

Solutions to Homework Set #5

Phys2414 – Fall 2005

Note: The numbers in the boxes correspond to those that are generated by WebAssign. The numbers on your individual assignment will vary. Any calculated quantities that involve these variable numbers will be boxed as well.

1. GRR1 5.P.002. A soccer ball of diameter 31 cm rolls without slipping at a linear speed of 2.6 m/s. Through how many revolutions has the soccer ball turned as it moves a linear distance of 16 m?

The distance the ball travels, x , is equal to the circumference of the ball, C , times the number of revolutions, α , it turns.

$$x = C \cdot \alpha$$

The circumference of a sphere at its equator with diameter, d , is πd . The number of revolutions the ball turned is then

$$\begin{aligned} x &= \pi d \cdot \alpha \\ \alpha &= \frac{x}{\pi d} \\ \alpha &= \frac{\text{16 m}}{\pi \text{0.31 m}} \\ \alpha &= \text{16.4 rev} \end{aligned}$$

2. GRR1 5.P.006. An elevator cable winds on a drum of radius 70.0 cm that is connected to a motor.

(a) If the elevator is moving down at 0.60 m/s, what is the angular speed of the drum?

The speed of the elevator must be the speed of the rope as it leaves the drum. Since the rope is unspooling from the drum, this is also the speed of the drum at its edge. The equation which relates angular speed with linear speed is

$$v = \omega r$$

The angular speed of the drum is then

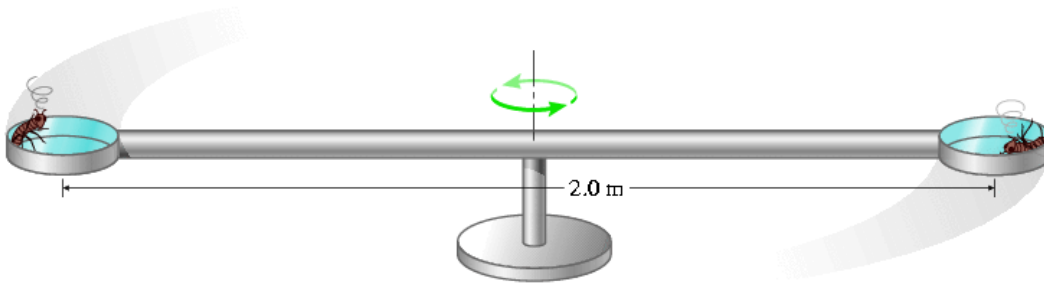
$$\begin{aligned} \omega &= \frac{v}{r} \\ \omega &= \frac{\text{0.60 m/s}}{\text{0.700 m}} \\ \omega &= 0.86 \text{ rad/s} \end{aligned}$$

(b) If the elevator moves down 9.0 m, how many revolutions has the drum made?

We may use the relationship we found in problem 1 to find the number of revolutions made.

$$\begin{aligned}x &= \pi d \cdot \alpha \\ \alpha &= \frac{x}{\pi d} \\ \alpha &= \frac{9.0 \text{ m}}{\pi(2 \cdot 0.700 \text{ m})} \\ \alpha &= 2.0 \text{ rev}\end{aligned}$$

3. GRR1 5.P.011. The apparatus of the figure below is designed to study insects at an acceleration of magnitude 960 m/s^2 ($= 98g$). The apparatus consists of a 2.0 m rod with insect containers at either end. The rod rotates about an axis perpendicular to the rod and at its center.



(a) How fast does an insect move when it experiences a centripetal acceleration of 960 m/s^2 ?

Using the relationship between centripetal acceleration, linear speed, and radius, we can find the linear speed of the insect as it swings around in this device.

$$\begin{aligned}a_c &= \frac{v^2}{r} \\ v &= \sqrt{a_c r} \\ v &= \sqrt{960 \text{ m/s}^2 \left(\frac{1}{2} \cdot 2.0 \text{ m} \right)} \\ v &= 31.0 \text{ m/s}\end{aligned}$$

(b) What is the angular speed of the insect?

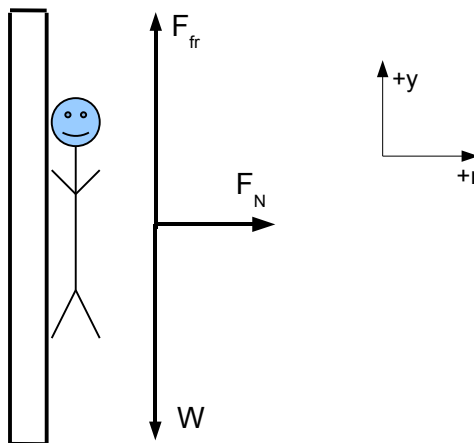
The angular speed can be calculated using the cousin of the relationship above.

$$\begin{aligned}a_c &= \omega^2 r \\ \omega &= \sqrt{\frac{a_c}{r}} \\ \omega &= \sqrt{\frac{960 \text{ m/s}^2}{1/2 \cdot 2.0 \text{ m}}} \\ \omega &= 31.0 \text{ rad/s}\end{aligned}$$

4. GRR1 5.P.012. The rotor (the figure below) is an amusement park ride where people stand against the inside of a cylinder. Once the cylinder is spinning fast enough, the floor drops out.



(a) What force keeps the people from falling out the bottom of the cylinder?
The free-body diagram of a person in the "rotor" looks like the following figure.



The force that acts against the weight of the person is the friction force on the person by the wall. Since the person is not sliding, we know it's static friction that is holding the person up.

(b) If the coefficient of friction is 0.42 and the cylinder has a radius of 2.3 m, what is the minimum angular speed of the cylinder so that the people don't fall out? (Normally the operator runs it considerably faster as a safety measure.)

The sums of the forces in the y and r directions are

$$\Sigma F_y = F_{fr} - W = 0$$

$$\Sigma F_r = F_N = ma_c$$

where a_c is the centripetal acceleration. The minimum angular speed of the cylinder would occur when the maximum frictional force is exactly equal to the weight of the person. Using the equation for static friction, $F_{fr} = \mu_s F_N$, we can combine the two above equations to get

$$\begin{aligned} F_{fr} &= W \\ \mu_s F_N &= mg \\ \mu_s (ma_c) &= mg \\ \mu_s a_c &= g \end{aligned}$$

Recognizing that the centripetal acceleration may be expressed as $a_c = \omega^2 r$, we can rewrite the last equation as

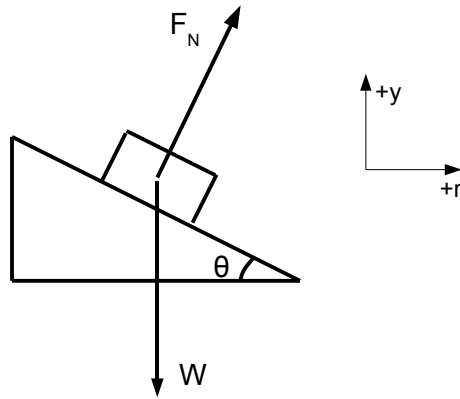
$$\begin{aligned} \mu_s \omega^2 r &= g \\ \omega^2 &= \frac{g}{\mu_s r} \\ \omega &= \sqrt{\frac{g}{\mu_s r}} \end{aligned}$$

The minimum angular velocity is then

$$\begin{aligned} \omega &= \sqrt{\frac{9.8 \text{ m/s}^2}{0.42 \cdot 2.3 \text{ m}}} \\ \omega &= 3.2 \text{ rad/s} \end{aligned}$$

5. GRR1 5.P.018. A highway curve has a radius of 112 m. At what angle should the road be banked so that a car traveling at 26.3 m/s has no tendency to skid sideways on the road? [Hint: No tendency to skid means the frictional force is zero.]

The free-body diagram for this situation looks like the following figure.



Notice that one of the coordinate axes is pointed in the direction of the acceleration. The sums of the forces in the y and r directions are

$$\begin{aligned}\Sigma F_y &= F_N \cos \theta - W = 0 \\ \Sigma F_r &= F_N \sin \theta = ma_c\end{aligned}$$

Solving for F_N in the first equation gives

$$\begin{aligned}F_N \cos \theta &= W \\ F_N \cos \theta &= mg \\ F_N &= \frac{mg}{\cos \theta}\end{aligned}$$

We can substitute this expression for F_N into the second equation above. This gives

$$\begin{aligned}F_N \sin \theta &= ma_c \\ \left(\frac{mg}{\cos \theta}\right) \sin \theta &= ma_c \\ mg \frac{\sin \theta}{\cos \theta} &= ma_c \\ \frac{\sin \theta}{\cos \theta} &= \frac{a_c}{g}\end{aligned}$$

The fraction on the left is equal to $\tan \theta$. We can also express a_c in terms of the linear speed, v , and the radius, r . We can now solve for theta.

$$\begin{aligned}\tan \theta &= \frac{v^2}{gr} \\ \theta &= \arctan \left(\frac{v^2}{gr} \right)\end{aligned}$$

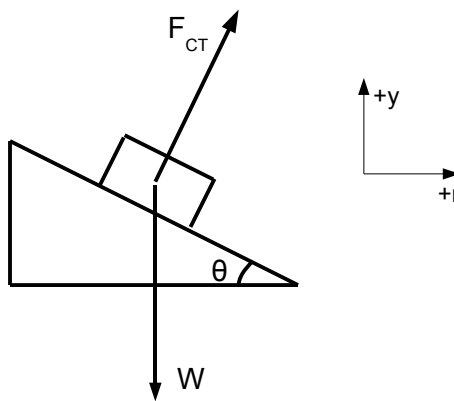
The angle at which the car does not skid sideways is then

$$\theta = \arctan\left(\frac{(\boxed{26.3} \text{ m/s})^2}{9.8 \text{ m/s}^2 \boxed{112} \text{ m}}\right)$$

$$\theta = \boxed{32.2}^\circ$$

6. GRR1 5.P.020. A roller coaster car of mass $\boxed{330}$ kg (including passengers) travels around a horizontal curve of radius $\boxed{37}$ m. Its speed is $\boxed{14}$ m/s. What is the magnitude and direction of the total force exerted on the car by the track?

The free-body diagram for this situation is very similar to that of the last problem. In this case we will consider the only two forces acting on the car, i.e. the force on the car by the track, \vec{F}_{CT} , and the force of gravity on the car, \vec{W} .



The sums of the forces in the y and r directions are then

$$\Sigma F_y = F_{CT,y} - W = 0$$

$$\Sigma F_r = F_{CT,r} = ma_c$$

The magnitude of the force on the car by the track is given by

$$|\vec{F}_{CT}| = \sqrt{(F_{CT,r})^2 + (F_{CT,y})^2}$$

Using the first two equations we know that $F_{CT,r}$ and $F_{CT,y}$ are equal to

$$F_{CT,r} = ma_c, \quad F_{CT,y} = mg$$

The magnitude is then

$$|\vec{F}_{CT}| = \sqrt{(ma_c)^2 + (mg)^2}$$

$$|\vec{F}_{CT}| = m\sqrt{a_c^2 + g^2}$$

$$|\vec{F}_{CT}| = m\sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$$

The angle above horizontal is then given by

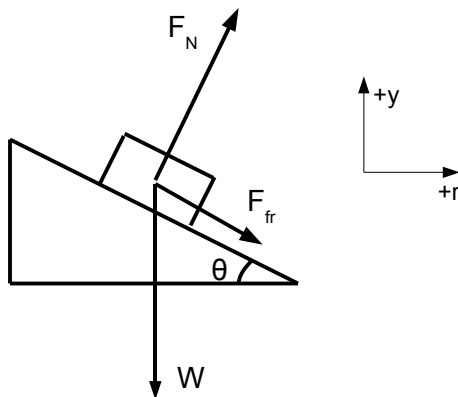
$$\begin{aligned}\tan \theta &= \frac{F_{CT,y}}{F_{CT,x}} \\ \tan \theta &= \frac{mg}{ma_c} \\ \tan \theta &= \frac{g}{a_c} \\ \tan \theta &= \frac{gr}{v^2} \\ \theta &= \arctan\left(\frac{gr}{v^2}\right)\end{aligned}$$

The magnitude and direction are then

$$\begin{aligned}|\vec{F}_{CT}| &= 330 \text{ kg} \sqrt{\left(\frac{(14 \text{ m/s})^2}{37 \text{ m}}\right)^2 + (9.8 \text{ m/s}^2)^2} \\ |\vec{F}_{CT}| &= 3700 \text{ N} \\ \theta &= \arctan\left(\frac{9.8 \text{ m/s}^2 (37 \text{ m})}{(14 \text{ m/s})^2}\right) \\ \theta &= 61.6^\circ\end{aligned}$$

7. GRR1 5.P.023. A car drives around a curve with radius 420 m at a speed of 33 m/s . The road is banked at 5.2° . The mass of the car is 1600 kg .

(a) What is the frictional force on the car? This problem is similar to problem 5, but this time there is a frictional force. The corresponding free-body diagram is given in the figure below.



The sums of the forces in the y and r directions are then

$$\begin{aligned}\Sigma F_y &= F_N \cos \theta - F_{fr} \sin \theta - W = 0 \\ \Sigma F_r &= F_N \sin \theta + F_{fr} \cos \theta = ma_c\end{aligned}$$

We would like to solve for the frictional force to get the answer. Comparing the two equations above we can see that both equations have F_N as the first term, but one is multiplied by $\cos \theta$ while the other is multiplied by $\sin \theta$. In order to solve this system of equations, we have to eliminate F_N . To do this we can multiply the first equation by $\sin \theta$ and the second equation by $\cos \theta$. Doing this gives the following two equations

$$\begin{aligned}F_N \cos \theta \sin \theta - F_{fr} \sin^2 \theta &= W \sin \theta \\ F_N \cos \theta \sin \theta + F_{fr} \cos^2 \theta &= ma_c \cos \theta\end{aligned}$$

Next we can multiply the first equation by -1 and add the equations together

$$\begin{aligned}-F_N \cos \theta \sin \theta + F_{fr} \sin^2 \theta &= -W \sin \theta \\ F_N \cos \theta \sin \theta + F_{fr} \cos^2 \theta &= ma_c \cos \theta\end{aligned}$$

Adding the equations together gives

$$F_{fr} \cos^2 \theta + F_{fr} \sin^2 \theta = ma_c \cos \theta - W \sin \theta$$

Factoring out F_{fr} on the left side we find

$$F_{fr}(\cos^2 \theta + \sin^2 \theta) = ma_c \cos \theta - W \sin \theta$$

The quantity in parentheses is a trigonometric identity and is equal to one. Finally we have

$$\begin{aligned}F_{fr} &= ma_c \cos \theta - W \sin \theta \\ F_{fr} &= m \frac{v^2}{r} \cos \theta - mg \sin \theta \\ F_{fr} &= m \left(\frac{v^2}{r} \cos \theta - g \sin \theta \right)\end{aligned}$$

The force of friction is then equal to

$$\begin{aligned}F_{fr} &= \boxed{1600} \text{ kg} \left(\frac{(\boxed{33} \text{ m/s})^2}{\boxed{420} \text{ m}} \cos(\boxed{5.2}^\circ) - 9.8 \text{ m/s}^2 \sin(\boxed{5.2}^\circ) \right) \\ F_{fr} &= \boxed{2710} \text{ N}\end{aligned}$$

(b) At what speed could you drive around this curve so that the force of friction is zero?

This speed may be found by setting the frictional force in our above expression equal to zero.

$$\begin{aligned}0 &= m \left(\frac{v^2}{r} \cos \theta - g \sin \theta \right) \\0 &= \frac{v^2}{r} \cos \theta - g \sin \theta\end{aligned}$$

Solving for v we get

$$\begin{aligned}\frac{v^2}{r} \cos \theta &= g \sin \theta \\v^2 &= gr \frac{\sin \theta}{\cos \theta} \\v &= \sqrt{gr \tan \theta}\end{aligned}$$

The velocity at which no frictional force occurs is then

$$\begin{aligned}v &= \sqrt{(9.8 \text{ m/s}^2)(420 \text{ m}) \tan(5.2^\circ)} \\v &= 19.6 \text{ m/s}\end{aligned}$$