Solutions to Homework Set #6

Phys2414 - Fall 2005

Please note: The numbers in the $\boxed{\text{boxes}}$ correspond to those that are generated by WebAssign. The numbers on your individual assignment will vary. Any calculated quantities that involve these variable numbers will be $\boxed{\text{boxed}}$ as well.

1. GRR1 5.P.023. A car drives around a curve with radius $\boxed{410}$ m at a speed of $\boxed{32}$ m/s. The road is banked at $\boxed{5.0}$ degrees. The mass of the car is $\boxed{1400}$ kg. (a) What is the frictional force on the car? (b) At what speed could you drive around this curve so that the force of friction is zero? *Let the x-axis point toward the center of curvature and the y-axis point upward. Use Newton's second law.*

(a)

$$\Sigma F_y = N\cos\theta - mg - f\sin\theta = 0$$

$$\Sigma F_x = N\sin\theta + f\cos\theta = m\frac{v^2}{r}$$

Solve for N in the first equation substitute in the second,

$$N = \frac{f\sin\theta + mg}{\cos\theta}$$

so:

$$\begin{aligned}
\frac{f\sin\theta + mg}{\cos\theta}\sin\theta + f\cos\theta &= m\frac{v^2}{r} \\
f\sin^2\theta + mg\sin\theta + f\cos^2\theta &= m\frac{v^2}{r}\cos\theta \\
f(\sin^2\theta + \cos^2\theta) &= m\frac{v^2}{r}\cos\theta - mg\sin\theta \\
f(1) &= m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right) \\
f &= (\boxed{1400}\,kg)\left[\frac{\left(\boxed{32}\frac{m}{s}\right)^2}{\boxed{410}\,m}\cos\boxed{5.0} - \left(9.8\frac{m}{s^2}\right)\sin\boxed{5.0}\right] \\
&= \boxed{2300}\,N
\end{aligned}$$

(b)

$$f = 0 = m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)$$

$$\begin{aligned}
\frac{v^2}{r}\cos\theta &= \sin\theta \\
v^2 &= gr\tan\theta \\
v &= \sqrt{gr\tan\theta} \\
&= \sqrt{\left(9.8\frac{m}{s^2}\right)(\boxed{410}\,m)\tan\boxed{5.0}} \\
&= \boxed{19}\,N
\end{aligned}$$

2. GRR1 5.P.027. Two satellites are in circular orbits around Jupiter. One, with orbital radius r, makes one revolution every $\boxed{17}$ h. The other satellite has orbital radius $\boxed{4.0}$r. How long does the second satellite take to make one revolution around Jupiter?
According to the Kepler's third law, $r^3 \propto T^2$.

$$\begin{aligned}
\left(\frac{\boxed{4.0}r}{r}\right)^3 &= \left(\frac{T_{4.0}}{T}\right)^2 \\
64 &= \frac{T_{4.0}^2}{T^2} \\
T_{4.0}^2 &= 64T^2 \\
T_{4.0} &= 8T \\
&= 8(\boxed{16}h) \\
&= \boxed{128}h
\end{aligned}$$

3. GRR1 5.P.030. A satellite travels around Earth in uniform circular motion at altitude $\boxed{35800}$ km above Earth's surface. The satellite is in geosynchronous orbit (that is, the time for it to complete one orbit is exactly one day). In the figure 5.18, the satellite moves counterclockwise (ABCDA). State directions in terms of the x- and y-axes. (a) What is the satellite's instantaneous velocity at point C? (b) What is the satellite's average velocity for one quarter of an orbit, starting at A and ending at B? (c) What is the satellite's average acceleration for one quarter of an orbit, starting at A and ending at B?
(a)

$$v = \frac{2\pi r}{T} = \frac{2\pi(\boxed{35800}km + 6371km)}{86400s} = 3.07km/s$$

and $\vec{\boldsymbol{v}} = \boxed{\textit{3.07 km/s in the -y-direction}}$ *at point C.*
(b)

$$|\vec{\boldsymbol{v}}_{av}| = \frac{|\Delta\vec{\boldsymbol{r}}|}{\Delta t} = \frac{r\sqrt{2}}{\frac{T}{4}} = \frac{4r\sqrt{2}}{T} = \frac{4(\boxed{35800}km + 6371km)\sqrt{2}}{86400s} = 2.76km$$

$\vec{\boldsymbol{v}}_{av}$ is in the same direction as $\Delta\vec{\boldsymbol{r}}$, which is 45 degrees above the -x-axis, so $\vec{\boldsymbol{v}}_{av} = \boxed{\textit{2.76 km/s at 45 deg. above the -x-axis}}$.
(c) $\vec{\boldsymbol{a}}_{av} = \frac{\Delta\vec{\boldsymbol{v}}}{\Delta t}$, so the average acceleration is in the same direction as $\Delta\vec{\boldsymbol{v}} = \vec{\boldsymbol{v}}_B - \vec{\boldsymbol{v}}_A$, which is 45 degrees below the -x-axis.

$$|\Delta\vec{\boldsymbol{v}}| = \sqrt{[(\Delta\vec{\boldsymbol{v}})_x]^2 + [(\Delta\vec{\boldsymbol{v}})_y]^2} = \sqrt{v^2 + v^2} = v\sqrt{2}$$

$$\frac{|\Delta\vec{\boldsymbol{v}}|}{\Delta t} = \frac{v\sqrt{2}}{\frac{T}{4}} = \frac{4v\sqrt{2}}{T} = \frac{4\sqrt{2}\left(3.07 \times 10^3 \frac{m}{s}\right)}{86400s} = 0.201m/s^2$$

So, $\vec{\boldsymbol{a}}_{av} = \boxed{\textit{0.201 } m/s^2 \textit{ at 45 degrees below the -x-axis}}$.
(d) The instantaneous acceleration at point D is in the +y-direction, since the acceleration is always directed radially inward for uniform circular motion and its magnitude is a_c. Use Newton's second law the law of universal gravitation:

$$\Sigma F_c = \frac{GmM_E}{r^2} = ma_c$$

$$a_c = \frac{GM_E}{r^2} = \frac{\left(6.673 \times 10^{-11} \frac{N.m^2}{kg^2}\right)(5.975 \times 10^{24} kg)}{(\boxed{35800} \times 10^3 + 6371 \times 10^3 kg)^2} = 0.224$$

So, $\vec{a} = \boxed{0.224\ m/s^2 \textit{ in the +y-direction}}$.

4. GRR1 5.P.031. A spacecraft is in orbit around Jupiter. The radius of the orbit is $\boxed{3.0}$ times the radius of Jupiter (which is $R_J = \boxed{71500}$ km). The gravitational field at the surface of Jupiter is $\boxed{23}$ N/kg. What is the period of the spacecraft's orbit? [Hint: You don't need to look up any more data about Jupiter to solve the problem.]
Use Newton's second law and the law of universal gravitation.

$$\Sigma F_c = \frac{GmM_J}{(\boxed{3.0}R_J)^2}$$

Now $g_J \frac{GM_J}{R_J^2}$*, so:*

$$\begin{aligned}
\frac{mv^2}{\boxed{3.0}R_J} &= \frac{mg_J}{9.0} \\
v^2 &= \frac{g_J R_J}{\boxed{3.0}} \\
\left(\frac{2\pi r}{T}\right)^2 &= \\
\frac{4\pi^2(\boxed{3.0}R_J)^2}{T^2} &= \\
\frac{4\pi^2(27R_J)}{g_J} &= T^2 \\
T &= 2\pi\sqrt{\frac{27R_J}{g_J}} \\
&= 2\pi\sqrt{\frac{27(\boxed{71500} \times 10^3 m)}{\boxed{23}\frac{N}{kg}}}\left(\frac{1h}{3600s}\right) \\
&= \boxed{16h}
\end{aligned}$$

5. GRR1 5.P.032. A roller coaster has a vertical loop with radius $\boxed{20.0}$ m. With what minimum speed should the roller coaster car be moving at the top of the loop so that the passengers do not lose contact with the seats?
Use Newton's second law.

$$\Sigma F_c = N + mg = ma_c = \frac{mv_{top}^2}{r}$$

So:

$$N = \frac{mv_{top}^2}{r} - mg$$

The normal force must be greater than or equal to zero so that the passengers do not lose contact *with their seats. The minimum speed is found when $N = 0$.*

$$\begin{aligned} 0 &= \frac{mv_{top}^2}{r} - mg \\ \frac{v_{top}^2}{r} &= g \\ v_{top} &= \sqrt{gr} \\ &= \sqrt{\left(9.8\frac{m}{s^2}\right)\left(\boxed{20.0}m\right)} \\ &= \boxed{14}m/s \end{aligned}$$

6. GRR1 5.P.057. What's the fastest way to make a U-turn at constant speed? Suppose that you need to make a 180-degree turn on a circular path. The minimum radius (due to the car's steering system) is $\boxed{5.0}$ m, while the maximum (due to the width of the road) is $\boxed{20.0}$ m. Your acceleration must never exceed $\boxed{3.0}$ m/s^2 or else you will skid. Should you use the smallest possible radius, so the distance is small, or the largest, so you can go faster without skidding? What is the minimum possible time for this U-turn?
Distance $\pi r = vt$ and $a_c = \frac{v^2}{r}$, so $v = \sqrt{a_c r}$

$$t = \frac{\pi r}{v} = \frac{\pi r}{\sqrt{a_c r}} = \pi\sqrt{\frac{r}{a_c}}$$

so the larger the radius the greater the time to complete the U-turn. Now the smallest possible radius should be used:

$$t_{min} = \pi\sqrt{\frac{\boxed{5.0}m}{\boxed{3.0}\frac{m}{s^2}}} = \boxed{4.1}s$$

7. GRR1 5.P.060. A coin is placed on a record that is rotating at $\boxed{33.3}$ rpm. If the coefficient of static friction between the coin and the record is $\boxed{0.1}$, how far from the center of the record can the coin be placed without having it slip off?
Use Newton's second law:
$\Sigma F_c = f = ma_c$
$\Sigma F_y = N - mg = 0$

$$\begin{aligned} f &= ma_c \\ \mu mg &= m\omega^2 r \\ \mu g &= \omega^2 r \\ r &= \frac{\mu g}{\omega^2} \\ &= \frac{\boxed{0.1}\left(9.8\frac{m}{s^2}\right)}{\left(\boxed{33.3}\frac{rev}{min}\right)^2\left(\frac{2\pi rad}{rev}\right)^2\left(\frac{1min}{60s}\right)^2} \\ &= \boxed{8}cm \end{aligned}$$

8. GRR1 5.P.061. Grace gives her dolls a merry-go-round ride on an old phonograph set at 33.3 rpm. The dolls are $\boxed{5.0}$ in. from the central axis. She changes the setting to $\boxed{45}$ rpm. (a) For this

new setting, what is the linear speed of a point on the turntable at the location of the dolls? (b) If the coefficient of static friction between the dolls and the turntable is $\boxed{0.13}$, do the dolls stay on the record?

(a)

$$v = \omega r = \left(\frac{45rev}{min}\right)\left(\frac{2\pi\, rad}{rev}\right)\left(\frac{1min}{60s}\right)(5.0in.)\left(\frac{0.0254m}{in.}\right) = \boxed{0.60}\, m/s$$

(b) Use Newton's second law:

$\Sigma F_c = f \geq ma_c$

$\Sigma F_y = N - mg = 0$

$$\begin{aligned} f &\geq ma_c \\ \mu mg &\geq m\omega^2 r \\ \mu g &\geq \omega^2 r \\ \boxed{0.13}\left(9.8\frac{m}{s^2}\right) &\geq \left(\frac{45rev}{min}\right)^2\left(\frac{2\pi\, rad}{rev}\right)^2\left(\frac{1min}{60s}\right)^2(5.0in.)\left(\frac{0.0254m}{in.}\right) \\ 1.3\frac{m}{s^2} &\geq 2.8\frac{m}{s^2}\, False! \end{aligned}$$

$\boxed{\textit{The dolls do not stay on the record.}}$

9. GRR1 5.TB.034. A pendulum (the figure below) is 0.800 m long and the bob has a mass of 1.00 kg. When the string makes an angle of θ= 15.0 with the vertical, the bob is moving at 1.40 m/s. Let g = 9.80 m/s2. Find the tangential and centripetal acceleration components and the tension in the string. [Hint: Draw an FBD for the bob. Choose the x-axis to be tangential to the motion of the bob and the y-axis to be centripetal. Apply Newton's second law.]

Use Newton's second law:

$\Sigma F_c = T - mg\cos\theta = ma_c$

$\Sigma F_t = mg\sin\theta = ma_t$

$$a_t = g\sin\theta = \left(9.80\frac{m}{s^2}\right)\sin 15.0 = \boxed{2.54}\, m/s^2$$

$$a_c = \frac{v^2}{r} = \frac{\left(1.40\frac{m}{s}\right)^2}{0.800m} = \boxed{2.45}\, m/s^2$$

$$T = m(a_c + g\cos\theta) = (100kg)\left[2.45\frac{m}{s^2} + \left(9.80\frac{m}{s^2}\right)\cos 15.0\right] = \boxed{11.9}\, N$$