Solutions to Homework Set #1
Phys2414 – Fall 2005

Note: The numbers in the $\boxed{\text{boxes}}$ correspond to those that are generated by WebAssign. The numbers on your individual assignment will vary. Any calculated quantities that involve these variable numbers will be $\boxed{\text{boxed}}$ as well.

1. GRR1 1.P.003. In cleaning out the artery of a patient, a doctor increases the radius of the opening by a factor of $\boxed{\text{five}}$. By what factor does the cross-sectional area of the artery change?

To find the percentage increase in the area of the artery we begin by calculating the original area of the artery.

$$A_{old} = \pi r_{old}^2$$

The problem states that the new radius is the old radius increased by a factor of $\boxed{\text{five}}$. The new radius is then

$$r_{new} = \boxed{5} r_{old}$$

The new area in terms of the new radius is given by

$$A_{new} = \pi r_{new}^2$$

Substituting in the expression for the new radius in terms of the old radius we have

$$A_{new} = \pi \left(\boxed{5} r_{old}\right)^2$$

Simplifying this expression we get

$$A_{new} = \left(\boxed{5}\right)^2 \pi r_{old}^2$$

$$A_{new} = \boxed{25} \pi r_{old}^2$$

Now we recognize that this is $\boxed{\text{twenty-five}}$ times the old area,

$$A_{new} = \boxed{25} A_{old}$$

The area of the artery is increased by a factor of $\boxed{\text{twenty-five}}$.

2. GRR1 1.P.006. The electrical power P drawn from a generator by a lightbulb of resistance R is given by the formula below, where V is the line voltage.

$$P = \frac{V^2}{R}$$

The resistance of bulb B is $\boxed{70}$% greater than the resistance of bulb A. What is the ratio $\mathbf{P_B/P_A}$ of the power drawn by bulb B to the power drawn by bulb A, if the line voltages are the same?

Using the relationship given, the power output of bulb A is given by the equation

$$P_A = \frac{V^2}{R_A}$$

Likewise, the power output of bulb B must be

$$P_B = \frac{V^2}{R_B}$$

We did not change the voltage. We only changed the bulb, and thus only the resistance has changed. Dividing these two quantities gives

$$\frac{P_B}{P_A} = \frac{\frac{V^2}{R_B}}{\frac{V^2}{R_A}}$$

To simplify this expression we notice that the voltage, V, divides out and we are left with

$$\frac{P_B}{P_A} = \frac{\frac{1}{R_B}}{\frac{1}{R_A}}$$

$$\frac{P_B}{P_A} = \frac{R_A}{R_B}$$

Since we do not have values for R_A or R_B, we express R_B in terms of R_A. The information in the question tells us that R_B is R_A increased by $\boxed{70}$%. Putting this in equation form we have

$$R_B = R_A + \boxed{0.70}R_A$$

$$R_B = \boxed{1.70}R_A$$

We may now substitute this expression for R_B into the expression above for the ratio of the powers in terms of R_A and R_B.

$$\frac{P_B}{P_A} = \frac{R_A}{\boxed{1.70}R_A}$$

Since the resistance, R_A, divides out, this expression becomes

$$\frac{P_B}{P_A} = \frac{1}{\boxed{1.70}}$$

$$\frac{P_B}{P_A} = \boxed{0.588}$$

The power of bulb B is almost half of that of bulb A.

3. GRR1 1.P.010. Write your answers to these problems with the appropriate number of significant figures.

(a) $\boxed{7.00} \times 10^{-5} + \boxed{2.7} \times 10^{-7}$
Putting the quantity on the right in a more suggestive form gives

$$\boxed{7.00} \times 10^{-5} + \boxed{0.027} \times 10^{-5}$$

The answer in the correct significant digits is

$$\boxed{7.00} \times 10^{-5} + \boxed{0.027} \times 10^{-5} = \boxed{7.03} \times 10^{-5}$$

(b) $\boxed{702.28} + \boxed{1818.624}$
The correct number of significant digits for this sum is two after the decimal. The answer is then

$$\boxed{702.28} + \boxed{1818.624} = \boxed{2520.90}$$

(c) $\boxed{5.0} \times \boxed{2.7}$
Both quantities have two significant digits, giving a result of

$$\boxed{5.0} \times \boxed{2.7} = \boxed{1.4 \times 10^1}$$

(d) $\boxed{0.08} \div \pi$
Only one significant digit is given, so the result is

$$\boxed{0.08} \div \pi = \boxed{0.03}$$

(e) $\boxed{0.080} \div \pi$
Now we are given two significant digits, giving a result of

$$\boxed{0.080} \div \pi = \boxed{0.025}$$

4. GRR1 1.P.028. The largest living creature on Earth is the blue whale, which has an average length of 70 ft. The largest blue whale on record (and therefore the largest animal ever found) was 1.10×10^2 ft long. The record blue whale has mass of $\boxed{1.7} \times 10^5$ kg. Assuming that its average density was 0.85 g/cm³, as has been measured for other blue whales, what was the volume of the whale in m³? (Average density is the ratio of mass to volume.)

The crux of this problem is to recognize that too much information is given. The final sentence says that the average density is the ratio of the mass to the volume. Expressing this in terms of a equation we have

$$\rho = \frac{m}{V}$$

where ρ is the average density, m is the mass, and V is the volume. The information in the problem tells us both the average density and the mass, and the question asks us for the volume. This equation relates those three things. Solving for the volume we get

$$V = \frac{m}{\rho}$$

We now have an equation for the volume, but we must take care to use the correct units in the calculation. We have the mass in kilograms, and the problem requests units of m^3, so we will put the density in units of kg/m^3

$$0.85 \ \frac{g}{cm^3} = \frac{0.85 \ g}{cm^3} \times \left(\frac{1 \ kg}{1000 \ g}\right) \times \left(\frac{100 \ cm}{m}\right)^3$$

$$0.85 \ \frac{g}{cm^3} = 850 \ \frac{kg}{m^3}$$

The volume is then

$$V = \frac{1.7 \times 10^5 \ kg}{850 \ kg/m^3} = 200 \ m^3$$

5. GRR1 1.P.031. The Space Shuttle astronauts use a "massing chair" to measure their mass. The chair is attached to a spring and is free to oscillate back and forth. The frequency of the oscillation is measured and that is used to calculate the total mass m attached to the spring. If the spring constant of the spring k is measured in kg/s^2 and the chair's frequency f is 0.50 s^{-1} for a $\boxed{52}$ kg astronaut, what is the chair's frequency for a $\boxed{77}$ kg astronaut? The chair itself has a mass of $\boxed{15.0}$ kg. [Hint: use dimensional analysis to find out how f depends on m and k.]

The key to solving this problem is to use the hint. We need to think of a way to combine the spring constant, the mass, and the frequency into an equation. The units of each are

$$\begin{aligned} [k] &= \frac{kg}{s^2} \\ [m] &= kg \\ [f] &= \frac{1}{s} \end{aligned}$$

One way that we can combine these three quantities to make an equation is

$$k = mf^2$$

We can see that on the left side of the equation we have units of kg/s^2, and we have the same units on the right side of the equation. Now we have a relationship for the spring constant, the mass, and the frequency.

We can apply this equation to find the frequency of the second astronaut. The spring constant in terms of the first astronaut's mass and frequency is then

$$k = (m_{chair} + m_1) f_1^2$$

Note that both the chair and the astronaut in it are oscillating so we must include both masses in the expression for the spring constant. Similarly, the spring constant in terms of the second astronaut's mass and frequency is

$$k = (m_{chair} + m_2) f_2^2$$

Setting the two equations equal to each other we get

$$(m_{chair} + m_1)f_1^2 = (m_{chair} + m_2)f_2^2$$

$$m_{chair}f_1^2 + m_1 f_1^2 = m_{chair}f_2^2 + m_2 f_2^2$$

$$m_1 f_1^2 = m_2 f_2^2$$

Notice that the mass of the chair actually drops out for the relationship of the masses of the astronauts and the frequencies. Solving for the second frequency, we obtain

$$f_2 = \sqrt{\frac{m_1}{m_2}} f_1$$

The frequency for the second astronaut is then

$$f_2 = \sqrt{\frac{\boxed{52}\ \text{kg}}{\boxed{77}\ \text{kg}}}\ \boxed{0.50}\ \text{s}^{-1}$$

$$f_2 = \boxed{0.41}\ \text{s}^{-1}$$

6. GRR1 1.P.047. If the radius of a circular garden plot is increased by $\boxed{25}$%, by what percentage does the area of the garden increase?

Similar to problem 1, we can write the new radius in terms of the old radius as

$$r_{new} = r_{old} + \boxed{0.25} r_{old}$$

$$r_{new} = \boxed{1.25} r_{old}$$

In order to write the new area as increase of the old area, just as we did for the radius, let's define a number, p, which is the percentage increase in the area. The new area in terms of the old area is then

$$A_{new} = A_{old} + pA_{old}$$

$$A_{new} = (1 + p)A_{old}$$

Now let's try to find an expression for the new area in terms of the old area using the relationship for the new radius. The area of the new circle in terms of the new radius is, of course,

$$A_{new} = \pi r_{new}^2$$

Substituting the expression for r_{new} in terms of r_{old} we get

$$A_{new} = \pi \left(\boxed{1.25} r_{old}\right)^2$$

$$A_{new} = \left(\boxed{1.25}\right)^2 \pi r_{old}^2$$

$$A_{new} = \boxed{1.56} \pi r_{old}^2$$

Again we recognize that this is just $\boxed{1.56}$ times the old area,

$$A_{new} = \boxed{1.56} A_{old}$$

Comparing this equation with the expression we have above for the new area in terms of the old area, we recognize that $\boxed{1.56}$ must be equal to $(1 + p)$. Solving for p we get

$$(1 + p) = \boxed{1.56}$$

$$p = \boxed{0.56}$$

Therefore the area increased by $\boxed{56}$%.

7. GRR1 1.P.049. A marathon race is about $\boxed{22}$ miles long. What is the length of the race in kilometers?

The conversion between miles and kilometers is

$$1 \text{ mi} = 1.609 \text{ km}$$

The length of a marathon in kilometers is then

$$\boxed{22} \text{ mi} \left(\frac{1.609 \text{ km}}{1 \text{ mi}} \right) = \boxed{35.4} \text{ km}$$

8. GRR1 1.TB.016. A furlong is 220 yards; a fortnight is 14 days. How fast is 1 furlong per fortnight (a) in μm/s? (b) in km/day?

The best approach to doing many unit conversions is to write out each of the conversions clearly and save computation until then end. To get to $\mu m/s$ we write out all of the conversions in between.

$$\begin{aligned} \frac{1 \text{ furlong}}{1 \text{ fortnight}} &= \left(\frac{1 \text{ furlong}}{1 \text{ fortnight}}\right) \times \left(\frac{220 \text{ yd}}{1 \text{ furlong}}\right) \times \left(\frac{3 \text{ ft}}{1 \text{ yd}}\right) \times \left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) \times \left(\frac{10^6 \ \mu\text{m}}{1 \text{ m}}\right) \\ &\quad \times \left(\frac{1 \text{ fortnight}}{14 \text{ d}}\right) \times \left(\frac{1 \text{ d}}{24 \text{ hr}}\right) \times \left(\frac{1 \text{ hr}}{60 \text{ min}}\right) \times \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \\ \frac{1 \text{ furlong}}{1 \text{ fortnight}} &= 166 \ \frac{\mu\text{m}}{\text{s}} \end{aligned}$$

Similarly, to get to km/day we write out all of the unit conversions and compute at the end.

$$\begin{aligned} \frac{1 \text{ furlong}}{1 \text{ fortnight}} &= \left(\frac{1 \text{ furlong}}{1 \text{ fortnight}}\right) \times \left(\frac{220 \text{ yd}}{1 \text{ furlong}}\right) \times \left(\frac{3 \text{ ft}}{1 \text{ yd}}\right) \times \left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) \times \left(\frac{1 \text{ km}}{1000 \text{ m}}\right) \\ &\quad \times \left(\frac{1 \text{ fortnight}}{14 \text{ d}}\right) \\ \frac{1 \text{ furlong}}{1 \text{ fortnight}} &= 0.0144 \text{ km/day} \end{aligned}$$

9. GRR1 1.TB.052. When a force acts over a distance, work is done. If the force is parallel to the motion, the work W is given by W = Fd, where F is the magnitude of the force and d is the distance over which the force acts. (Note that in this context, W stands for work, not for weight.) Find the SI unit of work. Write it two ways: once in terms of base units only and again using the newton ($\mathbf{1N = 1kg \cdot m/s^2}$).

Given that the unit of force is the newton

$$[F] = \text{ N}$$

The unit of force times distance must be

$$[F \cdot d] = \text{ N} \cdot \text{m}$$

Therefore the unit of work is **Nm**. We can also write this unit in terms of kilograms, meters, and seconds as

$$[W] = \text{ N} \cdot \text{m} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$

10. GRR1 1.TB.057. A patient's temperature was 97.0 °F at 8:05 a.m. and 101.0 °F at 12:05 p.m. If the temperature change with respect to elapsed time was linear throughout the day, what would the patient's temperature be at 3:35 p.m.?

The relationship between the temperature and the time is linear, so we can use the equation of a line to determine the patient's final temperature. Recall that the slope of a line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where (x_1, y_1) and (x_2, y_2) are any two points on that line. The slope of this line will be

$$m = \frac{T_2 - T_1}{t_2 - t_1}$$

where T_1 is the temperature at the first time, t_1, and T_2 is the temperature at the second time, t_2. Since the slope is the same for *any* two points we can also say that the slope is equal to

$$m = \frac{T_3 - T_1}{t_3 - t_1}$$

where T_3 is the temperature at the first time, t_3. We can now set the two equations for the slope equal to each other.

$$\frac{T_3 - T_1}{t_3 - t_1} = \frac{T_2 - T_1}{t_2 - t_1}$$

Solving for the final temperature, T_3, we get

$$T_3 - T_1 = \left(\frac{T_2 - T_1}{t_2 - t_1}\right)(t_3 - t_1)$$

$$T_3 = \left(\frac{T_2 - T_1}{t_2 - t_1}\right)(t_3 - t_1) + T_1$$

The final temperature is then

$$T_3 = \left(\frac{101.0\ ^\circ\mathrm{F} - 97.0\ ^\circ\mathrm{F}}{12:05\ \mathrm{pm} - 8:05\ \mathrm{am}}\right)(3:35\ \mathrm{pm} - 8:05\ \mathrm{am}) + 97.0\ ^\circ\mathrm{F}$$

$$T_3 = \left(\frac{4.0\ ^\circ\mathrm{F}}{4\ \mathrm{hr}}\right)(7.5\ \mathrm{hr}) + 97.0\ ^\circ\mathrm{F}$$

$$T_3 = \left(1.0\frac{^\circ\mathrm{F}}{\mathrm{hr}}\right)(7.5\ \mathrm{hr}) + 97.0\ ^\circ\mathrm{F}$$

$$T_3 = 104.5\ ^\circ\mathrm{F}$$