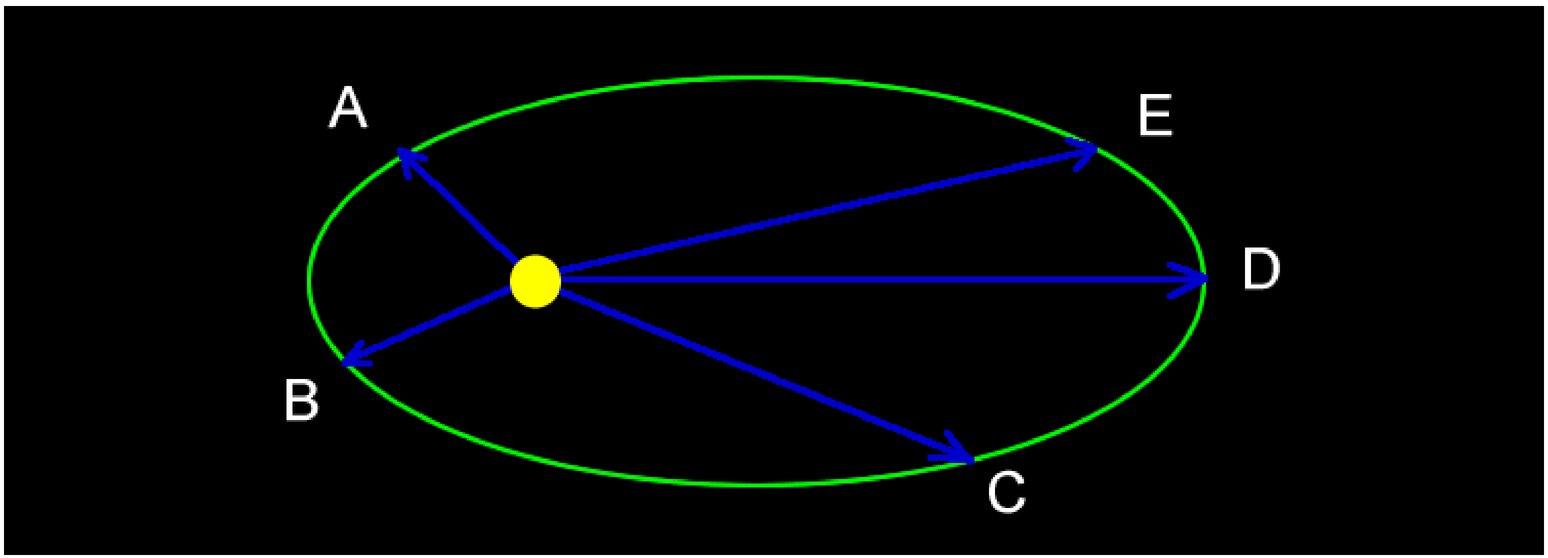


Is a circle an ellipse?
Absolutely Yes!

Interactive Question

The figure below shows the orbit of a comet about the sun. The comet has the greatest velocity when traveling

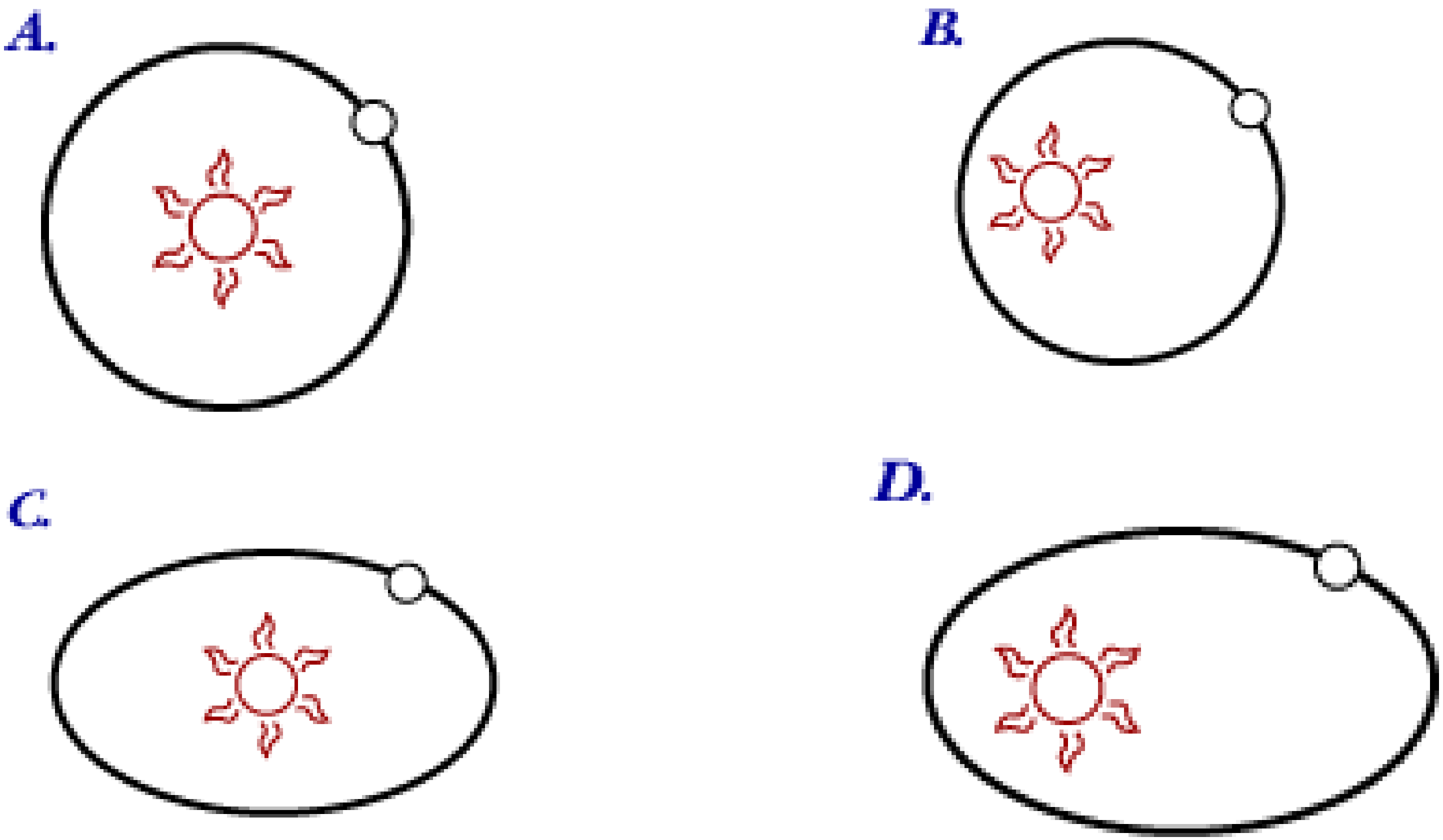
A) from A to B.
B) from B to C.
C) from C to D.
D) from D to E.

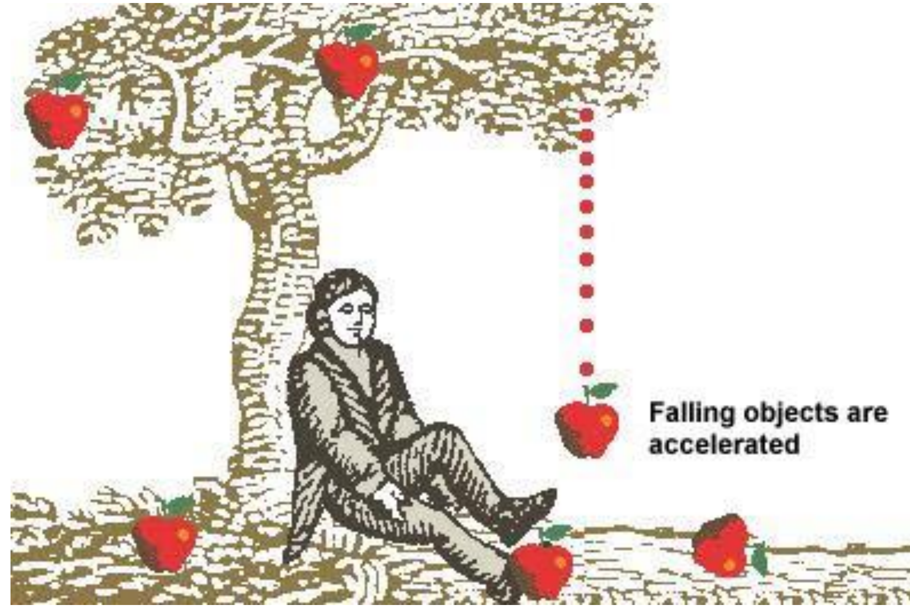


Problem: Saturn is about 9.7 times as far from the sun as the earth is. What is the length of Saturn's year?

Interactive Question

Which of the following diagrams most accurately depicts the shape of Earth's orbit around the Sun?




Falling objects are
accelerated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Newton's Law of Universal Gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

- Using this law, Newton explained why Kepler's laws worked (which had been shown to be correct), and explained why objects are attracted to the Earth.
- F is the force between any two objects, one with mass m_1 and one mass m_2
- r is the distance between the two objects.
- The direction of the force points from one object toward the other.
- G is Newton's universal gravitational constant with a value of $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Observations Regarding Universal Gravitation

1. The force acts over all space. Every object in the universe feels gravity from every other object in the universe.
2. Force is proportional to mass.
3. The force that one object feels from another is equal and opposite to the force the other object feel on the first one. (This is the same as Newton's third law). The acceleration of the two objects would be different if their mass is different.

When the objects are spherical, the distance "between" the objects is measured from their centers. One of the reasons Newton developed calculus to demonstrate that this was true.

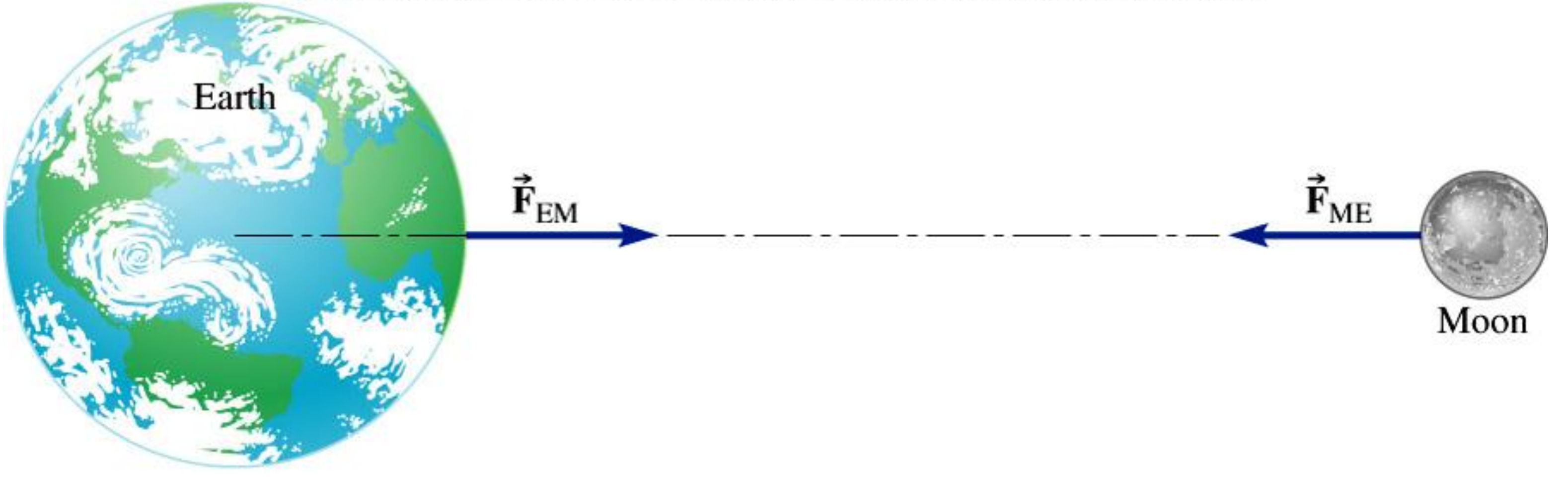
Interactive Question

A spaceship is traveling to the moon. At what point is it beyond the pull of the earth's gravity? The mass of the moon is 1/80 the mass of the earth, and the surface gravity of the moon is 1/6 that of the earth.

A) When it gets out of the atmosphere.
B) When it is half-way there.
C) When it is 5/6 of the way there.
D) When it is 79/80 of the way there.
E) It is never beyond the pull of earth's gravity.

Interactive Question

Which is stronger, the Earth's pull on the Moon, or the Moon's pull on the Earth?



A) The Earth’s pull on the Moon
B) The Moon’s pull on the Earth
C) They are the same
D) It depends on other factors

Problem: (a) What is the magnitude and direction of the gravitational force on the Moon from the Earth? (b) What is the magnitude and direction of the gravitational force on the Earth from the Moon?

Interactive Question

If the distance to the moon were halved, then the force of attraction between the earth and moon would be

A) quartered.
B) halved.
C) unchanged
D) doubled.
E) quadrupled.

Interactive Question

Three objects with equal masses are located as shown. What is the direction of the total force acting on the object labeled m_2?

A) To the left.
B) To the right.
C) The forces cancel such that the total force is zero.
D) It is impossible to determine from the figure.

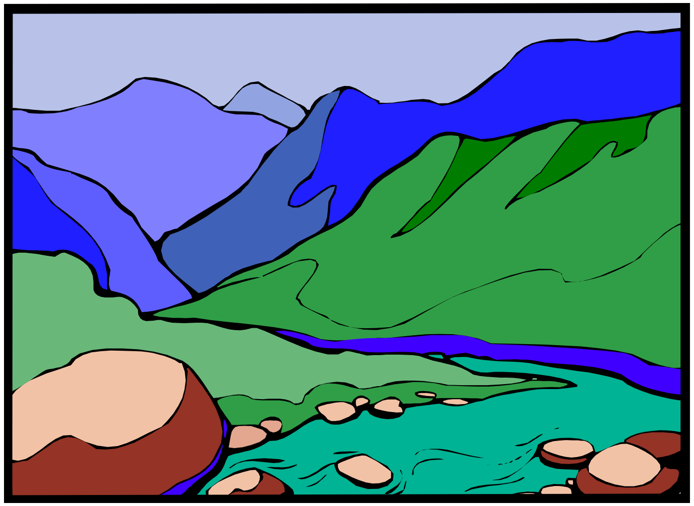
Gravity at the surface of the Earth

$W = mg = Gmm_E/r_E^{\,2}$

$g = Gm_E\,/r_E^{\,2}$

$= (6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.97\times10^{24}\mathrm{kg}) \div (6.38\times10^{6}\ \mathrm{m})^2$

$= 9.8\ \mathrm{m/s^2}$



Problem: How far above the earth must you go for the force you feel to be 1/2 of what it is on the surface of the earth?

Interactive Question

The radius of plant Sooner is identical to that of Earth, yet the mass is twice that of Earth. Your weight on Sooner, compared to your weight on Earth, is

A) quartered.
B) halved.
C) the same.
D) doubled.
E) quadrupled.

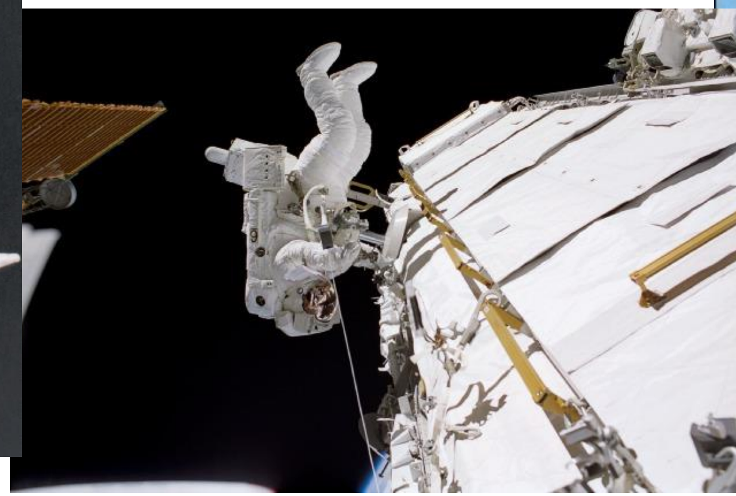
Interactive Question

The radius of plant Boomer is twice that of Earth, yet the two planets have identical masses. Your weight on Boomer, compared to your weight on Earth, is

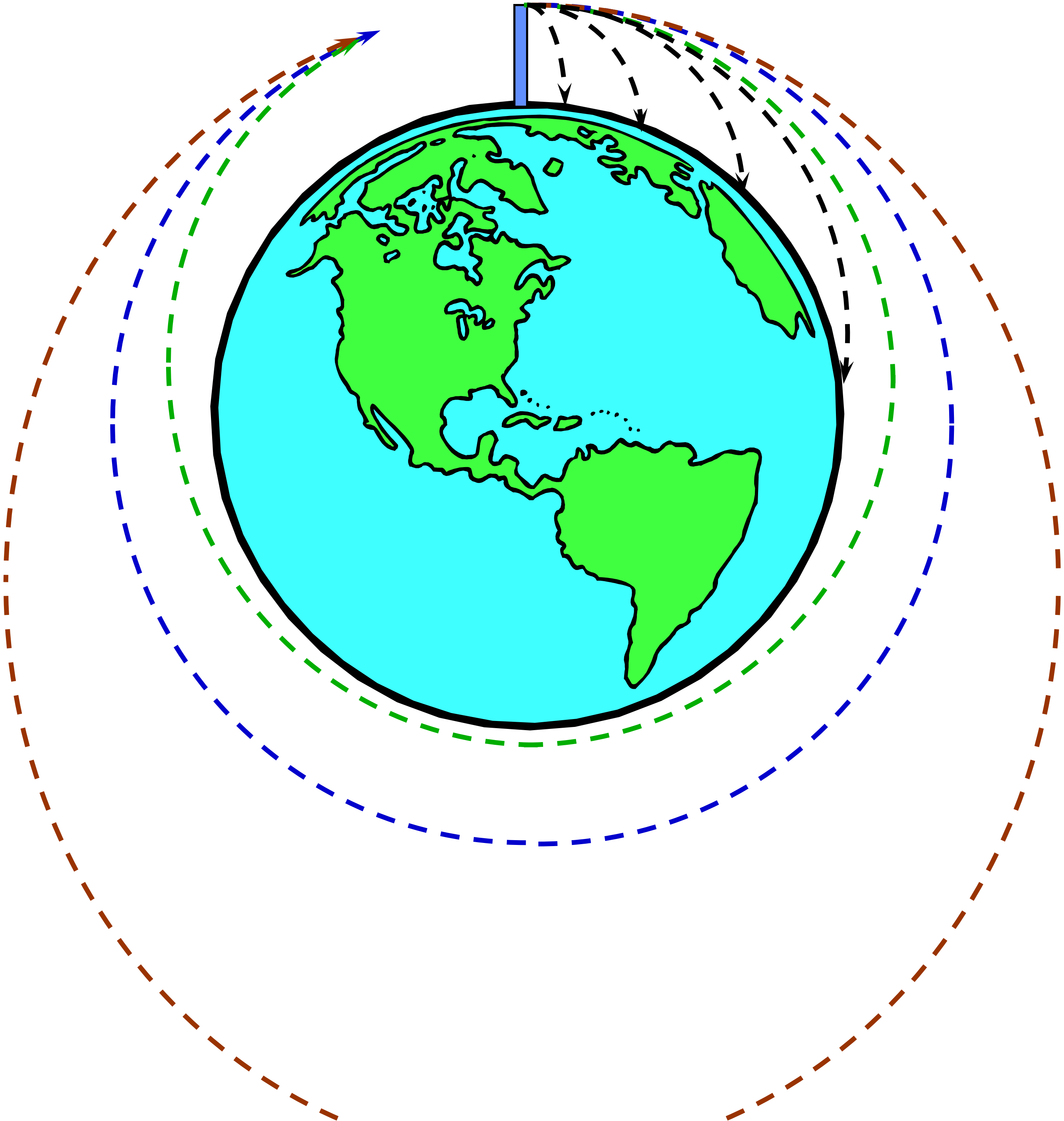
A) quartered.
B) halved.
C) the same.
D) doubled.
E) quadrupled.

Satellites and "Weightlessness"

Examples of objects having only the force of gravity acting on them. They are in freefall.



When an object is in freefall, we call it "weightless" because a scale would not read any weight. This is really "apparent weightlessness" because there are no normal or other forces acting on the object. In reality, the object still has a weight which is defined as the force of gravity acting on it.



The Moon and Other Satellites

The moon and man-made satellites orbit the Earth following the same physical principles as those for that the planets orbiting the sun. For instance,

- All satellites orbit the earth in elliptical orbits.
- Satellites orbiting closer to Earth always move faster than satellites orbiting farther from Earth.
- The ratio $T_1^2/T_2^2 = r_1^3/r_2^3$ still applies.
  - This relationship holds when comparing two different objects that are both orbiting the same central object (e.g. two planets orbiting the sun, or two satellites orbiting the earth.)

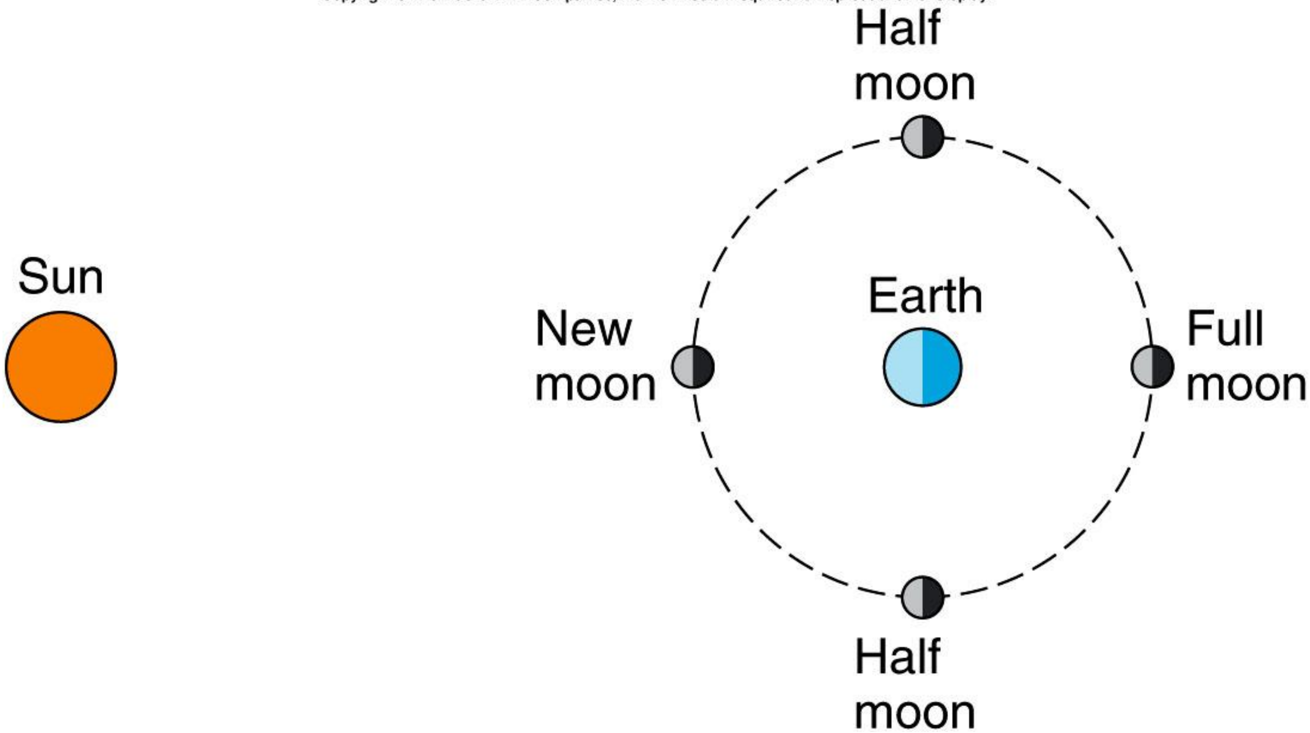
Interactive Question

If an artificial satellite is orbiting about the Earth, between the Earth and the moon, how does its period of rotation compare to the moon's period?

A) It is greater.
B) It is the same.
C) It is less.
D) We don't have enough information because it depends on the satellite's speed
E) We don't have enough information because it depends on the satellite's mass

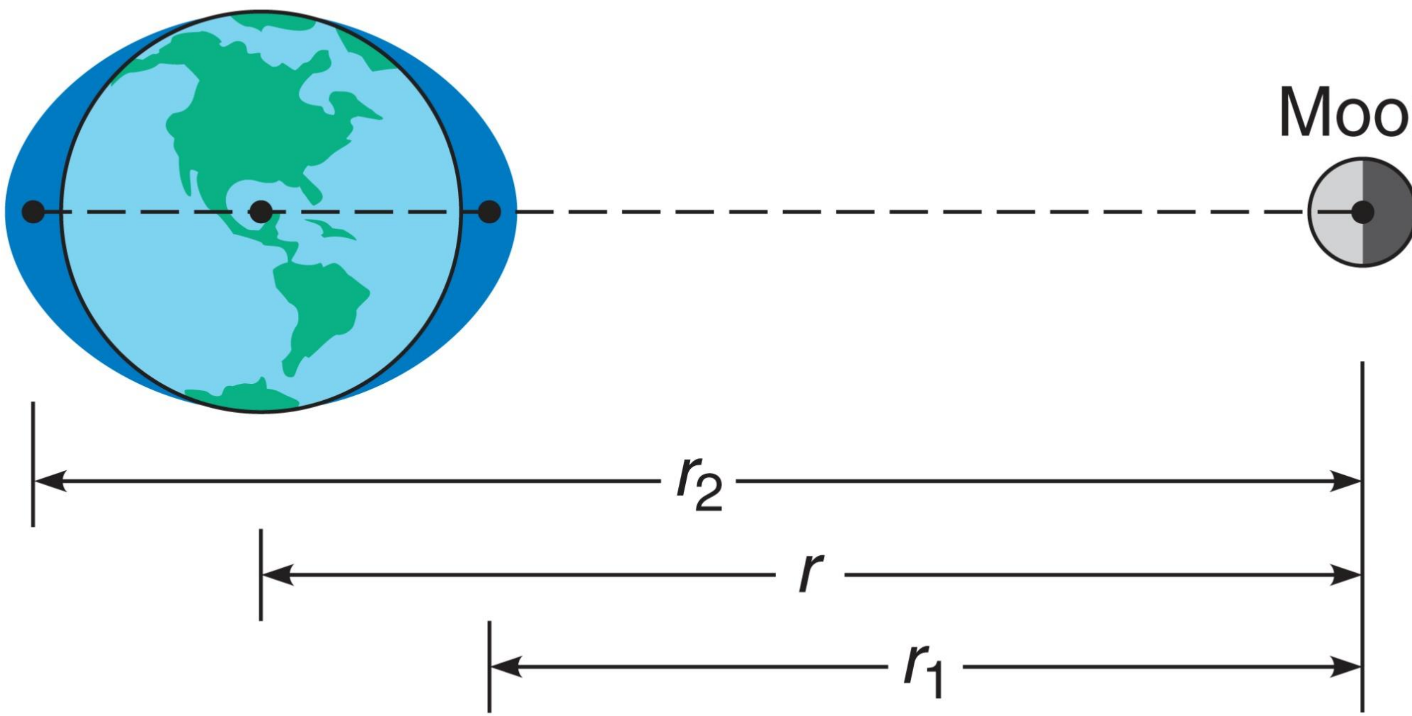
Phases of the Moon

Phases of the moon result from the changes in the positions of the moon, Earth, and sun.



Tides and the Moon

Tides occur because of Newton's law of Universal Gravity. Water on the side of the Earth closest to the moon feels a stronger gravitational force than water on the side of the Earth farther from the moon, causing the water to bulge on both sides of the Earth aligned with the Moon. The water doesn't bulge on the sides perpendicular to the moon.



Interactive Question

Consider this statement: The Sun exerts a greater gravitational force on the Earth than the moon does, yet the moon is primarily responsible for the tides.
Is this true or not true, and why?

A) It's not true. The moon is closer, so the gravitational force it exerts is stronger than the Sun's.
B) It's not true. The Sun does exert a greater gravitational force so the Sun is responsible for the tides.
C) It's not true. Tides are created by both the Sun and the Moon only when the they line up correctly.
D) It is true. The variation of the gravitational force on the different sides of the Earth from the Moon is greater than the variation on the different sides of the Earth from the Sun because the Moon is closer to the Earth than the Sun.