Inhomogeneities and the Intergalactic Distance-Redshift Relation

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Abstract. Modifications in Friedmann-Lemaître-Robertson-Walker (FLRW) Hubble diagrams caused by mass density inhomogeneities are used to illustrate possible effects on a determination of the mass parameter Ω_m and the cosmological constant Λ. The values of these parameters inferred from a given set of observations depend on the fractional amount of matter in inhomogeneities and can differ from those obtained by using standard FLRW Hubble diagrams by as much as a factor of two.

1. Introduction

The pressure-free FLRW models of GR have, for a very long time, been assumed to adequately describe the large scale geometry of our universe but only recently have observational techniques become available which promise to determine values for all three of their requisite parameters $\{H_0,\ \Omega_m,$ and $\Omega_\Lambda \equiv \Lambda c^2/(3H_0^2)\}$ and implicitly test this assumption. As an example, corrected magnitudes and redshifts (m-z) for Type Ia supernovae (SNe Ia) are measured, plotted, and compared with theoretical $m(H_0,\ \Omega_m, \Omega_\Lambda; z)$ curves computed for the FLRW models [1–3]. Because these models are isotropic and homogeneous, and our universe appears quite inhomogeneous, modifications in the FLRW predictions have long been proposed and estimated [4,5]. When a wide angle measurement of the CMB is made the average mass density of the universe (the FLRW value for ρ_0) likely exists within the radio beam collected by the antenna. However, when small objects such as SNe Ia are observed at $z < 1$, a mass density significantly less than the average is 'likely' to be in the observing optical path. In particular, if the underlying mass density approximately follows luminous matter (*i.e.,* associated with bounded galaxies) then effects of a diminished mass density in the observing beams on relations like $m(\Omega_m, \Omega_\Lambda; z)$ are important. The majority of currently observed SNe Ia are not being seen through foreground galaxies and whether or not this is due to selection (rather than statistics) is not important. If the objects observed do not have the average FLRW mass density ρ_0 in their foregrounds then the FLRW m-z relation does not apply to them. Ultimately some SNe Ia should exist behind foreground galaxies and for these, m-z should be computed using the lensing formulas. These formulas [6,7] contain source-observer, deflector-observer, and source-deflector distances, respectively D_s, D_d, and D_{ds}, all of which depend on the mass density in the observing beam, **excluding** the deflector. These distances will not be given by the standard FLRW result if the observing beam contains less than the average FLRW mass density, but instead given by the 'intergalactic' distance discussed here.

2. Inhomogeneous Optics

Hubble diagrams (or equivalently distance-redshift relations) for inhomogeneous universes are found by integrating the appropriate equation for the cross sectional area $A(z)$ of a beam propagating through them (see [5,8,9]). When the only inhomogeneities are galaxies and their converging/lensing effects are taken into account by lensing formulas as indicated above, the appropriate equation to solve is:

$$(1+z)^3\sqrt{1+\Omega_m z+\Omega_\Lambda[(1+z)^{-2}-1]}\times \frac{d}{dz}(1+z)^3\sqrt{1+\Omega_m z+\Omega_\Lambda[(1+z)^{-2}-1]}\,\frac{d}{dz}\sqrt{A(z)} + \frac{(3+\nu)(2-\nu)}{4}\Omega_m(1+z)^5\sqrt{A(z)}=0, \tag{1}$$

where the parameter ν is defined by the fraction of matter $\rho_I/\rho_0 = \nu(\nu+1)/6$ in the universe existing in inhomogeneities and excluded from the observing beams (*e.g.,* galactic matter). The luminosity distance-redshift relation is given by integrating (1) from source to observer and evaluating $D_\ell^2 \equiv (A|_0/\delta\Omega)(1+z)^2$, *i.e.,* the area at the observer divided by the solid angle at the source and multiplied by two factors of the redshift. Eqn. (1) was recently recognized [9] as a Lame$'$ equation and analytic solutions were given in terms of Heun functions. Because this $D_\ell(z)$ comes from neglecting optical effects of galactic mass it might be reasonably dubbed the 'intergalactic' luminosity distance-redshift relation thus giving on a log scale, the 'intergalactic' Hubble diagram.

3. Errors Made by Using the Wrong Hubble Diagram

With a little effort the reader can see from (1) how increasing from the FLRW value of $\nu = 0$ (100% homogeneous) to $\nu = 2$ (100% clumpy) increases the area at observation and hence decreases the apparent luminosity of an object. If the wrong Hubble diagram is used, incorrect values of two of the three cosmic parameters will be obtained. Figures 1 and 2 are intended to illustrate just how large an error can be made.

If the distance modulus of a source such as SN 1997ap at $z = 0.83$ were **precisely** known (*e.g.,* see the two sample horizontal lines) then a determination of Ω_m could be made from Figure 1, assuming Ω_Λ were somehow known; or from Figure 2, a determination of Ω_Λ could be made if Ω_m were somehow known. From Fig. 1 the reader can easily see that the determined value of Ω_m depends on the clumping parameter ν. The Ω_m value will be about 95% larger for a $\nu = 2$ completely clumpy universe than it will be for a $\nu = 0$ completely smooth FLRW universe. Equivalently, Ω_m could be underestimated by as much as 50% if FLRW is used. The maximum underestimate is reduced to 33% at the smaller redshift of $z = 0.5$ (see a similar result for $\Omega_\Lambda = 0$ in [8]). These conclusions are not sensitive to the value of Ω_Λ.

Slightly different conclusions follow from Fig. 2 about Ω_Λ. The discrepancy in the determined value of Ω_Λ is $\Delta\Omega_\Lambda \sim -0.14$ for $\nu = 2$ compared to $\nu = 0$, and is not sensitive to the distance modulus. The discrepancy is halved, $\Delta\Omega_\Lambda \sim -0.07$, at a smaller redshift of $z = 0.5$.

4. Concluding Remarks

Numerous arguments have been made since the '60s against the existence of any effect on apparent magnitudes such as given here (see [9] for current refs. to dissenting opinions). These are almost all weak-lensing arguments, valid as long as density perturbations don't produce multiple images (or absorb photons), *i.e.,* the FLRW result will coincide with the theoretical mean of weak lensing observations. Even if weak lensing arguments can be extended to strong lensing perturbations, *e.g.,* by not resolving separate images, the theoretical mean of a distribution of magnitudes may be of little use in determining Ω_m or Ω_Λ. If mass is as inhomogeneous as luminosity, the distribution of magnitudes is expected to be so skewed as to make the mean statistically insignificant. The 'most likely' value should be far more useful in a determination of the cosmic parameters. However, to determine the most likely m-z not only requires knowledge of the average mass density ρ_0, it requires the modeling of galaxies masses etc. What can be determined by fixing a single additional parameter ν (which proportions total FLRW mass density into an intergalactic component and a galactic component) is the 'intergalactic' Hubble curve. This m-z can be much more useful in determining Ω_m or Ω_Λ than the mean Hubble curve, *e.g.,* if galaxies are not larger than they appear optically, the distribution of magnitudes (at a given redshift) is expected to peak much closer to the intergalactic value than to the mean value (see the numerical work [10]). Additionally the effects of galaxy lensing can be controlled by simple selection. All that is required is that the observed SNe Ia are separated into those with foreground galaxies and those without. Those without should be fit to the intergalactic Hubble curve and those with (when any are found) could be included by correcting for lensing and/or by averaging in with the others to see if the weak-lensing FLRW value can be obtained. Only with enough unbiased and absorption corrected data will the FLRW Hubble curve be useful.

The intergalactic Hubble curve contains the additional parameter ν; however, if luminous matter is essentially the whole story most matter is in galaxies and one can put $\nu = 2$ as a good approximation. Use of the intergalactic Hubble curve is then, in principle, no more involved than use of the standard FLRW Hubble curve. The division of ρ_0 into galactic and intergalactic parts for gravitational-optics purposes seems simplistic but it is certainly less simplistic than ignoring optical effects of inhomogeneities altogether as is done by using FLRW.

References

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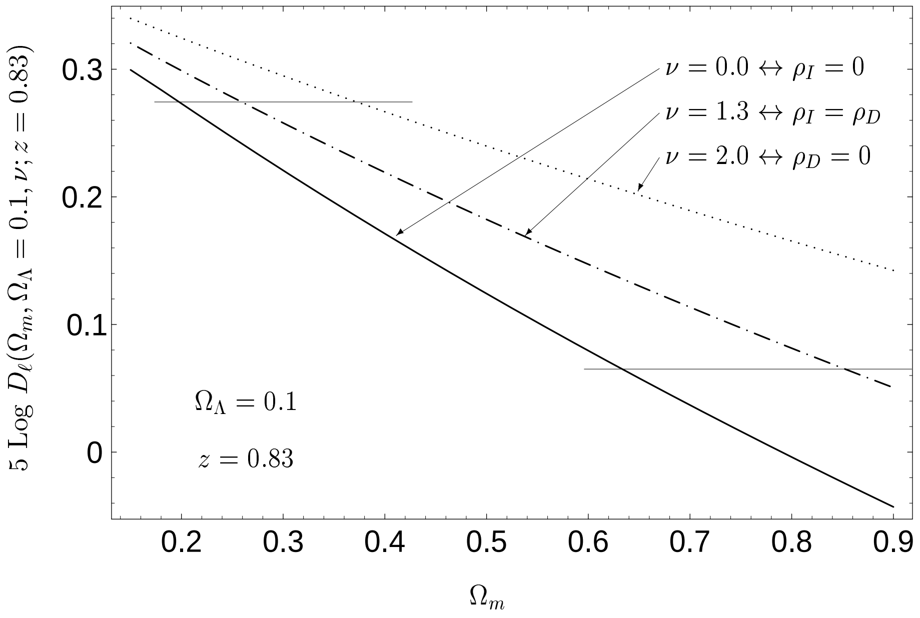

Figure 1. Magnitude, $5 \log_{10} \frac{H_0}{c} D_\ell(\Omega_m, \Omega_\Lambda = 0.1, \nu; z = 0.83)$, as a function of Ω_m for three values of ν.

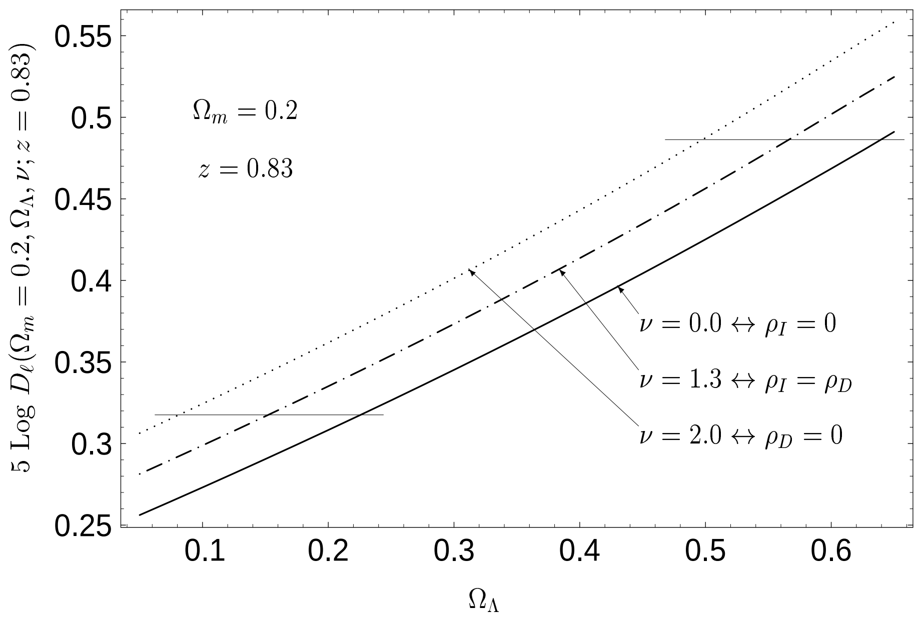

Figure 2. Magnitude, $5 \log_{10} \frac{H_0}{c} D_\ell(\Omega_m = 0.2, \Omega_\Lambda, \nu; z = 0.83)$, as a function of Ω_Λ for three values of ν.