Chapter 26 Lecture Notes

Physics 2424 - Strauss

Formulas:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \qquad L = L_0\sqrt{1 - v^2/c^2}$$

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$E = m_0c^2 + \text{KE} = mc^2 \qquad \text{KE} = mc^2 - m_0c^2$$

$$p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

$$E^2 = p^2c^2 + m_0^2c^4$$

$$u = \frac{v + u'}{1 + vu'/c^2}$$

There were two revolutions in physics in the twentieth century. One of them is quantum mechanics. Quantum mechanics has its major impact when dealing with objects on a very small scale. The other is special relativity. Special relativity has its major impact when velocities are very fast.

1. Reference Frames

The theory of special relativity deals with how two different observers in different reference frames would view the same events. The reference frame is the point of view of the observer. For instance, if I threw a ball up on a train, it would appear to go straight up and come straight down. But to an observer who wasn't on the train, it would appear to follow a parabolic path. Special relativity only deals with observers who are in an *inertial* reference frame. An inertial reference frame is a reference frame that is not accelerating. It may be moving, but it can not be accelerating. So riding in a car at a steady velocity is an inertial reference frame, but sitting on a merry-go-round is not because as you are sitting, you are experiencing centripetal acceleration.

2. Postulates of Special Relativity

The special theory of relativity is, in some sense, a misnomer, for this theory is based on two absolute principles. Because these two principles are completely absolute, there are other things that were once considered absolute, but are now relative, like length, time, mass, momentum, and energy. The two absolute principles are the two postulates of the special theory of relativity. They are

1. The laws of physics are the same in all inertial reference frames. There is no preferred, or absolute, inertial reference frame. The laws themselves are absolute.
2. The speed of light in a vacuum is always measured to be the same value in any inertial reference frame, independent of the speed of the source or the observer.

So the laws of physics, and the speed of light in a vacuum are absolutes. Length, time, mass, momentum, and energy are dependent on the reference frame of the observer. Therefore, it is a postulate of the special theory of relativity, that if I am in a spaceship traveling at the speed of light, and I turn on my headlights, the light still moves away from me at the speed of light.

3. CONSEQUENCES OF SPECIAL RELATIVITY

3.1 Simultaneity

What does it mean for two events to happen simultaneously? Suppose that I am standing at the 50 yard line of a football stadium, and I notice that two people on opposite goal lines catch a pass at the same time. I conclude that the two events must have happened at the same time since I am equal distance from each goal line and I know that $\Delta t = x/v = x/c$. The two events were simultaneous. Now suppose that I see someone at one goal line catch a pass before someone at the other goal line. I conclude that the events were not simultaneous since I am the same distance from the two events, and I see one happen before the other.

Now I do the same experiment but instead of standing on the 50 yard line, I am standing in the middle of a railroad car. As one railroad car on one train passes another railroad car on a different train lightning strikes each end of the car simultaneously, from my perspective. That is, the light from each of the lightning strikes reaches my eyes at the same time. However, because a person on the other train is moving relative to me, the light from one of the lightning strikes reaches him before the light from the other lightning strike. (See figure 26-6). He concludes that the events were not simultaneous since the light from one event reaches him before the light from the other event, and they both occurred at equal distances from him. So I think the events were simultaneous, and he thinks they were not. We see that the idea of whether two events were simultaneous or not does not have an absolute answer in special relativity.

3.2 Time Dilation

In fact, time itself does not have an absolute meaning in special relativity. Suppose I am sitting on a moving rocket ship and put a light source on the floor. I then bounce the light off of a mirror on the ceiling which is a distance D from the floor. From my perspective it took the light the amount of time $\Delta t_0 = 2D/c$ to go that distance. Now suppose someone is not on the rocket ship is watching the light bounce off the mirror. To a person who is not in the rocket ship, the rocket ship appears to be moving at a velocity of v. This person will say that the light did not go a distance D, but instead went a distance $2\sqrt{D^2 + L^2}$ = $2\sqrt{D^2 + v^2\Delta t^2/4}$ since $L = v\Delta t/2$. So the time for that person is

$$\Delta t = 2\sqrt{D^2 + v^2\Delta t^2/4}/c,$$

and squaring both sides, dividing by $(\Delta t)^2$, and solving for Δt gives

$$(\Delta t)^2 = (4D^2 + v^2\Delta t^2)/c^2$$
$$1 = (4D^2/\Delta t^2 + v^2)/c^2$$
$$\Delta t^2 = 4D^2/\{c^2(1-v^2/c^2)\}$$
$$\Delta t = 2D/\{c\sqrt{1 - v^2/c^2}\}$$

Because $\Delta t_0 = 2D/c$, this becomes

$$\Delta t = \Delta t_0 / \sqrt{1 - v^2/c^2}$$

where Δt is the time measured by an observer in a reference frame where the two events do not occur at the same location and Δt_0 is called the *proper time* and is the time measured by an observer in a reference frame where the two events do occur in the same location. (Whenever we see the word "proper" it means in a reference frame where the two events appear to happen in the same place.) Note that Δt is greater than Δt_0 so the phenomena is known as *time dilation.* This states that **according to a stationary observer, a moving clock runs more slowly than an identical stationary clock.**

PROBLEM: An astronaut on the earth has a heartbeat rate of 70 beats/min. When the astronaut is traveling in a spaceship at $0.90c$, what will be the rate of his heart (a) measured by an observer in the spaceship, and (b) measured by an observer on the earth.

Note that if the speed is relatively slow, even 100,000 miles/hour (59600 m/s = $2.0 \times 10^{-4}c$). (This speed would take one to the moon in about 2.5 hours), then 1.00000000 minute becomes 1.00000002 minutes. These effects do not become noticeable until one gets VERY close to the speed of light. Even at a speed of 10% of the speed of light, 3.0×10^7 m/s, then 1 minute becomes 1 minute and 0.302 seconds. However, time dilation has been shown to be true using either atomic clocks at relatively slow speeds, or elementary particles moving very close to the speed of light.

The key to all of relativity will be this relationship $1/\sqrt{1-v^2/c^2}$ which is often given the symbol γ. At 2 ten-thousandths of the speed of light (59600 m/s) we find that γ is only 1.00000002 and that normal quantities we measure do not change much. When the velocity of an object is as high as one hundredth of the speed of light γ is still close to 1. It is only 1.00005. When the velocity is 1 tenth of the speed of light γ is still only 1.005. (So even at 1/10 the speed of light, clocks only slow down by 0.5%). However, if the velocity is 95% of the speed of light, then γ is 3.203 (so the time measured would increase by a factor of 3.2). This quantity (γ) becomes large only when the velocity is very close to the speed of light.

3.3 Length Contraction

If time is relative, but the speed of light is an absolute, then one would expect that distance (which is speed times time) is also relative. Indeed, it is. When two reference frames are moving at a speed of v relative to each other we find that

$$v = L/\Delta t = L_0/\Delta t_0$$
$$L = L_0\sqrt{1-v^2/c^2}$$

So length is not absolute. We find that **according to a stationary observer, a moving object is shorter than an identical stationary object.** This is called length contraction. We find that *length contraction only works in the direction of the relative velocity of the two reference frames*. Again, L_0 is the length of the object in a reference frame that is stationary with respect to the object and L is the length of the same object when viewed from a reference frame that is moving a velocity v with respect to the first reference frame.

PROBLEM: A UFO moves by an observer at a speed of $0.90c$. If the observer measures the UFO to be 150 m long along the direction of motion and 50 m high, how big is the UFO according to the pilot?

3.4 Mass Increase

It can also be shown that the mass of an object increases as its relative velocity increases. We find that

$m = m_0/\sqrt{1-v^2/c^2}$.

PROBLEM: Suppose I throw a ball with a mass of 0.50 kg at a speed of 0.80*c*. What is the mass of the ball in flight?

4. RELATIVISTIC MOMENTUM

Since momentum is defined as mass time velocity, we find that the true momentum of an object can only be calculated using its relativistic mass. That is,

$p = mv = m_0 v/\sqrt{1-v^2/c^2}$.

Of course, unless the velocity is extremely great, then $m \approx m_0$ and these two equations are the same. As an object's speed increases its mass increases. Since the force it takes to accelerate an object is $F = ma$, as its mass increases, the force it takes to accelerate the object even more also increases. As v approaches c, the mass gets closer and closer to infinity. For v to actually reach c, the mass would be infinite, and the corresponding force required to reach that speed would have to be an infinite amount of force. Consequently, nothing with mass can ever actually reach the speed of light. Only objects without mass can reach the speed of light. Consequently, photons, which are light themselves, and therefore, move at the speed of light, must be massless particles.

5. RELATIVISTIC MASS AND ENERGY

In relativity, Einstein showed that the kinetic energy of an object is given by

$\text{KE} = mc^2 - m_0c^2$.

The total energy of the object is given by

$E = \text{KE} + m_0c^2 = mc^2 = m_0c^2/\sqrt{1-v^2/c^2}$

This equation for energy is always correct. Even when an object has no kinetic energy it still has an energy equal to m_0c^2 which is called the rest energy of an object. It is the energy that the object has even when it is not moving. We have seen that it is this rest energy (the energy inherent in the mass of the particle) which is released during nuclear reactions. We can rewrite this equation for energy in the following way (the derivation is on page 765).

$E^2 = m_0^2c^4 + p^2c^2$

This equation is widely used for the total energy. Like the equation for energy above, both of these equations are valid regardless of the speed of the object. Page 764 shows that for low speeds the equation

$$E = \text{KE} + m_0c^2 = m_0c^2/\sqrt{1 - v^2/c^2}$$

reduces to the more familiar equation $\text{KE} = (1/2)m_0v^2$.

<u>PROBLEM:</u> (a) What is the total energy and the kinetic energy of an electron moving at $0.850c$? (b) What would the kinetic energy be if relativity was not considered? The rest energy of an electron is 0.511 MeV. (Remember that when the rest energy is given in electron volts, it has already been multiplied by c^2).

This is very different than the true relativistic answer because the speed is so high. (Note however that if the speed were only $0.10c$ the relativistic answer would be 0.00257 MeV, and the nonrelativistic answer would be 0.00256 MeV. So they would be almost identical even at 10% of the speed of light).

6. ADDITION OF VELOCITIES

Finally, we discuss the relativistic addition of velocities. Suppose that you are riding in a rocket ship going a certain speed v with respect to a stationary observer. You throw a ball out that has a speed of u' relative to you. What speed will the observer think the ball is going. You would first think that the stationary observer sees the ball going $u = v + u'$, but that is not correct. Again Einstein showed that the correct relationship should be

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

We see that when v and u' are both less than or equal to c then the sum of these two velocities, u, can never be greater than the speed of light itself. When solving problems, remember that u' is the velocity as viewed in a frame of reference that is at rest relative to the second object, and v is the velocity of that frame of reference relative to one in which both objects are moving. (In problem 46 u and v are given, but not u').

<u>PROBLEM:</u> On a spaceship traveling at $0.80c$ a bullet is shot which travels at $0.70c$ relative to the spaceship. What speed does an observer on earth measure the bullet to be traveling?

PROBLEM: On a spaceship traveling at $0.80c$ headlights are turned on which have a velocity of c relative to the spaceship. What speed does an observer on earth measure the light from the headlights to be traveling?