Solutions to Homework Set #3
Phys2414 – Fall 2005

Note: The numbers in the $\boxed{\text{boxes}}$ correspond to those that are generated by WebAssign. The numbers on your individual assignment will vary. Any calculated quantities that involve these variable numbers will be $\boxed{\text{boxed}}$ as well.

1. GRR1 3.P.001. Two cars, a Porsche and a Honda, are traveling in the same direction, although the Porsche is $\boxed{186}$ m behind the Honda. The speed of the Porsche is $\boxed{24.2}$ m/s and the speed of the Honda is $\boxed{19.0}$ m/s. How much time does it take for the Porsche to catch the Honda? [Hint: What must be true about the displacements of the two cars when they meet?]

If we call the time when the cars are $\boxed{186}$ m apart t_i, and the time when the Porshe catches the Honda t_f, the equation of motion for the Porshe is given by

$$x_{f,p} - x_{f,p} = v_p\,(t_f - t_i)$$

where $x_{f,p}$ and $x_{i,p}$ are the positions of the Porsche at times t_f and t_i, respectively, and v_p is the speed of the Porshe. Likewise the equation of motion for the Honda is

$$x_{f,h} - x_{i,h} = v_h\,(t_f - t_i)$$

The hint for this problem wants us to recognize that at the time t_f that the displacement of the cars is zero. In terms of our variables, this would be

$$x_{f,h} - x_{f,p} = 0$$

We can use this property if we subtract the equation of motion of the Porsche from the equation of motion of the Honda. Doing this gives us

$$(x_{f,h} - x_{i,h}) - (x_{f,p} - x_{i,p}) = [v_h\,(t_f - t_i)] - [v_p\,(t_f - t_i)]$$

Rearranging the terms we get

$$(x_{f,h} - x_{f,p}) - (x_{i,h} - x_{i,p}) = (v_h - v_p)\,(t_f - t_i)$$

The first set of parentheses is the property we would like to invoke. Doing so and solving for the change in time we have

$$-\,(x_{i,h} - x_{i,p}) = (v_h - v_p)\,(t_f - t_i)$$

$$-\frac{x_{i,h} - x_{i,p}}{v_h - v_p} = (t_f - t_i)$$

Letting $\Delta t = (t_f - t_i)$ be the change in time we have

$$\Delta t = -\frac{x_{i,h} - x_{i,p}}{v_h - v_p}$$

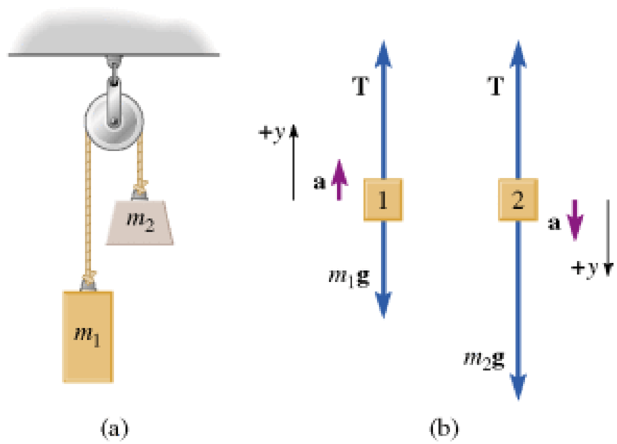
The quantity in the numerator is the initial displacement, and the quantity in the denominator is the difference in speeds of the two cars. The change in time is then

$$\Delta t = -\frac{\boxed{186}\text{ m}}{\boxed{19.0}\text{ m/s} - \boxed{24.2}\text{ m/s}}$$

$$\Delta t = \frac{\boxed{186}\text{ m}}{\boxed{5.2}\text{ m/s}}$$

$$\Delta t = \boxed{35.8}\text{ s}$$

2. GRR1 3.P.020. In the figure below (part a), two blocks are connected by a lightweight, flexible cord which passes over a frictionless pulley.



(a) If $m_1 = \boxed{3.2}$ kg and $m_2 = \boxed{5.4}$ kg, what are the accelerations of each block?
The net force on the two-block system is

$$F_{net} = -m_1 g + m_2 g$$

The acceleration of the blocks is then

$$\begin{aligned} F_{net} &= (m_1 + m_2)a \\ (m_1 + m_2)a &= -m_1 g + m_2 g \\ a &= \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g \\ a &= \left(\frac{\boxed{5.4}\text{ kg} - \boxed{3.2}\text{ kg}}{\boxed{5.4}\text{ kg} + \boxed{3.2}\text{ kg}}\right) 9.8\text{ m/s}^2 \\ a &= \boxed{2.51}\text{ m/s}^2 \end{aligned}$$

(b) What is the tension in the cord?
The net force on the first block must be equal to the tension of the cord minus its weight, i.e.

$$F_{net(1)} = T - m_1 g$$

The net force on the first block is also just its mass times the acceleration we found in part (a). Putting this information together we get

$$m_1 a = T - m_1 g$$

Solving for the tension gives us

$$T = m_1(a + g)$$
$$T = \boxed{3.2}\ \text{kg}\left(\boxed{2.5}\ \text{m/s}^2 + 9.8\ \text{m/s}^2\right)$$
$$T = \boxed{39.4}\ \text{N}$$

3. GRR1 3.P.036. The minimum stopping distance of a car moving at $\boxed{38.6}$ mi/h is $\boxed{12}$ m. Under the same conditions (so that the maximum braking force is the same), what is the minimum stopping distance for $\boxed{56.4}$ mi/h? Work by proportions to avoid converting units.

The kinematic equation that relates distance, velocity, and acceleration is

$$v_f^2 - v_i^2 = 2a(x_f - x_i)$$

where x_i and v_i are the initial position and velocity and x_f and v_f are the final postion and velocity with a being the constant acceleration. Solving for the acceleration we get

$$2a = \frac{v_f^2 - v_i^2}{x_f - x_i}$$

Since the maximum braking force is the same for both stops, we know that the right side of the above equation must be true for both stops. This means that

$$\frac{v_{a_f}^2 - v_{a_i}^2}{x_{a_f} - x_{a_i}} = \frac{v_{b_f}^2 - v_{b_i}^2}{x_{b_f} - x_{b_i}}$$

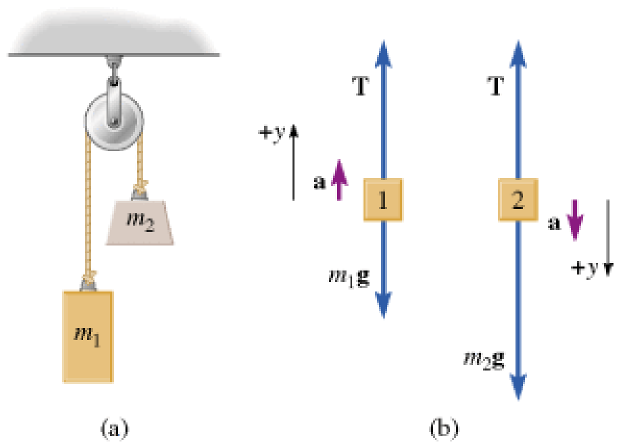
where the subscript a indicates the first stop, and b indicates the second stop. Letting each of the stopping distances equal a Δx and solving for the stopping distance of the second case, we get

$$\begin{aligned}\frac{v_{a_f}^2 - v_{a_i}^2}{\Delta x_a} &= \frac{v_{b_f}^2 - v_{b_i}^2}{\Delta x_b} \\ \Delta x_b &= \frac{v_{b_f}^2 - v_{b_i}^2}{v_{a_f}^2 - v_{a_i}^2}\Delta x_a\end{aligned}$$

The stopping distance in the second case is then

$$\begin{aligned}\Delta x_b &= \frac{(0)^2 - (\boxed{56.4}\ \text{mi/h})^2}{(0)^2 - (\boxed{38.6}\ \text{mi/h})^2}(\boxed{12}\ \text{m}) \\ \Delta x_b &= \boxed{25.6}\ \text{m}\end{aligned}$$

4. GRR1 3.P.067. In the figure below (part a), two blocks are connected by a lightweight, flexible cord which passes over a frictionless pulley. If m_1 is $\boxed{3.6}$ kg and m_2 is $\boxed{9.0}$ kg, and block 2 is initially at rest $\boxed{130}$ cm above the floor, how long does it take block 2 to reach the floor?



From problem 2, we know that the acceleration of the blocks is given by

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

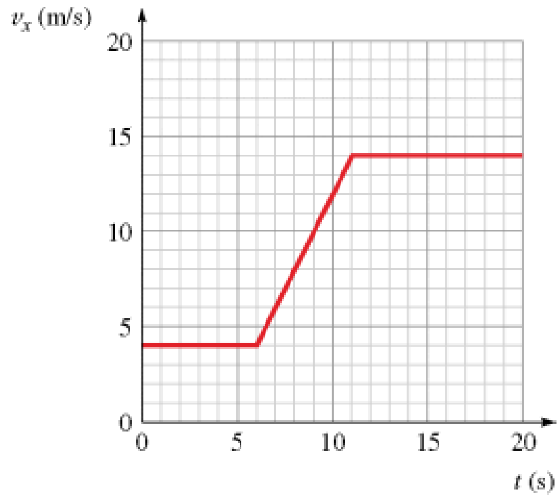
The equation of motion of an object starting at rest is

$$x = \frac{1}{2}at^2 + x_0$$

Taking $x - x_0 = \Delta x$, substituting in the equation for the acceleration, and solving for time we get

$$\begin{aligned}
\Delta x &= \frac{1}{2}at^2 \\
\Delta x &= \frac{1}{2}\left(\frac{m_2 - m_1}{m_2 + m_1}g\right)t^2 \\
t^2 &= 2\Delta x\left(\frac{m_2 + m_1}{m_2 - m_1}\right)\frac{1}{g} \\
t &= \sqrt{2\Delta x\left(\frac{m_2 + m_1}{m_2 - m_1}\right)\frac{1}{g}} \\
t &= \sqrt{2(\boxed{1.30}\text{ m})\left(\frac{\boxed{9.0}\text{ kg} + \boxed{3.6}\text{ kg}}{\boxed{9.0}\text{ kg} - \boxed{3.6}\text{ kg}}\right)\frac{1}{9.8\text{ m/s}^2}} \\
t &= \boxed{0.787}\text{ s}
\end{aligned}$$

5. GRR1 3.TB.016. The figure below shows a plot of $v_x(t)$ for a car. (a) What is $a_{av,x}$ between $t = 6$ s and $t = 11$ s? (b) What is $v_{av,x}$ for the same time interval? (c) How far does the car travel from time $t = 10$ s to time $t = 15$ s?



(a) The acceleration of the car, $a_{av,x}$, is the change in velocity per unit time. In terms of an equation this means

$$a_{av,x} = \frac{\Delta v_x}{\Delta t}$$

or

$$a_{av,x} = \frac{v_{f,x} - v_{i,x}}{t_f - t_i}$$

In this case the acceleration is then

$$\begin{aligned} a_{av,x} &= \frac{14 \text{ m/s} - 4 \text{ m/s}}{11 \text{ s} - 6 \text{ s}} \\ a_{av,x} &= 2 \text{ m/s}^2 \end{aligned}$$

(b) Since the relationship between the velocities between times $t = 6$ s and $t = 11$ s is linear, the average velocity between these two times is just the average of the two velocities at those times. That is,

$$v_{av,x} = \frac{1}{2}(v_f + v_i)$$

The average velocity is then

$$\begin{aligned} v_{av,x} &= \frac{1}{2}(14 \text{ m/s} + 4 \text{ m/s}) \\ v_{av,x} &= 9 \text{ m/s} \end{aligned}$$

(c) Since the velocity changes between $t = 10$ s and $t = 11$ s we must be careful with our calculation. The change in position is just the change in time time the average velocity during that time, i.e.

$$\Delta x = v_{av,x} \cdot \Delta t$$

To find the distance travelled between the times $t = 10$ s and $t = 11$ s, we may just use the average velocity during that time. Therefore the total distance travelled is then

$$\begin{aligned}\Delta x &= 13 \text{ m/s}\,(11 \text{ s} - 10 \text{ s}) + 14 \text{ m/s}\,(15 \text{ s} - 11 \text{ s}) \\ \Delta x &= 69 \text{ m}\end{aligned}$$

6. GRR1 3.TB.028. A crate of oranges weighing 180 N rests on a flatbed truck 2.0 m from the back of the truck. The coefficients of friction between the crate and the bed are $\mu_s = 0.30$ and $\mu_k = 0.20$. The truck drives on a straight level highway at a constant 8.0 m/s. (a) What is the force of friction acting on the crate? (b) If the truck speeds up with an acceleration of 1.0 m/s^2, what is the force of friction on the crate? (c) What is the maximum acceleration the truck can have without the crate starting to slide?

(a) Since the truck is not accelerating, there is no external force on the crate in the horizontal direction. Therefore there is no force of friction acting on the crate. $F_{fr} = 0$ N.

(b) If the truck accelerates at $a_{truck} = 1.0$ m/s^2, the force on the crate in the horizontal direction is then

$$F_h = m_{crate} \cdot a_{truck}$$

Assuming that the crate does not slip on the truck's flatbed, the force of static friction must be equal to this horizontal force, i.e.

$$F_{fr} = F_h$$

We can get the mass of the crate from the weight given in the problem. Therefore the force of friction on the crate is

$$\begin{aligned}F_{fr} &= \frac{180 \text{ N}}{9.8 \text{ m/s}^2} \cdot 1.0 \text{ m/s}^2 \\ F_{fr} &= 18 \text{ N}\end{aligned}$$

(c) The maximum acceleration is found by setting F_{fr} equal to the maximum force of friction.

$$F_{fr,max} = \mu_s F_N$$

where F_N is the normal force on the crate. Setting this equal to the horizontal force and solving for the acceleration gives

$$\begin{aligned}F_h &= F_{fr,max} \\ m_{crate} a_{truck} &= \mu_s F_N \\ a_{truck} &= \mu_s \frac{F_N}{m_{crate}} \\ a_{truck} &= \mu_s \frac{m_{crate} g}{m_{crate}} \\ a_{truck} &= \mu_s \cdot g\end{aligned}$$

The maximum acceleration is then

$$\begin{aligned}a_{truck} &= 0.30(9.8 \text{ m/s}^2) \\ a_{truck} &= 2.9 \text{ m/s}^2\end{aligned}$$

Notice that the maximum acceleration does not depend on the mass of the crate, and it depends just on the coefficient of static friction.

7. GRR1 3.TB.032. A train is traveling south at 24.0 m/s when the brakes are applied. It slows down at a constant rate to a speed of 6.00 m/s in a time of 9.00 s. (a) What is the acceleration of the train during the 9.00 s interval? (b) How far does the train travel during the 9.00 s?

(a) The acceleration is the change in velocity per unit time, i.e.

$$\begin{aligned} a &= \frac{\Delta v}{\Delta t} \\ a &= \frac{v_f - v_i}{\Delta t} \end{aligned}$$

The acceleration is then

$$a = \frac{6.00 \text{ m/s} - 24.0 \text{ m/s}}{9.00 \text{ s}}$$
$$a = -2.00 \text{ m/s}^2$$

Since the train is traveling south the negative sign we got must mean that the acceleration is north.

$$\vec{a} = 2.00 \text{ m/s}^2 \text{ to the north}$$

(b) The change in position is then given by

$$\Delta x = v_{ave} \Delta t$$

where v_{ave} is the ave velocity during the acceleration. The change in position is then

$$\begin{aligned} \Delta x &= \left(\frac{1}{2}(v_f + v_i)\right) \Delta t \\ \Delta x &= \left(\frac{1}{2}(24.0 \text{ m/s} + 6.00 \text{ m/s})\right) \cdot 9.00 \text{ s} \\ \Delta x &= 15.0 \text{ m/s} \cdot 9.00 \text{ s} \\ \Delta x &= 135. \text{ m} \end{aligned}$$

8. GRR1 3.TB.044. A student, looking toward his fourth-floor dormitory window, sees a flower pot with nasturtiums (originally on a window sill above) pass his 2.0-m-high window in 0.093 s. The distance between floors in the dormitory is 4.0 m. From a window on which floor did the flower pot fall?

The flower pot, after being released from a higher floor, is accelerated by gravity. The velocity of the flower pot as a function of time (assuming it starts at rest) is then

$$v_y = -gt$$

At some time t_w it passed by the student's window. The velocity at this time was

$$v_{w,y} = -gt_w$$

As measured by the student, the average velocity of the flower pot as it passed his window was

$$v_{w,y} = \frac{\Delta h}{\Delta t}$$

where Δh is the height of the window and Δt is the time it took to pass it. Combining this equation with the above expression and solving for t_w we get

$$\begin{aligned} v_{w,y} &= -gt_w \\ \frac{\Delta y}{\Delta t} &= -gt_w \\ t_w &= -\frac{\Delta h}{g\Delta t} \end{aligned}$$

The equation of motion for the flower pot as it falls under gravity is just

$$y = -\frac{1}{2}gt^2$$

The distance the flower pot fell to the window is then

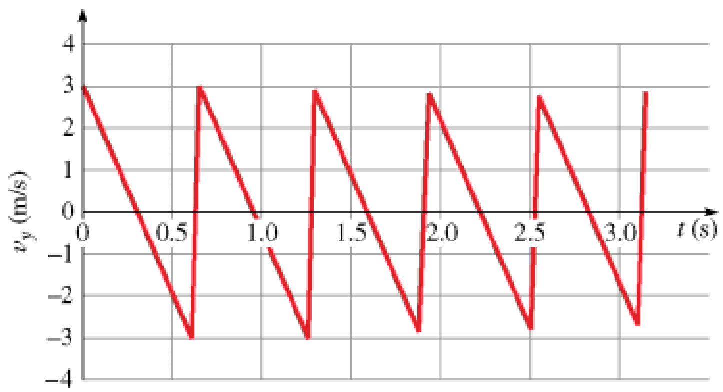
$$\begin{aligned} y_w &= -\frac{1}{2}gt_w^2 \\ y_w &= -\frac{1}{2}g\left(-\frac{\Delta h}{g\Delta t}\right)^2 \\ y_w &= -\frac{1}{2}\frac{\Delta h^2}{g\Delta t^2} \end{aligned}$$

The flower pot then fell

$$\begin{aligned} y_w &= -\frac{1}{2}\frac{(2.0\ \text{m})^2}{9.8\ \text{m/s}^2(0.093\ \text{s})^2} \\ y_w &= -24\ \text{m} \end{aligned}$$

Given that each floor is 4.0 m tall, the flower pot must have fallen from 6 floors above the student. Since the student is on the fourth floor, the flower pot fell from the tenth floor.

9. GRR1 3.TB.074. The graph in the figure below is the vertical velocity component v_y of a bouncing ball as a function of time. The y-axis points up. Answer the following based on the data in the graph. (a) At what time does the ball reach its maximum height? (b) For how long is the ball in contact with the floor? (c) What is the maximum height of the ball?



(a) Since a postive velocity indicates a ball moving upward, and a negative velocity indicates a ball moving downward, the ball must reach the apex when its velocity is zero. Therefore the time when it reaches its maximum height occurs first at $t \approx 0.3$ s.

(b) The ball is in contact with the floor from the time the velocity is negative to when it is positive again. From the graph the ball must be in contact with the floor for about half of a tenth of a second ≈ 0.05 s. (Looking right after 2.5 s helped me determine this.)

(c) The maximum height of the ball may be determined by taking the average of the velocities between when the ball first moves upward and when it is at the apex and multiplying by the time it takes to do that. The ball starts upward with a velocity of 3.0 m/s. We know from part (a) that it reaches the apex in about 0.3 s. The maximum height is then

$$\begin{aligned}
\Delta y &= v_{ave}\Delta t \\
\Delta y &= \left(\frac{1}{2}(3.0\text{ m/s} + 0\text{ m/s})\right) 0.3\text{ s} \\
\Delta y &= 0.45\text{ m}
\end{aligned}$$

10. GRR1 3.TB.075. [219493] A rocket engine can accelerate a rocket launched from rest vertically up with an acceleration of 20.0 m/s^2. However, after 50.0 seconds of flight the engine fails. Neglect air resistance. (a) What is the rocket's elevation when the engine fails? (b) When does it reach its maximum height? (c) What is the maximum height reached? [Hint: a graphical solution may be easiest.]

(a) The equation of motion for the rocket is given by

$$y = \frac{1}{2}a_{engine}t^2 + y_0$$

where a_{engine} is the accelereation due to the engine, and y_0 is the initial height. Assuming the rocket is launched from sea level, the elevation when the rocket fails is

$$\begin{aligned}
y_f &= \frac{1}{2} a_{engine} \Delta t_{engine}^2 \\
y_f &= \frac{1}{2} 20.0\ \mathrm{m/s^2}\ (50.0\ \mathrm{s})^2 \\
y_f &= 10.0\ \mathrm{m/s^2} \cdot 2500\ \mathrm{s^2} \\
y_f &= 25000\ \mathrm{m} \\
y_f &= 25.0\ \mathrm{km}
\end{aligned}$$

(b) After the engine fails, the rocket is simply under the influence of gravity. Therefore it will undergo constant acceleration, but downward this time. The velocity as a function of time for an object under constant acceleration is given by

$$v_y = a_y t + v_{f,y}$$

In this case the acceleration, a_y, is that due to gravity, and the initial velocity, $v_{0,y}$, is the velocity the rocket had reached when the engine failed. The velocity that the rocket had reached is given by

$$v_{f,y} = a_{engine} \Delta t_{engine}$$

where Δt_{engine} is the time the rocket engine was on. The equation for the velocity after the engine fails then becomes

$$v_y = -gt + a_{engine} \Delta t_{engine}$$

From Problem 9, we know that the rocket will have reached its maximum height when the vertical velocity is equal to zero. Setting the equation for the velocity equal to zero and solving for the time gives

$$\begin{aligned}
0 &= -g t_{apex} + a_{engine} \Delta t_{engine} \\
g t_{apex} &= a_{engine} \Delta t_{engine} \\
t_{apex} &= \frac{a_{engine}}{g} \Delta t_{engine}
\end{aligned}$$

The time when the rocket reaches its maximum height *after the engine fails* is then

$$\begin{aligned}
t_{apex} &= \frac{20.0\ \mathrm{m/s^2}}{9.8\ \mathrm{m/s^2}} 50.0\ \mathrm{s} \\
t_{apex} &= 102\ \mathrm{s}
\end{aligned}$$

Therefore the rocket reaches its maximum height $102\ \mathrm{s} + 50.0\ \mathrm{s} = 152\ \mathrm{s}$ into the flight.

(c) After the engine fails the equation of motion for the rocket is

$$y = -\frac{1}{2} g t^2 + v_{f,y} t + y_f$$

where $v_{f,y}$ and y_f are the velocity and elevation it reached when the engine failed. Substituting in these quantities, which we found in parts (a) and (b), we get

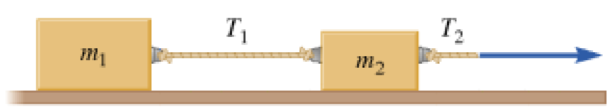
$$\begin{aligned}
y_{apex} &= -\frac{1}{2}gt_{apex}^2 + v_{f,y}t_{apex} + y_f \\
y_{apex} &= -\frac{1}{2}g\left(\frac{a_{engine}}{g}\Delta t_{engine}\right)^2 + (a_{engine}\Delta t_{engine})\left(\frac{a_{engine}}{g}\Delta t_{engine}\right) + \frac{1}{2}a_{engine}\Delta t_{engine}^2 \\
y_{apex} &= -\frac{1}{2}\frac{(a_{engine}\Delta t_{engine})^2}{g} + \frac{(a_{engine}\Delta t_{engine})^2}{g} + \frac{1}{2}a_{engine}\Delta t_{engine}^2 \\
y_{apex} &= \frac{1}{2}\frac{(a_{engine}\Delta t_{engine})^2}{g} + \frac{1}{2}a_{engine}\Delta t_{engine}^2 \\
y_{apex} &= \frac{1}{2}a_{engine}\Delta t_{engine}^2\left(\frac{a_{engine}}{g} + 1\right)
\end{aligned}$$

The maximum height the rocket reaches is then

$$\begin{aligned}
y_{apex} &= \frac{1}{2}20.0 \text{ m/s}^2 \cdot (50.0 \text{ s})^2\left(\frac{20.0 \text{ m/s}^2}{9.8 \text{ m/s}^2} + 1\right) \\
y_{apex} &= 76020 \text{ m} \\
y_{apex} &= 76.0 \text{ km}
\end{aligned}$$

The rocket reaches a maximum height of 76.0 km.

11. GRR1 3.TB.082. Two blocks, masses $\mathbf{m_1}$ and $\mathbf{m_2}$, are connected by a massless cord (the figure below). If the two blocks are accelerated uniformly on a frictionless surface by applying a force of magnitude $\mathbf{T_2}$ to a second cord connected to $\mathbf{m_2}$, what is the ratio of the tensions in the two cords in terms of the masses? $\mathbf{T_1/T_2}$ = ?



The net force on the two-block system is

$$F_{net} = (m_1 + m_2)a$$

where a is the acceleration. The tension in the second rope must be equal to this net force.

$$\begin{aligned}
T_2 &= F_{net} \\
T_2 &= (m_1 + m_2)a
\end{aligned}$$

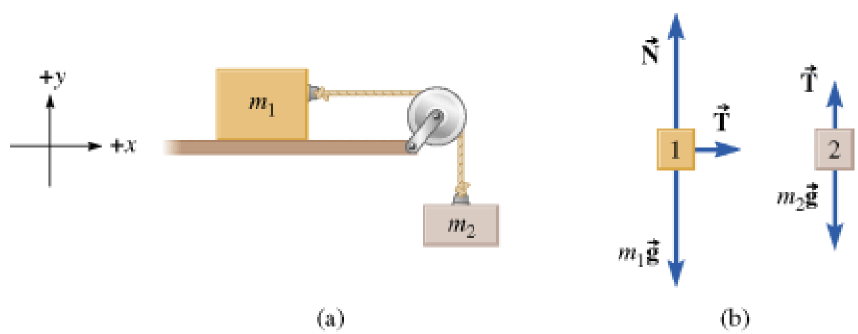
The tension in the first rope is equal to the force required to accelerate the first block at an acceleration of a, i.e.

$$T_1 = m_1 a$$

Therefore the ratio of the tensions is given by dividing the two equations for each of the tensions

$$\frac{T_1}{T_2} = \frac{m_1 a}{(m_1+m_2)a}$$
$$\frac{T_1}{T_2} = \frac{m_1}{m_1+m_2}$$

12. GRR1 3.TB.062. In the figure below (part a), the block of mass $\mathbf{m_1}$ slides to the right with coefficient of kinetic friction μ_k on a horizontal surface. The block is connected to a hanging block of mass $\mathbf{m_2}$ by a light cord that passes over a light, frictionless pulley. (a) Find the acceleration of each of the blocks and the tension in the cord. (b) Check your answers in the special cases $\mathbf{m_1 \ll m_2}$, $\mathbf{m_1 \gg m_2}$, and $\mathbf{m1 = m2}$. (c) For what value of $\mathbf{m_2}$ (if any) do the two blocks slide at constant velocity? What is the tension in the cord in that case?



(a) The net force on the two-block system is given by

$$\vec{F}_{net} = \vec{F}_{fr} + \vec{F}_W$$

where $\vec{F}_{fr}$ is the force of friction on the first block, and $\vec{F}_W$ is the weight of the second block. Since $\vec{F}_{fr}$ opposes $\vec{F}_W$, we will call rightward and downward positive. The acceleration of the blocks is then

$$(m_1+m_2)a = -\mu_k m_1 g + m_2 g$$
$$(m_2+m_1)a = (m_2 - \mu_k m_1)g$$
$$a = \frac{m_2 - \mu_k m_1}{m_2+m_1} g$$

The net force on the first block is equal to the tension plus the force of friction

$$\vec{F}_{net,1} = \vec{T} + \vec{F}_{fr}$$
$$F_{net,1} = T - F_{fr}$$
$$m_1 a = T - \mu_k m_1 g$$

Solving for the tension we get

$$\begin{aligned} T &= m_1 a + \mu_k m_1 g \\ T &= m_1(a + \mu_k g) \end{aligned}$$

Now we can substitute in what we found the acceleration to be

$$T = m_1 \left(\frac{m_2 - \mu_k m_1}{m_2 + m_1} g + \mu_k g \right)$$

Simplifying this we get

$$\begin{aligned} T &= m_1 \left(\frac{m_2 - \mu_k m_1}{m_2 + m_1} g + \frac{\mu_k g(m_2 + m_1)}{m_2 + m_1} \right) \\ T &= m_1 \left(\frac{m_2 g - \mu_k m_1 g + \mu_k m_2 g + \mu_k m_1 g}{m_2 + m_1} \right) \\ T &= m_1 \left(\frac{m_2 g + \mu_k m_2 g}{m_2 + m_1} \right) \\ T &= (1 + \mu_k) \frac{m_1 m_2}{m_2 + m_1} g \end{aligned}$$

(b) To ensure that our answers make sense, we consider the cases when the first block is really light, when it is really heavy, and when the masses are equal. To say that the mass of the first block is really light compared to the second block mathematically, we write

$$m_1 \ll m_2$$

or

$$\frac{m_1}{m_2} \ll 1$$

This means when we encounter the ratio m_1/m_2 we may treat it as zero. We can find this ratio in our expression for the acceleration if we rearrange a bit,

$$\begin{aligned} a &= \frac{m_2 - \mu_k m_1}{m_2 + m_1} g \\ a &= \frac{m_2 g}{m_2 + m_1} - \frac{\mu_k m_1 g}{m_2 + m_1} \\ a &= \frac{m_2 g}{m_2(1 + \frac{m_1}{m_2})} - \frac{\mu_k m_1 g}{m_2(1 + \frac{m_1}{m_2})} \\ a &= \frac{m_2 g}{m_2(1 + \frac{m_1}{m_2})} - \frac{\mu_k g \frac{m_1}{m_2}}{(1 + \frac{m_1}{m_2})} \end{aligned}$$

Now if we treat every instance of m_1/m_2 as zero we get

$$\begin{aligned} a &\approx \frac{m_2 g}{m_2(1+0)} - \frac{\mu_k g \cdot 0}{(1+0)} \\ a &\approx \frac{m_2 g}{m_2} - 0 \\ a &\approx g \end{aligned}$$

This result makes good sense because if the first block were very light, the second block would fall as if it were not there.

Similarly, the tension can be rewritten as

$$\begin{aligned}
T &= (1+\mu_k)\frac{m_1 m_2}{m_2+m_1}g \\
T &= (1+\mu_k)\frac{m_1 m_2}{m_2(1+\frac{m_1}{m_2})}g \\
T &= (1+\mu_k)\frac{m_1}{m_2}\frac{m_2}{(1+\frac{m_1}{m_2})}g \\
T &\approx (1+\mu_k)(0)\cdot\frac{m_2}{1+0}g \\
T &\approx 0
\end{aligned}$$

The tension would be very small if the mass of the first block was very small.

For the case of the first block being much heavier than the second block, we look for the ratio m_2/m_1 and treat it as zero. The acceleration then becomes

$$\begin{aligned}
a &= \frac{m_2-\mu_k m_1}{m_2+m_1}g \\
a &= \frac{m_2 g}{m_2+m_1}-\frac{\mu_k m_1 g}{m_2+m_1} \\
a &= \frac{m_2 g}{m_1(\frac{m_2}{m_1}+1)}-\frac{\mu_k m_1 g}{m_1(\frac{m_2}{m_1}+1)} \\
a &= \frac{m_2}{m_1}\frac{g}{\frac{m_2}{m_1}+1}-\frac{\mu_k m_1 g}{m_1(\frac{m_2}{m_1}+1)} \\
a &\approx (0)\frac{g}{(0)+1}-\frac{\mu_k m_1 g}{m_1((0)+1)} \\
a &\approx -\frac{\mu_k m_1 g}{m_1} \\
a &\approx -\mu_k g
\end{aligned}$$

This result is somewhat nonsense. The first block would not accelerate to the left if it was very heavy, but it does point out that the force of friction dominates over the weight of the second block. It that sense, it does have meaning. So if the first block has a large mass compared to the second block, the acceleration is effectively zero.

The tension would then act like

$$\begin{aligned}
T &= (1+\mu_k)\frac{m_1 m_2}{m_2+m_1}g \\
T &= (1+\mu_k)\frac{m_1 m_2}{m_1(\frac{m_2}{m_1}+1)}g \\
T &= (1+\mu_k)\frac{m_2}{\frac{m_2}{m_1}+1}g \\
T &\approx (1+\mu_k)\frac{m_2}{(0)+1}g \\
T &\approx (1+\mu_k)m_2 g
\end{aligned}$$

It makes good sense that the tension in the cord just becomes the weight of the second block, because the two-block system should not move if the first block is large enough.

For the case of the masses being equal, if we let $m_1 = m_2 = m$, the acceleration becomes

$$\begin{aligned} a &= \frac{m - \mu_k m}{m + m} g \\ a &= (1 - \mu_k) \frac{m}{2m} g \\ a &= \frac{1}{2} g (1 - \mu_k) \end{aligned}$$

We can see that if μ_k were small the acceleration would be like $\frac{1}{2}g$ or half of the acceleration due to gravity.

The tension would become

$$\begin{aligned} T &= (1 + \mu_k) \frac{m \cdot m}{m + m} g \\ T &= (1 + \mu_k) \frac{m^2}{2m} g \\ T &= \frac{1}{2} mg (1 + \mu_k) \end{aligned}$$

Again, if the coefficient of friction were small, the tension would just be half the weight of one of the blocks.

(c) The blocks slide at constant velocity when the net force is zero or when the acceleration is zero. Setting the acceleration equal to zero and solving for m_2 we get

$$\begin{aligned} 0 &= \frac{m_2 - \mu_k m_1}{m_2 + m_1} g \\ 0 &= \frac{m_2 g}{m_2 + m_1} - \frac{\mu_k m_1 g}{m_2 + m_1} \\ \frac{m_2 g}{m_2 + m_1} &= \frac{\mu_k m_1 g}{m_2 + m_1} \\ m_2 g &= \mu_k m_1 g \\ m_2 &= \mu_k m_1 \end{aligned}$$

Therefore the blocks would slide at a constant velocity if $m_2 = \mu_k m_1$.

The tension in the cord for this value of m_2 is then

$$\begin{aligned} T &= (1 + \mu_k) \frac{m_1 (\mu_k m_1)}{(\mu_k m_1) + m_1} g \\ T &= (1 + \mu_k) \frac{m_1^2 \mu_k}{m_1 (\mu_k + 1)} g \\ T &= \mu_k m_1 g \end{aligned}$$