Solutions to Homework Set #9
Phys2414 – Fall 2005

Note: The numbers in the $\boxed{\text{boxes}}$ correspond to those that are generated by WebAssign. The numbers on your individual assignment will vary. Any calculated quantities that involve these variable numbers will be $\boxed{\text{boxed}}$ as well.

1. GRR1 7.P.046. A hockey puck moving at $\boxed{0.40}$ m/s collides into another puck that was at rest. The pucks have equal mass. The first puck is deflected $\boxed{38}^\circ$ to the right and moves off at $\boxed{0.38}$ m/s. Find the speed and direction of the second puck after the collision.

As with all of these linear momentum problems, we will start with

$$\begin{aligned}\vec{p}_{i,tot} &= \vec{p}_{f,tot}\\ \vec{p}_{1,i}+\vec{p}_{2,i} &= \vec{p}_{1,f}+\vec{p}_{2,f}\end{aligned}$$

where $\vec{p}_{1,i}$, $\vec{p}_{1,f}$, $\vec{p}_{2,i}$, and $\vec{p}_{2,f}$ are the initial and final momenta of the first and second puck, respectively. The equation above is a vector equation. This means that the x and y components of the vector can be solved separately. Therefore we have two equations: one for x and one for y,

$$\begin{aligned}p_{1x,i}+p_{2x,i} &= p_{1x,f}+p_{2x,f}\\ p_{1y,i}+p_{2y,i} &= p_{1y,f}+p_{2y,f}\end{aligned}$$

The second puck is at rest initially, so we know that $\vec{p}_{2,i}=0$. We can also make our coordinate system so that the first puck initially slides only in the x-direction. So in the x-direction we have

$$\begin{aligned}p_{1x,i}+0 &= p_{1x,f}+p_{2x,f}\\ p_{2x,f} &= p_{1x,f}-p_{1x,i}\end{aligned}$$

Since the masses are equal, this equation becomes a bit simpler

$$\begin{aligned}p_{2x,f} &= p_{1x,f}-p_{1x,i}\\ mv_{2x,f} &= mv_{1x,f}-mv_{1x,i}\\ v_{2x,f} &= v_{1x,f}-v_{1x,i}\end{aligned}$$

And in the y-direction we get

$$\begin{aligned}0+0 &= p_{1y,f}+p_{2y,f}\\ p_{2y,f} &= -p_{1y,f}\\ mv_{2y,f} &= -mv_{1y,f}\\ v_{2y,f} &= -v_{1y,f}\end{aligned}$$

The magnitude of the second puck's final velocity, $|\vec{v}_{2,f}|$, is then

$$|\vec{v}_{2,f}| = \sqrt{(v_{2x,f})^2 + (v_{2y,f})^2}$$

Substituting in the relationships we found above, we get

$$|\vec{v}_{2,f}| = \sqrt{(v_{1x,f} - v_{1x,i})^2 + (-v_{1y,f})^2}$$
$$|\vec{v}_{2,f}| = \sqrt{(v_{1x,f} - v_{1x,i})^2 + (v_{1y,f})^2}$$

We know the initial and final magnitudes of the first puck as well as the angle of deflection. The first started out moving only in the x-direction, and we can use trigonometry to find the x and y components of $\vec{v}_{1,f}$. We can then substitute this into the above equation and get

$$|\vec{v}_{2,f}| = \sqrt{(v_{1,f}\cos\theta_1 - v_{1,i})^2 + (v_{1,f}\sin\theta_1)^2}$$
$$|\vec{v}_{2,f}| = \sqrt{((\boxed{0.38}\text{ m/s})\cdot\cos\boxed{38^\circ} - \boxed{0.40}\text{ m/s})^2 + ((\boxed{0.38}\text{ m/s})\cdot\sin\boxed{38^\circ})^2}$$
$$|\vec{v}_{2,f}| = \boxed{0.255}\text{ m/s}$$

2. GRR1 7.P.048. A projectile of mass $\boxed{2.0}$ kg approaches a stationary target body at $\boxed{4.8}$ m/s. The projectile is deflected through an angle of $\boxed{59.7}^\circ$ and its speed after the collision is $\boxed{2.8}$ m/s. What is the magnitude of the momentum of the target body after the collision?

This problem is very similar to the last, but this time the masses are different. Assuming the projectile only moves initially in the x-direction, we can start with the relationships for the momenta we obtained above. Namely,

$$p_{2x,f} = p_{1x,f} - p_{1x,i}$$

and

$$p_{2y,f} = -p_{1y,f}$$

The magnitude of the momtentum of the target body after the collision is then given by

$$|\vec{p}_{2,f}| = \sqrt{(p_{2x,f})^2 + (p_{2y,f})^2}$$
$$|\vec{p}_{2,f}| = \sqrt{(p_{1x,f} - p_{1x,i})^2 + (-p_{1y,f})^2}$$
$$|\vec{p}_{2,f}| = \sqrt{(p_{1x,f} - p_{1x,i})^2 + (p_{1y,f})^2}$$
$$|\vec{p}_{2,f}| = \sqrt{(m_1 v_{1x,f} - m_1 v_{1x,i})^2 + (m_1 v_{1y,f})^2}$$
$$|\vec{p}_{2,f}| = m_1\sqrt{(v_{1,f}\cos\theta_1 - v_{1,i})^2 + (v_{1,f}\sin\theta_1)^2}$$

$$|\vec{p}_{2,f}| = \boxed{2.0}\text{ kg}\sqrt{((\boxed{2.8}\text{ m/s})\cdot\cos\boxed{59.7^\circ} - \boxed{4.8}\text{ m/s})^2 + ((\boxed{2.8}\text{ m/s}\cdot\sin\boxed{59.7^\circ})^2}$$
$$|\vec{p}_{2,f}| = \boxed{8.32}\text{ kg}\cdot\text{m/s}$$

3. GRR1 7.P.050. A firecracker is tossed straight up into the air. It explodes into three pieces of equal mass just as it reaches the highest point. Two pieces move off at $\boxed{130}$ m/s at right angles to each other. How fast is the third piece moving?

Let's say that the first piece goes in the x-direction, and the second goes in the y-direction. Conservation of momentum implies that the momentum of the first piece is equal to the momentum in the x-direction of the third piece, and the momtentum of the second piece is equal to the momentum in th y-direction of the third piece. That is,

$$|\vec{p}_{3,f}| = \sqrt{(p_{3x,f})^2 + (p_{3y,f})^2}$$
$$|\vec{p}_{3,f}| = \sqrt{(-p_{1x,f})^2 + (-p_{2y,f})^2}$$
$$|\vec{p}_{3,f}| = \sqrt{(p_{1x,f})^2 + (p_{2y,f})^2}$$

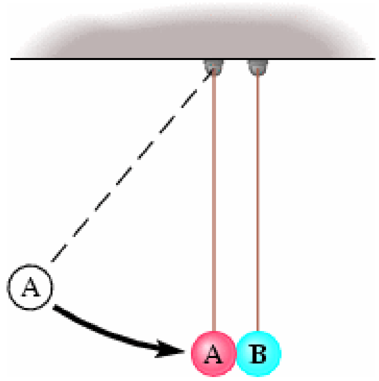
Substituting in the masses gives

$$m|\vec{v}_{3,f}| = \sqrt{(mv_{1x,f})^2 + (mv_{2y,f})^2}$$
$$m|\vec{v}_{3,f}| = m\sqrt{(v_{1x,f})^2 + (v_{2y,f})^2}$$
$$|\vec{v}_{3,f}| = \sqrt{(v_{1x,f})^2 + (v_{2y,f})^2}$$

Since $v_{1x,f} = v_{2y,f}$, we can just say that $v_{1x,f} = v_{2y,f} = v$. The magnitude of the final velocity of the third piece is then

$$|\vec{v}_{3,f}| = \sqrt{(v_{1x,f})^2 + (v_{2y,f})^2}$$
$$|\vec{v}_{3,f}| = \sqrt{v^2 + v^2}$$
$$|\vec{v}_{3,f}| = v\sqrt{2}$$
$$|\vec{v}_{3,f}| = \boxed{130}\text{ m/s}\sqrt{2}$$
$$|\vec{v}_{3,f}| = \boxed{184.}\text{ m/s}$$

4. GRR1 7.TB.066. The pendulum bobs in the figure below are made of soft clay so that they stick together after impact. The mass of bob A is half that of bob B. Bob B is initially at rest. What is the ratio of the kinetic energy of the combined bobs, just after impact, to the kinetic energy of bob A just before impact?



The kinetic energy before the collision is given by

$$K_i = \frac{p_i^2}{2m_A}$$

where p_i is the magnitude of the initial momentum and m_A is the mass of bob A. After the collision, the kinetic energy is

$$K_f = \frac{p_f^2}{2(m_A + m_B)}$$

Since momentum is conserved we know that

$$\vec{p_i} = \vec{p_f}$$
$$(p_i)^2 = (p_f)^2$$

Therefore the final kinetic energy can be written in terms of the initial momentum.

$$K_f = \frac{p_i^2}{2(m_A + m_B)}$$

The ratio of final kinetic energy to the initial kinetic energy is then

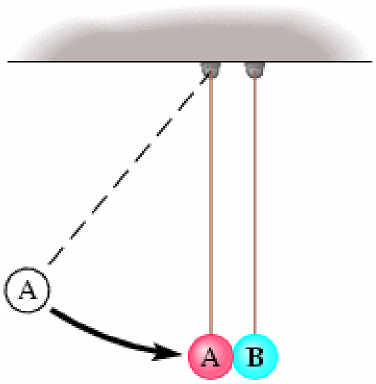
$$\frac{K_f}{K_i} = \frac{p_i^2/2(m_A + m_B)}{p_i^2/2m_A}$$
$$\frac{K_f}{K_i} = \frac{2m_A}{2(m_A + m_B)}$$
$$\frac{K_f}{K_i} = \frac{m_A}{m_A + m_B}$$

Since $m_A = \frac{1}{2}m_B$, we find the ratio to be

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}m_B}{\frac{1}{2}m_B + m_B}$$

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}m_B}{m_B(\frac{1}{2}+1)}$$
$$\frac{K_f}{K_i} = \frac{\frac{1}{2}}{\frac{3}{2}}$$
$$\frac{K_f}{K_i} = \frac{1}{3}$$

5. GRR1 7.TB.067. The pendulum bobs in the figure below are made of soft clay so that they stick together after impact. The mass of bob A is half that of bob B. Bob B is initially at rest. If bob A is released from a height h above its lowest point, what is the maximum height attained by bobs A and B after the collision?



The initial energy of bob A is

$$E = m_A gh$$

This energy is converted completely to kinetic energy at the bottom of the swing. So the kinetic energy of bob A just before it hits bob B is

$$K_i = m_A gh$$

After the two stick together, they will have kinetic energy which we will call K_f. This kinetic energy will then be completely converted to potential energy when the two bobs swing together to a height of h'. The height, h', is then related to the kinetic energy just after the collision by the equation

$$K_f = (m_A + m_B)gh'$$

We can now use the result of the last problem, and get

$$\frac{K_f}{K_i} = \frac{m_A}{m_A + m_B}$$
$$\frac{(m_A + m_B)gh'}{m_A gh} = \frac{m_A}{m_A + m_B}$$

Solving for the final height, h', we get

$$\begin{aligned}\frac{(m_A+m_B)gh'}{m_Agh} &= \frac{m_A}{m_A+m_B}\\ \frac{h'}{h} &= \left(\frac{m_A}{m_A+m_B}\right)^2\\ h' &= h\cdot\left(\frac{m_A}{m_A+m_B}\right)^2\end{aligned}$$

We found above that this mass ratio is equal to 1/3. Substituting this in, we find the final height to be

$$\begin{aligned}h' &= h\cdot\left(\frac{1}{3}\right)^2\\ h' &= h\cdot\frac{1}{9}\end{aligned}$$

6. GRR1 7.TB.070. Two identical gliders, each with elastic bumpers and mass 0.10 kg, are on a horizontal air track. Friction is negligible. Glider 2 is stationary. Glider 1 moves toward glider 2 from the left with a speed of 0.20 m/s. They collide. After the collision, what are the velocities of glider 1 and glider 2?

Starting with

$$\vec{p}_{i,tot} = \vec{p}_{f,tot}$$

we can use the properties of the system to simplify this equation. First, the air track confines the gliders to one dimension, so that the momentum equation can be rewritten as

$$p_{1x,i}+p_{2x,i} = p_{1x,f}+p_{2x,f}$$

The second glider is initially at rest so we can write

$$\begin{aligned}p_{1x,i}+0 &= p_{1x,f}+p_{2x,f}\\ p_{1x,i} &= p_{1x,f}+p_{2x,f}\end{aligned}$$

The masses are identical so that we have

$$\begin{aligned}mv_{1x,i} &= mv_{1x,f}+mv_{2x,f}\\ v_{1x,i} &= v_{1x,f}+v_{2x,f}\end{aligned}$$

We also know that energy is conserved in elastic collisions. The energy of the system before the collision is

$$K_i = \frac{1}{2}mv_{1x,i}^2$$

The energy of the system after the collision is just the sum of the energies of the two gliders. So that we have

$$K_f = \frac{1}{2}mv_{1x,f}^2 + \frac{1}{2}mv_{2x,f}^2$$

Since energy is conserved in elastic collisions, $K_i = K_f$. Starting with this and substituting in the result from conservation of momentum gives us

$$\begin{aligned}
K_i &= K_f \\
\frac{1}{2}mv_{1x,i}^2 &= \frac{1}{2}mv_{1x,f}^2 + \frac{1}{2}mv_{2x,f}^2 \\
v_{1x,i}^2 &= v_{1x,f}^2 + v_{2x,f}^2 \\
(v_{1x,f} + v_{2x,f})^2 &= v_{1x,f}^2 + v_{2x,f}^2 \\
v_{1x,f}^2 + v_{2x,f}^2 + 2v_{1x,f}v_{2x,f} &= v_{1x,f}^2 + v_{2x,f}^2 \\
2v_{1x,f}v_{2x,f} &= 0
\end{aligned}$$

This can only be true if $v_{1x,f}$ or $v_{2x,f}$ is zero. Since we know we observed a collision, and the first glider cannot travel through the second glider, we conclude that the final velocity of the first glider is zero after the collision. That is $v_{1x,f} = 0$. Using the result from the conservation of momentum, the velocity of the second glider is then

$$\begin{aligned}
v_{1x,i} &= 0 + v_{2x,f} \\
v_{1x,i} &= v_{2x,f}
\end{aligned}$$

After the collision, the first glider is stationary and the second glider moves off at a velocity equal to that of the first glider before the collision.