

Solutions to Homework Set #10

Phys2414 – Fall 2005

Note: The numbers in the boxes correspond to those that are generated by WebAssign. The numbers on your individual assignment will vary. Any calculated quantities that involve these variable numbers will be boxed as well.

1. GRR1 8.P.007. A bicycle has wheels of radius 0.23 m. Each wheel has a rotational inertia of 0.095 kg · m² about its axle. The total mass of the bicycle including the wheels and the rider is 72 kg. When coasting at constant speed, what fraction of the total kinetic energy of the bicycle (including rider) is the rotational kinetic energy of the wheels?

The total kinetic energy of this system is the sum of the kinetic energies of the system. The bicycle is moving so it has translational kinetic energy equal to $KE_{trans} = \frac{1}{2}mv^2$. The wheels are turning so they both have rotational kinetic energy equal to $KE_{wheel} = \frac{1}{2}I\omega^2$. The total rotational energy is then $KE_{rot} = 2 \cdot KE_{wheel}$. The fraction of the total energy that is the rotational energy is then given by

$$\begin{aligned} f &= \frac{KE_{rot}}{KE_{tot}} \\ f &= \frac{2 \cdot \frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + 2 \cdot \frac{1}{2}I\omega^2} \\ f &= \frac{I\omega^2}{\frac{1}{2}mv^2 + I\omega^2} \end{aligned}$$

Since the bicycle is moving at a speed of v , the angular velocity of the wheels must be equal to $\omega = \frac{v}{r}$. Substituting this into the fraction above gives

$$\begin{aligned} f &= \frac{I(v/r)^2}{\frac{1}{2}mv^2 + I(v/r)^2} \\ f &= \frac{(I/r^2)v^2}{\frac{1}{2}mv^2 + (I/r^2)v^2} \\ f &= \frac{(I/r^2)v^2}{v^2(\frac{1}{2}m + I/r^2)} \\ f &= \frac{I/r^2}{(\frac{1}{2}m + I/r^2)} \end{aligned}$$

The speed cancels out and we find

$$\begin{aligned} f &= \frac{\text{0.095 kg} \cdot \text{m}^2 / (\text{0.23 m})^2}{\left(\frac{1}{2}\text{72 kg} + \text{0.095 kg} \cdot \text{m}^2 / (\text{0.23 m})^2\right)} \\ f &= \text{0.0475} \end{aligned}$$

2. GRR1 8.P.020. The radius of a wheel is 0.470 m. A rope is wound around the outer rim of the wheel. The rope is pulled with a force of magnitude 3.10 N, unwinding the rope and making the wheel spin counterclockwise about its central axis. Ignore the mass of the rope.

(a) How much rope unwinds while the wheel makes 1.00 revolution?

The amount of rope unwound is given by the circumference of the wheel times the number of revolutions made, i.e.

$$\begin{aligned}x_{rope} &= 2\pi R \cdot (\text{number of revolutions}) \\x_{rope} &= 2\pi \boxed{0.470} \text{ m} \cdot (1.00) \\x_{rope} &= \boxed{2.95} \text{ m}\end{aligned}$$

(b) How much work is done by the rope on the wheel during this time?

The work done by the rope on the wheel is given by the torque applied to the wheel times the angle through which it turns.

$$W = \tau\theta$$

The torque is given by the radius of the wheel times the magnitude of the force the rope is pulled with, since they are perpendicular. θ is the angle it turned *in radians*. One revolution is equal to 2π radians, so the work done is

$$\begin{aligned}W &= rF\theta \\W &= \boxed{0.470} \text{ m} \boxed{3.10} \text{ N} \cdot 2\pi \\W &= \boxed{9.15} \text{ J}\end{aligned}$$

(c) What is the torque on the wheel due to the rope?

As we said before, the torque is given by multiplying the radius of the wheel by the magnitude of the force.

$$\begin{aligned}\tau &= rF \\\tau &= \boxed{0.470} \text{ m} \boxed{3.10} \text{ N} \\\tau &= \boxed{1.46} \text{ N} \cdot \text{m}\end{aligned}$$

3. GRR1 8.P.025. A sign painter is standing on a uniform, horizontal platform that is held in equilibrium at the tenth-story level by two cables attached to supports on the roof of the building (the figure below). The painter has a mass of $\boxed{77}$ kg and the mass of the platform is $\boxed{20.4}$ kg. The distance from the left end of the platform to where the painter is standing is $d = \boxed{2.0}$ m and the total length of the platform is $\boxed{5.2}$ m.



(a) How large is the force exerted by the left-hand cable on the platform?

There are four forces in the platform: (1) the force exerted by the left-hand cable, F_l , (2) the force exerted by the right-hand cable, F_r , (3) the weight of the painter, W_p , and (4) the weight of the platform itself, W_g . The platform is in equilibrium, so we know that both the sum of the forces on the platform and the sum of

the torques are both zero. We can exploit the fact that the sum of the torques must be zero around any axis by placing the axis so that one of the torques is eliminated. That is, the sum of the torques is in general given by

$$\sum \vec{\tau} = \vec{\tau}_l + \vec{\tau}_r + \vec{\tau}_p + \vec{\tau}_g$$

If we place the axis for this torque equation at the force exerted by the right-hand cable we get

$$\begin{aligned}\sum \vec{\tau} &= -\tau_l + \tau_p + \tau_g \\ 0 &= -\tau_l + \tau_p + \tau_g \\ \tau_l &= \tau_p + \tau_g\end{aligned}$$

All of the forces are perpendicular to the platform, so we can write

$$\begin{aligned}\tau_l &= \tau_p + \tau_g \\ r_l F_l &= r_p W_p + r_g W_g \\ F_l &= \frac{1}{r_l} (r_p W_p + r_g W_g)\end{aligned}$$

And we can find the force exerted by the left-hand cable directly.

$$\begin{aligned}F_l &= \frac{1}{r_l} (r_p W_p + r_g W_g) \\ F_l &= \frac{1}{5.2 \text{ m}} \left((5.2 \text{ m} - 2.0 \text{ m}) 77 \text{ kg} \cdot 9.8 \text{ m/s}^2 + (5.2 \text{ m})/2 \cdot 20.4 \text{ kg} \cdot 9.8 \text{ m/s}^2 \right) \\ F_l &= 564 \text{ N}\end{aligned}$$

(b) How large is the force exerted by the right-hand cable?

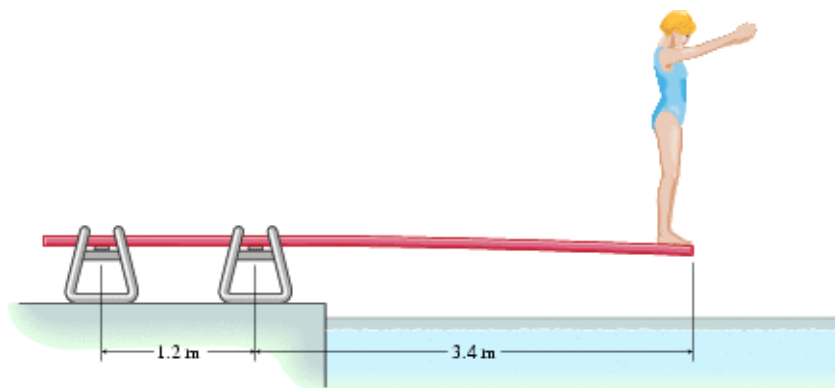
We can use the same trick here, except this time we should put the axis at the left cable. The sum of the torques is then

$$\begin{aligned}\sum \vec{\tau} &= \tau_r - \tau_p - \tau_g \\ 0 &= \tau_r - \tau_p - \tau_g \\ \tau_r &= \tau_p + \tau_g\end{aligned}$$

And solving for the force exerted by the right-hand cable gives

$$\begin{aligned}r_r F_r &= r_p W_p + r_g W_g \\ F_r &= \frac{1}{r_r} (r_p W_p + r_g W_g) \\ F_r &= \frac{1}{5.2 \text{ m}} \left((2.0 \text{ m}) 77 \text{ kg} \cdot 9.8 \text{ m/s}^2 + (5.2 \text{ m})/2 \cdot 20.4 \text{ kg} \cdot 9.8 \text{ m/s}^2 \right) \\ F_r &= 390 \text{ N}\end{aligned}$$

4. GRR1 8.P.028. A uniform diving board, of length 5.0 m and mass 52 kg, is supported at two points; one support is located 3.4 m from the end of the board and the second is at 4.6 m from the end (see the figure below). What are the forces acting on the board due to the two supports when a diver of mass 62 kg stands at the end of the board over the water? Assume that these forces are vertical. [Hint: In this problem, consider using two different torque equations about different rotation axes. This may help you determine the directions of the two forces.]



We can use the trick that we learned in the previous problem to solve this problem. There are four forces on the diving board: (1) the force of the left support on the board, F_l , (2) the force of the right support on the board, F_r , (3) the force of the diver on the board, W_d , and, finally, (4) the weight of the board itself, W_b . In general, the sum of the torques is given by

$$\sum \vec{\tau} = \vec{\tau}_l + \vec{\tau}_r + \vec{\tau}_d + \vec{\tau}_b$$

We can eliminate the torque due to the right support by putting the axis we are considering at the right support. We then get

$$\begin{aligned} \sum \vec{\tau} &= \vec{\tau}_l - \tau_d - \tau_b \\ 0 &= \vec{\tau}_l - \tau_d - \tau_b \\ \vec{\tau}_l &= \tau_d + \tau_b \\ r_l \vec{F}_l &= r_d W_d + r_b W_b \\ \vec{F}_l &= \frac{1}{r_l} (r_d W_d + r_b W_b) \end{aligned}$$

Notice that we have left $\vec{F}_l$ as a vector. This is because we don't know which direction it points. Since we do know the directions of the other torques, we can infer if it is upward or downward from the sign of the answer we get. Evaluating this equation gives

$$\begin{aligned} \vec{F}_l &= \frac{1}{r_l} (r_d W_d + r_b W_b) \\ \vec{\tau}_l &= \frac{1}{1.2 \text{ m}} \left((3.4 \text{ m}) \cdot (62 \text{ kg}) \cdot 9.8 \text{ m/s}^2 + (3.4 \text{ m} - (5.0 \text{ m})/2) (52 \text{ kg}) \cdot 9.8 \text{ m/s}^2 \right) \\ \vec{\tau}_l &= 2.10 \text{ kN} \end{aligned}$$

Since this sign of the answer is positive and counter-clockwise is positive, the this force must point downward.

Likewise, to determine the magnitude and direction of the right support, we place the axis at the left support. The sum of the torques is then

$$\sum \vec{\tau} = \vec{\tau}_r - \tau_d - \tau_b$$

$$\begin{aligned}
0 &= \vec{\tau}_r - \tau_d - \tau_b \\
\vec{\tau}_r &= \tau_d + \tau_b \\
r_r \vec{F}_r &= r_d W_d + r_b W_b \\
\vec{F}_r &= \frac{1}{r_r} (r_d W_d + r_b W_b)
\end{aligned}$$

Again we have left $\vec{F}_r$ as a vector. We will determine the direction of $\vec{F}_r$ from the sign of the answer we get.

$$\begin{aligned}
\vec{F}_r &= \frac{1}{r_r} (r_d W_d + r_b W_b) \\
\vec{F}_r &= \frac{1}{1.2 \text{ m}} \left((4.6 \text{ m}) \cdot (62 \text{ kg}) \cdot 9.8 \text{ m/s}^2 + ((4.6 \text{ m}) - (5.0 \text{ m})/2) (52 \text{ kg}) \cdot 9.8 \text{ m/s}^2 \right) \\
\vec{F}_r &= 3.22 \text{ kN}
\end{aligned}$$

The sign of this answer is positive, and since counter-clockwise is taken to be positive, this force must point upward.

5. GRR1 8.P.044. A grinding wheel, with a mass of 24.8 kg and a radius of 23.3 cm , is a uniform cylindrical disk.

(a) Find the rotational inertia of the wheel about its central axis.

The rotational inertia of a cylindrical disk is given by

$$\begin{aligned}
I &= \frac{1}{2} M R^2 \\
I &= \frac{1}{2} (24.8 \text{ kg}) (0.233 \text{ m})^2 \\
I &= 0.673 \text{ kg} \cdot \text{m}^2
\end{aligned}$$

(b) When the grinding wheel's motor is turned off, friction causes the wheel to slow from 1200 rpm to rest in 59.8 s . What torque must the motor provide to accelerate the wheel from rest to 1200 rpm in 4.00 s ? Assume that the frictional torque is the same regardless of whether the motor is on or off.

The net torque on the wheel is given by

$$\tau_{net} = \tau_{motor} - \tau_{fric}$$

Solving for τ_{motor} we get

$$\begin{aligned}
\tau_{motor} &= \tau_{net} + \tau_{fric} \\
\tau_{motor} &= I \alpha_{net} + I \alpha_{fric} \\
\tau_{motor} &= I (\alpha_{net} + \alpha_{fric})
\end{aligned}$$

Using the fact that average angular acceleration is given by

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

we find the torque of the motor needs to be

$$\begin{aligned}
 \tau_{motor} &= I \left(\frac{\Delta\omega}{\Delta t_{start}} + \frac{\Delta\omega}{\Delta t_{stop}} \right) \\
 \tau_{motor} &= I \Delta\omega \left(\frac{1}{\Delta t_{start}} + \frac{1}{\Delta t_{stop}} \right) \\
 \tau_{motor} &= (\boxed{0.673} \text{ kg} \cdot \text{m}^2) \cdot \left(1200 \frac{\text{rev}}{\text{m}} \frac{1 \text{ m}}{60 \text{ s}} \frac{2\pi}{1 \text{ rev}} \right) \left(\frac{1}{\boxed{4.00} \text{ s}} + \frac{1}{\boxed{59.8} \text{ s}} \right) \\
 \tau_{motor} &= \boxed{22.6} \text{ N} \cdot \text{m}
 \end{aligned}$$

6. GRR1 8.TB.016. A tower outside the Houses of Parliament in London has a famous clock commonly referred to as Big Ben, the name of its 13-ton chiming bell. The hour hand of each clock face is 2.7 m long and has a mass of 60.0 kg. Assume the hour hand to be a uniform rod attached at one end. (a) What is the torque on the clock mechanism due to the weight of one of the four hour hands when the clock strikes noon? The axis of rotation is perpendicular to a clock face and through the center of the clock. (b) What is the torque due to the weight of one hour hand about the same axis when the clock tolls 9:00 a.m.?

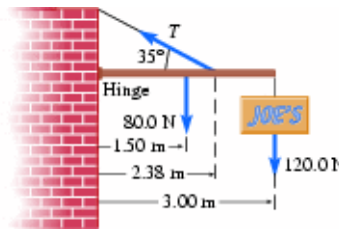
The rods that are the clock hands are uniform, so the weight of the hand acts at the center of mass. At noon, the center of mass is directly over the axis so that the angle between the rod and the force of the weight is equal to zero. The torque about that axis is then

$$\begin{aligned}
 \tau_{12:00} &= rW \sin \theta \\
 \tau_{12:00} &= rW \sin 0^\circ \\
 \tau_{12:00} &= 0
 \end{aligned}$$

At nine o'clock, the force of the weight is perpendicular to the rod. The torque at that time is

$$\begin{aligned}
 \tau_{9:00} &= (2.7 \text{ m})/2 \cdot 60.0 \text{ kg} \cdot 9.8 \text{ m/s}^2 \sin 90^\circ \\
 \tau_{9:00} &= 764 \text{ N} \cdot \text{m}
 \end{aligned}$$

7. GRR1 8.TB.032. A sign is supported by a uniform horizontal boom of length 3.00 m and weight 80.0 N. A cable, inclined at an angle of 35° with the boom, is attached at a distance of 2.38 m from the hinge at the wall (the figure below). The weight of the sign is 120 N. What is the tension in the cable and what are the horizontal and vertical forces F_x and F_y exerted on the boom by the hinge?



The sign is at equilibrium so we know that the both the sum of the forces and the sum of the torques are zero. There are four forces on the boom: (1) the force of the wall on the boom, F_w , (2) the force of the cable on the boom, T , (3) the weight of the sign, W_s , and (4) the weight of the boom itself, W_b . Let's start by using the torque equation to find the tension in the cable. Because the sum of the torques must be zero

for any axis, we can place the axis so that one of the torques is zero. In general the sum of the torques is given by

$$\sum \vec{\tau} = \vec{\tau}_w + \vec{\tau}_T + \vec{\tau}_s + \vec{\tau}_b$$

Placing the axis at the wall gives

$$\begin{aligned}\sum \vec{\tau} &= \tau_T - \vec{\tau}_s - \vec{\tau}_b \\ 0 &= \tau_T - \tau_s - \tau_b \\ \tau_T &= \tau_s + \tau_b\end{aligned}$$

The component of tension perpendicular to the boom is $T_{\perp} = T \sin 35^\circ$. The other two forces are perpendicular to the boom. The tension is then

$$\begin{aligned}r_T T \sin 35^\circ &= r_s W_s + r_b W_b \\ T &= \frac{1}{r_t \sin 35^\circ} (r_s W_s + r_b W_b) \\ T &= \frac{1}{2.38 \text{ m} \sin 35^\circ} (3.00 \text{ m} \cdot 120 \text{ N} + 1.50 \text{ m} \cdot 80.0 \text{ N}) \\ T &= 350 \text{ N}\end{aligned}$$

To find F_x , we consider the sum of the forces in the horizontal direction

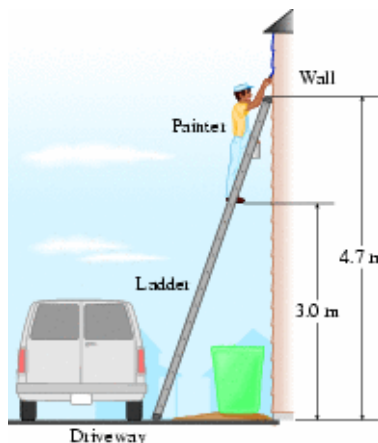
$$\begin{aligned}\sum F_h &= F_x - T \cos 35^\circ \\ 0 &= F_x - T \cos 35^\circ \\ F_x &= T \cos 35^\circ \\ F_x &= 350 \text{ N} \cos 35^\circ \\ F_x &= 290 \text{ N}\end{aligned}$$

The force in the y -direction can be found by using the torque equation and placing the axis where the cable connects to the boom. The sum of the torques is then

$$\begin{aligned}\sum \vec{\tau} &= \vec{\tau}_w - \tau_s + \tau_b \\ 0 &= \vec{\tau}_w - \tau_s + \tau_b \\ \vec{\tau}_w &= \tau_s - \tau_b \\ r_w \vec{F}_y &= r_s W_s - r_b W_b \\ \vec{F}_y &= \frac{1}{r_w} (r_s W_s - r_b W_b) \\ \vec{F}_y &= \frac{1}{2.38 \text{ m}} ((3.00 \text{ m} - 2.38 \text{ m}) \cdot 120 \text{ N} - (2.38 \text{ m} - 1.50 \text{ m}) \cdot 80.0 \text{ N}) \\ \vec{F}_y &= 2 \text{ N}\end{aligned}$$

Like the diving board problem, the sign in front of this answer indicates the direction. The sign is positive and counter-clockwise is positive, so this force must point *downward*.

8. GRR1 8.TB.029. A house painter stands 3.0 m above the ground on a 5.0-m-long ladder that leans against the wall at a point 4.7 m above the ground (the figure below). The painter weighs 680 N and the ladder weighs 120 N. Assuming no friction between the house and the upper end of the ladder, find the force of friction that the driveway exerts on the bottom of the ladder.



There are five forces on the ladder: (1) the normal force on the ladder due to the wall, N_h , (2) the normal force on the ladder due to the ground, N_v , (3) the frictional force, F_f , (4) the weight of the painter, W_p , and (5) the weight of the ladder itself, W_l . Since we don't know the first three forces, we should try to eliminate one or more of them from the torque equation like in problems 3, 4, and 7. The sum of the torques on the ladder is then

$$\sum \vec{\tau} = \vec{\tau}_h + \vec{\tau}_v + \vec{\tau}_f + \vec{\tau}_p + \vec{\tau}_l$$

We can eliminate *two* of the torques by placing the axis at the ground where the F_r and N_v act on the ladder. The sum of the torques is then

$$\begin{aligned} \sum \vec{\tau} &= \vec{\tau}_h - \vec{\tau}_p - \vec{\tau}_l \\ 0 &= \tau_h - \tau_p - \tau_l \\ \tau_h &= \tau_p + \tau_l \end{aligned}$$

Let the angle that the ladder makes with the wall be θ . The perpendicular components of the three forces in the torque equation are then

$$\begin{aligned} \tau_h &= \tau_p + \tau_l \\ r_h N_h \cos \theta &= r_p W_p \sin \theta + r_l W_l \sin \theta \\ N_h &= \frac{\sin \theta}{r_h \cos \theta} (r_p W_p + r_l W_l) \\ N_h &= \frac{\tan \theta}{r_h} (r_p W_p + r_l W_l) \end{aligned}$$

We can find θ by using the length of the ladder and the distance up the wall it touches.

$$\begin{aligned} \cos \theta &= \frac{4.7 \text{ m}}{5.0 \text{ m}} \\ \theta &= 19.9^\circ \end{aligned}$$

The normal force due to the wall is then

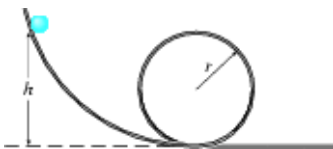
$$\begin{aligned} N_h &= \frac{\tan \theta}{r_h} (r_p W_p + r_l W_l) \\ N_h &= \frac{\tan 19.9^\circ}{5.0 \text{ m}} (3.0 \text{ m} \cdot 680 \text{ N} + (5.0 \text{ m})/2 \cdot 120 \text{ N}) \\ N_h &= 170 \text{ N} \end{aligned}$$

The sum of the forces in the horizontal direction must be zero, i.e.

$$\begin{aligned} \sum F_h &= F_f - N_h \\ 0 &= F_f - N_h \\ F_f &= N_h \end{aligned}$$

So the frictional force must be equal to the normal force, $F_f = 170 \text{ N}$, and it points toward the wall.

9. GRR1 8.TB.052. A solid sphere of radius R and mass M slides without friction down a loop-the-loop track (the figure below). The sphere starts from rest at a height of h above the horizontal. Assume that the radius of the sphere is small compared to the radius r of the loop. (a) Find the minimum value of h in terms of r so that the sphere remains on the track all the way around the loop. (b) Find the minimum value of h if, instead, the sphere rolls without slipping on the track.



We start by considering the forces on the sphere at the top of the loop-the-loop. There are only two forces on the sphere; the normal force on the sphere by the track and the weight of sphere itself. The sum of these two forces must be equal to Ma_c because it is traveling in circular motion, i.e.

$$\begin{aligned} \sum \vec{F} &= N + W \\ Ma_c &= N + W \\ M \frac{v^2}{R} &= N + Mg \end{aligned}$$

By analyzing this equation we can see that the only variables that can change are v and N . The mass, M , and gravity, g , are always the same. So if N gets smaller, v has to get smaller. The normal force cannot be negative, so the minimum value for v occurs when $N = 0$, or

$$M \frac{v_{min}^2}{R} = Mg$$

Solving for v_{min} we get

$$\begin{aligned} M \frac{v_{min}^2}{R} &= Mg \\ \frac{v_{min}^2}{R} &= g \\ v_{min} &= \sqrt{gR} \end{aligned}$$

Now we have what the minimum speed must be so that the sphere will make it around the loop-the-loop. The initial energy the sphere starts with is equal to $E_i = Mgh$, where h is the initial height. At the top of the loop-the-loop the energy of the sphere is

$$E_f = \frac{1}{2}Mv^2 + Mg(2R)$$

At the top of the loop-the-loop, the sphere has a speed equal to v and a height of $2R$. Using conservation of energy we know that $E_i = E_f$ and the minimum height is given by

$$\begin{aligned} Mgh_{min} &= \frac{1}{2}Mv_{min}^2 + 2MgR \\ h_{min} &= \frac{1}{2g}v_{min}^2 + 2R \\ h_{min} &= \frac{1}{2g}(\sqrt{gR})^2 + 2R \\ h_{min} &= \frac{1}{2g}gR + 2R \\ h_{min} &= \frac{1}{2}R + 2R \\ h_{min} &= \frac{5}{2}R \end{aligned}$$

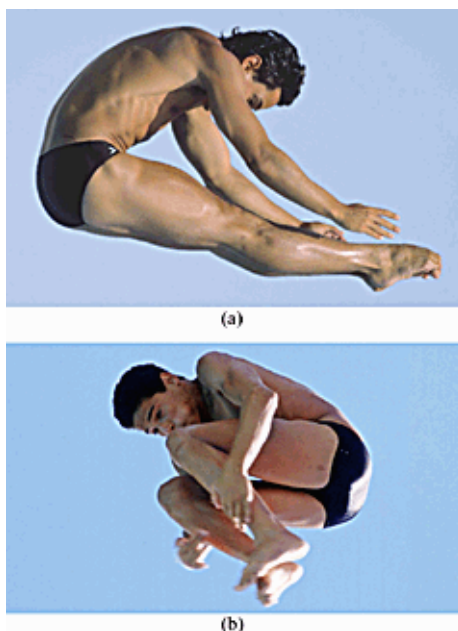
If the sphere is allowed to roll, only one detail changes. At the top of the loop-the-loop, the sphere has not only translational and potential energy but also rotational energy. That is

$$E_f = \frac{1}{2}Mv^2 + 2MgR + \frac{1}{2}I\omega^2$$

The angular velocity must be equal to $\omega = v/R$ if the ball rolls without slipping. Applying conservation of energy, we have

$$\begin{aligned} Mgh_{min} &= \frac{1}{2}Mv_{min}^2 + 2MgR + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{min}}{R}\right)^2 \\ h_{min} &= \frac{1}{2g}(\sqrt{gR})^2 + 2R + \frac{1}{2}\left(\frac{2}{5g}R^2\right)\left(\frac{\sqrt{gR}}{R}\right)^2 \\ h_{min} &= \frac{1}{2g}gR + 2R + \frac{1}{2}\left(\frac{2}{5g}R^2\right)\left(\frac{g}{R}\right) \\ h_{min} &= \frac{1}{2}R + 2R + \frac{1}{5}R \\ h_{min} &= \frac{1}{2}R + 2R + \frac{1}{5}R \\ h_{min} &= \frac{27}{10}R \end{aligned}$$

10. GRR1 8.TB.064. The rotational inertia for a diver in a pike position is about $15.5 \text{ kg} \cdot \text{m}^2$; it is only $8.0 \text{ kg} \cdot \text{m}^2$ in a tucked position (the figure below). (a) If the diver gives himself an initial angular momentum of $106 \text{ kg} \cdot \text{m}^2/\text{s}$ as he jumps off the board, how many turns can he make when jumping off a 10.0-m platform in a tuck position? (b) How many in a pike position? [Hint: Gravity exerts no torque on the person as he falls; assume he is rotating throughout the 10.0 m dive.]



Using kinematics, the diver is in the air for a time, t , given by

$$\begin{aligned}\Delta y &= -\frac{1}{2}gt^2 \\ t &= \sqrt{\frac{-2\Delta y}{g}}\end{aligned}$$

The angular velocity, ω of the diver is related to the angular momentum, L , by

$$\begin{aligned}L &= I\omega \\ \omega &= \frac{L}{I}\end{aligned}$$

The number of turns the diver make in the air is then given by

$$\begin{aligned}\#ofturns &= \omega \cdot t \cdot \left(\frac{1 \text{ rev}}{2\pi}\right) \\ \#ofturns &= \frac{L}{I} \cdot \sqrt{\frac{-2\Delta y}{g}} \frac{1 \text{ rev}}{2\pi}\end{aligned}$$

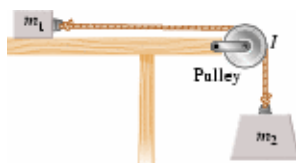
The number of turns the diver makes in the tuck position is

$$\begin{aligned}\#ofturns &= \frac{106 \text{ kg} \cdot \text{m}^2/\text{s}}{8.0 \text{ kg} \cdot \text{m}^2} \cdot \sqrt{\frac{-2(-10.0 \text{ m})}{9.8 \text{ m/s}^2}} \frac{1 \text{ rev}}{2\pi} \\ \#ofturns &= 3.0\end{aligned}$$

The number of turns the diver makes in the pike position is then

$$\begin{aligned}\#ofturns &= \frac{106 \text{ kg} \cdot \text{m}^2/\text{s}}{15.5 \text{ kg} \cdot \text{m}^2} \cdot \sqrt{\frac{-2(-10.0 \text{ m})}{9.8 \text{ m/s}^2}} \frac{1 \text{ rev}}{2\pi} \\ \#ofturns &= 1.6\end{aligned}$$

11. GRR1 8.TB.079. A block of mass m_2 hangs from a string. The string wraps around a pulley of rotational inertia I and then attaches to a second block of mass m_1 , which sits on a frictionless table (the figure below). What is the acceleration of the blocks when they are released?



To solve this problem we look at the forces and torques on each piece. The sum of the forces on the first mass is

$$\begin{aligned}\sum F_1 &= T_1 \\ m_1 a_1 &= T_1\end{aligned}$$

where T_1 is the tension in the string between m_1 and the pulley.

The sum of the forces on the second mass is

$$\begin{aligned}\sum F_2 &= T_2 - m_2 g \\ -m_2 a_2 &= T_2 - m_2 g\end{aligned}$$

where T_2 is the tension in the string between m_2 and the pulley.

The sum of the torques on the pulley is (taking counter-clockwise to be positive)

$$\begin{aligned}\sum \tau &= T_1 R - T_2 R \\ -I \alpha &= T_1 R - T_2 R\end{aligned}$$

The angular acceleration is related to the acceleration of the string by $\alpha = a/R$. Since everything is connected together, then the linear accelerations must be the same, i.e. $a = a_1 = a_2$. The equation of motion of the pulley is then

$$\begin{aligned}-I \alpha &= T_1 R - T_2 R \\ -I \frac{a}{R^2} &= T_1 - T_2\end{aligned}$$

Using the above equations for m_1 and m_2 , we can substitute in for the tensions and get

$$-I \frac{a}{R^2} = (m_1 a) - (m_2 g - m_2 a)$$

Solving for a gives

$$\begin{aligned} -I \frac{a}{R^2} &= (m_1 a) - (m_2 a + m_2 g) \\ -I \frac{a}{R^2} - m_1 a - m_2 a &= -m_2 g \\ a \left(\frac{I}{R^2} + m_1 + m_2 \right) &= m_2 g \\ a &= \frac{m_2 g}{\frac{I}{R^2} + m_1 + m_2} \end{aligned}$$