Solutions to Homework Set #11
Phys2414 – Fall 2005

Note: The numbers in the $\boxed{\text{boxes}}$ correspond to those that are generated by WebAssign. The numbers on your individual assignment will vary. Any calculated quantities that involve these variable numbers will be $\boxed{\text{boxed}}$ as well.

1. GRR1 9.P.012. How high can you suck water up a straw? The pressure in the lungs can be reduced to about $\boxed{6}$ kPa below atmospheric pressure.

Given that the maximum difference in pressure we can apply to the straw is $\Delta P = \boxed{6}$ kPa and force is related to pressure by $\Delta P = F/A$, we can see that this maximum difference in pressure must be equal to a maximum force. This maximum force is equal to the weight of the amount of water that is pulled up the straw. The weight of this water in terms of the cross sectional area of the straw, A, and the distance up the straw the water travels, d, is

$$\begin{aligned} W_{H_2O} &= \rho_{H_2O} V g \\ W_{H_2O} &= \rho_{H_2O} A d g \end{aligned}$$

Using $\Delta P = W_{H_2O}/A$, where we've substituted $F = W_{H_2O}$ we have

$$\begin{aligned} \Delta P &= W_{H_2O}/A \\ \Delta P &= (A d \rho_{H_2O} g)/A \\ \Delta P &= d \rho_{H_2O} g \\ d &= \frac{\Delta P}{\rho_{H_2O} g} \\ d &= \frac{\boxed{6000}\ \text{Pa}}{1000\ \text{kg/m}^3 9.8\ \text{m/s}^2} \\ d &= 0.612\ \text{m} \end{aligned}$$

2. GRR1 9.P.015. In the Netherlands a dike, holding back the sea from a town below sea level, springs a leak $\boxed{4.0}$ m below the water surface. If the area of the hole in the dike is $\boxed{1.0}$ cm^2, what force must the Dutch boy exert to save the town?

The pressure difference between the sea level and a distance, d, below sea level is equal to

$$\Delta P = \rho_{H_2O} d g$$

Given that $\Delta P = F/A$, we can equate the two and find

$$\begin{aligned} \frac{F}{A} &= \rho_{H_2O} d g \\ F &= \rho_{H_2O} A d g \\ F &= \left(1 \frac{\text{kg}}{\text{L}}\right)\left(\frac{1\ \text{L}}{1000\ \text{mL}}\right) \cdot \boxed{400}\ \text{cm} \cdot \boxed{1.0}\ \text{cm}^2 \cdot 9.8\ \text{m/s}^2 \\ F &= \boxed{3.9}\ \text{N} \end{aligned}$$

3. GRR1 9.P.028. A block of wood floats in oil with $\boxed{88.3}$% of its volume submerged. What is the density of the oil? The density of the block of wood is $\boxed{0.67}$ g/cm^3.

The bouyant force is equal to

$$\begin{aligned} F_b &= \rho_{oil} V_{disp} g \\ &= \rho_{oil} \boxed{0.883} V_{wood} g \end{aligned}$$

Since the block is floating comfortably, the bouyant force must be equal to the weight of the wood. The weight of the wood in terms of its density and volume is given by

$$W = \rho_{wood} V_{wood} d g$$

Setting the two forces equal to each other, we get

$$\begin{aligned} F_b &= W \\ \rho_{oil} \boxed{0.883} V_{wood} g &= \rho_{wood} V_{wood} g \\ \rho_{oil} \boxed{0.883} &= \rho_{wood} \\ \rho_{oil} &= \frac{1}{\boxed{0.883}} \rho_{wood} \\ \rho_{oil} &= \frac{1}{0.883} \cdot \boxed{0.67} \text{ g/cm}^3 \\ \rho_{oil} &= \boxed{0.759} \text{ g/cm}^3 \end{aligned}$$

4. GRR1 9.P.034. The average density of a fish can be found by first weighing it in air and then finding the apparent weight of the fish as it hangs completely immersed in water. If a fish has weight $\boxed{208.8}$ N and apparent weight $\boxed{15.5}$ N in water, what is the average density of the fish?

The apparent weight under water must be difference of the actual wieght and the bouyant force on the fish.

$$\begin{aligned} W_a &= W - F_b \\ F_b &= W - W_a \end{aligned}$$

The bouyant force must be equal to

$$F_b = \rho_{H_2O} V_{fish} g$$

We also know that the weight of the fish can be expressed in terms of its density and volume as

$$\begin{aligned} W &= \rho_{fish} V_{fish} g \\ V_{fish} &= \frac{W}{\rho_{fish} g} \end{aligned}$$

Substituting this into the bouyant force gives

$$F_b = \rho_{H_2O} \left(\frac{W}{\rho_{fish}} \right)$$

Now putting this into the equation above gives

$$
\begin{aligned}
F_b &= W - W_a \\
\rho_{H_2O}\left(\frac{W}{\rho_{fish}}\right) &= W - W_a \\
\frac{1}{\rho_{fish}} &= \frac{W - W_a}{\rho_{H_2O} W} \\
\rho_{fish} &= \frac{W}{W - W_a}\rho_{H_2O} \\
\rho_{fish} &= \frac{\boxed{208.8}\ \mathrm{N}}{\boxed{208.8}\ \mathrm{N} - \boxed{15.5}\ \mathrm{N}} \cdot 1000\ \mathrm{kg/m^3} \\
\rho_{fish} &= \boxed{1080}\ \mathrm{kg/m^3}
\end{aligned}
$$

5. GRR1 9.P.036. If the average volume flow of blood through the aorta is $\boxed{\mathbf{9.5}}\times\mathbf{10^{-5}m^3/s}$ and the cross-sectional area of the aorta is $\boxed{\mathbf{3.0}}\times\mathbf{10^{-4}m^2}$, what is the average speed of blood in the aorta?

This is a standard dimensional analysis problem. Given that the dimensions of the volume flow, f, are $\mathrm{m^3/s}$, the dimensions of area, A, are $\mathrm{m^2}$, and the dimensions of velocity, v, are $\mathrm{m/s}$, We can combine two of these quantities to make the third quantity, for example, by multiplying A and v to make f. That is

$$
\begin{aligned}
f &= Av \\
v &= \frac{f}{A} \\
v &= \frac{\boxed{9.5} \times 10^{-5}\ \mathrm{m^3/s}}{\boxed{3.0} \times 10^{-4}\ \mathrm{m^2}} \\
v &= \boxed{0.317}\ \mathrm{m/s} \\
v &= \boxed{31.7}\ \mathrm{cm/s}
\end{aligned}
$$

6. GRR1 8.TB.085. A merry-go-round (radius R, rotational inertia I_0) spins with negligible friction. Its initial angular velocity is ω_0. A child (mass m) on the merry-go-round moves from the center out to the rim. (a) Calculate the angular velocity after the child moves out to the rim. (b) Calculate the rotational kinetic energy and angular momentum of the system (merry-go-round + child) before and after.

The merry-go-round is the only thing rotating initially. So the initial angular momentum is just

$$L_i = I_0\omega_0$$

and the initial kinetic energy is then

$$\mathrm{KE}_i = \frac{1}{2}I_0\omega_0^2$$

After the kid moves to the edge of the merry-go-round, the merry-go-round turns at a different angular velocity, ω_f. The angular momentum of the system is the sum of the angular momenta of the two objects

$$
\begin{aligned}
L_f &= I_0\omega_f + mR^2\omega_f \\
L_f &= (I_0 + mR^2)\omega_f
\end{aligned}
$$

The final kinetic energy is likewise

$$\begin{aligned} \mathrm{KE}_f &= \frac{1}{2}I_0\omega_f^2 + \frac{1}{2}mR^2\omega_f^2 \\ \mathrm{KE}_f &= \frac{1}{2}\omega_f^2\left(I_0 + mR^2\right) \end{aligned}$$

The angular momentum must be conserved, so we can write the new angular momentum in terms of the initial angular momentum using

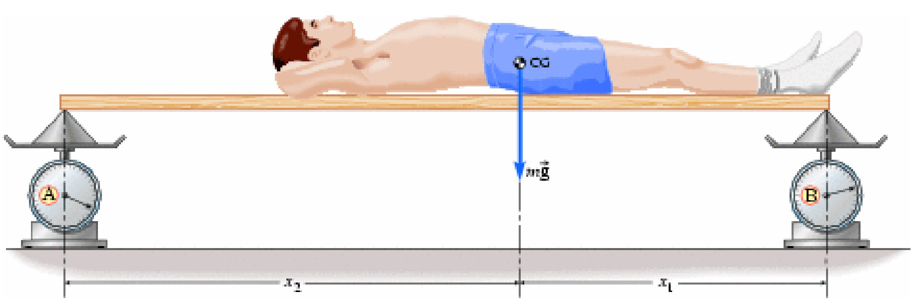
$$\begin{aligned} L_i &= L_f \\ I_0\omega_0 &= (I_0 + mR^2)\omega_f \\ \omega_f &= \frac{I_0}{I_0 + mR^2}\omega_0 \end{aligned}$$

The final kinetic energy can then be expressed in terms of the given variables by substituting this expression for ω_f into the equation above for KE_f.

$$\begin{aligned} \mathrm{KE}_f &= \frac{1}{2}\left(\frac{I_0}{I_0 + mR^2}\omega_0\right)^2\left(I_0 + mR^2\right) \\ \mathrm{KE}_f &= \frac{1}{2}\frac{I_0^2}{I_0 + mR^2}\omega_0^2 \end{aligned}$$

As we can see, the final kinetic energy is not conserved in this process.

7. GRR1 8.TB.087. Since humans are generally not symmetrically shaped, the height of our center of gravity is generally not half of our height. One way to determine the location of the center of gravity is shown in the figure below . A 2.2-m-long uniform plank is supported by two bathroom scales, one at either end. Initially the scales each read 100.0 N. A 1.60-m-tall student then lies on top of the plank, with the soles of his feet directly above scale B. Now scale A reads 394.0 N and scale B reads 541.0 N. (a) What is the student's weight? (b) How far is his center of gravity from the soles of his feet? (c) When standing, how far above the floor is his center of gravity, expressed as a fraction of his height?



Before the student lies on the plank, the scales each read 100.0 N. The plank must weigh 200.0 N. After the students lies on the plank, the sum of the forces must be equal to zero. There are four forces on the plank: (1) the force on the plank by the scale on the left, (2) the force on the plank by the scale on the right, (3) the weight of the student, and (4) the weigth of the plank itself. So we can find

$$\begin{aligned} \sum \vec{F} &= F_L + F_R - W_s - W_p \\ 0 &= F_L + F_R - W_s - W_p \\ W_s &= F_L + F_R - W_p \\ W_s &= 394.0\ \mathrm{N} + 541.0\ \mathrm{N} - 200.0\ \mathrm{N} \\ W_s &= 735.0\ \mathrm{N} \end{aligned}$$

To determine the distance from the student's center of gravity to his feet, we may use the fact that the sum of the torques is also zero. Choosing the axis to be at the scale on the right, the sum of the torques gives

$$\begin{aligned}
\sum \vec{\tau} &= -r_L F_L + R_s W_s + R_p W_p \\
0 &= -r_L F_L + R_s W_s + R_p W_p \\
R_s W_s &= r_L F_l - R_p W_p \\
R_s &= \frac{1}{W_s}\left(r_L F_l - R_p W_p\right) \\
R_s &= \frac{1}{735.0 \text{ N}}\left(2.2 \text{ m} \cdot 394.0 \text{ N} - 1.1 \text{ m}200.0 \text{ N}\right) \\
R_s &= 0.88 \text{ m}
\end{aligned}$$

This is $\frac{0.88 \text{ m}}{1.6 \text{ m}} = 0.55$ or 55% of the student's height.