Cosmology With A Dark Refraction Index

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(Dated: July 21, 2008)

Abstract

We review Gordon's optical metric and the transport equations for the amplitude and polarization of a geometrical optics wave traveling in a gravity field. We apply the theory to the FLRW cosmologies by associating a refraction index with the cosmic fluid. We then derive an expression for the accumulated effect of a refraction index on the distance redshift relations and fit the Hubble curve of current supernova observations with a non-accelerating cosmological model. We also show that some observational effects caused by inhomogeneities, e.g., the Sachs-Wolfe effect, can be interpreted as being caused by an effective index of refraction, and hence this theory could extend to other speed of light communications such as gravitational radiation and neutrino fluxes.

PACS numbers: 04.40.Nr, 98.80.-k, 95.36.+x, 95.35+d

Keywords: Cosmology; Distance Redshift; General Relativity

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I. INTRODUCTION

The study of the intergalactic medium (IGM) has long been an important field in both astrophysics and cosmology. Current study of the influence of the IGM on distance redshift relation is mainly focused on light absorption, e.g., the magnitudes of supernovae are corrected for light absorption by the IGM when drawing a Hubble diagram. We know that light paths in a dielectric medium are different from those in vacuum. According to classical electrodynamics light is altered in both speed and direction (light refraction). The impact of light refraction on the cosmological distance redshift relation is interesting from both a theoretical and observational point of view. An interesting and useful theoretical tool to study light refraction in curved spacetime is Gordon's optical metric [1]. The idea of the optical metric is simple, any solution to Maxwell's equations in a curved spacetime filled with a fluid whose electromagnetic properties can be described by a permittivity $\epsilon(x)$ and a permeability $\mu(x)$ can be found by solving a slightly modified version of Maxwell's equations in a related spacetime with vacuum values for the permittivity and permeability, i.e., with $\epsilon(x) = 1$ and $\mu(x) = 1$.

Our motive for undertaking this work was two fold. First we wanted to revive the old optical metric theory of Gordon [1] and show how one applies it to a modern cosmological model. The Second was to join in current efforts to find alternative explanations of the recently observed cosmic acceleration besides the existence of an exotic $p = -\rho c^2$ material, i.e., besides the cosmological constant (see [2–14]). In Section II we introduce the reader to Gordon's optical metric. In Section III we use the WKB approximation to derive the transport equations for the amplitude and polarization vector of waves moving through a spacetime possessing an optical metric. Readers not interested in the mathematical details can skip this section. In Section IV we construct the optical metric for Friedmann-Lemaître-Robertson-Walker (FLRW) models whose cosmological fluid possesses a spatially homogeneous and isotropic refraction index $n(t)$. We use the optical metric to derive both apparent-size, $d_A(z_n)$, and luminosity, $d_L(z_n)$, distance-redshift relations. In Section VI we fit the $d_L(z_n)$ of our index of refraction model to current supernova data. Without a cosmological constant Λ, we also estimate the value of $n(t)$ required to produce roughly the apparent size distance $d_A(z)$ at last scattering required by the position of the acoustic peaks in the angular power spectrum of the WMAP data. In Section VII we draw our conclusions.

II. THE OPTICAL METRIC

In GR type theories, a gravity field is described by a metric g_{ab} on a four dimensional manifold (we use a +2 signature here). We additionally assume the presence of an arbitrarily moving medium with normalized 4-velocity $u^a u_a = -1$ that fills spacetime. For simplicity, we also assume that the fluid's electromagnetic properties are linear, isotropic, transparent, and non-dispersive; and can be summarized by two scalar functions: a permittivity $\epsilon(x^a)$ and a permeability $\mu(x^a)$. Following Ehlers [16] in this section, we write the two electromagnetic bivectors as $F^{ab}(B, E)$ and $H^{ab}(H, D)$. They satisfy Maxwell's Equations[1]

$$\begin{aligned} \partial_{[a} F_{bc]} &= 0, \\ \nabla_b H^{ba} &= \frac{4\pi}{c} J^a, \end{aligned} \tag{1}$$

with constitutive relations

$$\begin{aligned} H^{ab} u_b &= \epsilon F^{ab} u_b, \\ F_{[ab} u_{c]} &= \mu H_{[ab} u_{c]}. \end{aligned} \tag{2}$$

(For a familiar example, take Minkowski spacetime with the fluid at rest, $u^a = (1, 0, 0, 0)$.)

The optical metric of Gordon [1] is defined as

$$\bar{g}_{ab} = g_{ab} + (1 - \frac{1}{\epsilon\mu}) u_a u_b, \tag{3}$$

with inverse

$$\bar{g}^{ab} = g^{ab} + (1 - \epsilon\mu) u^a u^b. \tag{4}$$

The combination $\epsilon\mu$ is related to the usually defined refraction index $n(x) \equiv \sqrt{\epsilon\mu}$. To relate covariant derivatives of the two metrics and to obtain Eq. (9) below the relationship of the two determinants is needed

$$\det\{\bar{g}_{ab}\} = \frac{1}{\epsilon\mu} \det\{g_{ab}\}. \tag{5}$$

At this point we have one differentiable manifold with two metrics or equivalently two related spacetimes, the physical and the optical. In this paper we are primarily interested in the dynamics of a particular type of physical field (radiation) in the optical spacetime. All physical objects are described by tensor fields in physical spacetime and those of interest here

[1] Square brackets [ ] symbolize complete anti-symmetrization of the enclosed indices.

have associated fields in optical spacetime. Where necessary we denote optical spacetime fields with a bar. The optical equivalent of the physical covariant Maxwell field F_{ab} is F_{ab} itself; hence the homogeneous Maxwell equations are satisfied in both spacetimes, and both share covariant 4-potentials. Using the optical metric the two constitutive equations can be written as a single equation

$$H^{ab} = \frac{1}{\mu}\bar{F}^{ab} \equiv \frac{1}{\mu}\bar{g}^{ac}\bar{g}^{bd}F_{cd}, \tag{6}$$

as can be seen by first expressing the contravariant Maxwell field in optical spacetime as

$$\bar{F}^{ab} = [F^{ab} - (1-\epsilon\mu)u^a F^b{}_c u^c + (1-\epsilon\mu)u^b F^a{}_c u^c], \tag{7}$$

and then contracting separately with u_b and $\epsilon_{abcd}u^c$, where ϵ_{abcd} is the completely anti-symmetric Levi-Civita symbol. Here the metric dependent bivector $\bar{F}^{ab}$ is the metric independent 2-form F_{ab} raised using the optical metric Eq. (4) rather than the physical metric g_{ab}. By using the identity relating the contracted Christoffel symbols to the metric's determinant

$$\left\{ {d \atop c\,d} \right\} = \partial_c \log\sqrt{-\det\{\mathrm{g_{ab}}\}}\ , \tag{8}$$

and Eq. (5), Maxwell's equations (1) can be written using the optical metric as

$$\begin{aligned} \partial_{[a}F_{bc]} &= 0, \\ \bar{\nabla}_b\left(e^{2\phi}\,\bar{F}^{ba}\right) &= \frac{4\pi}{c}\bar{J}^a \equiv \frac{4\pi}{c}\sqrt{\epsilon\mu}\,J^a, \end{aligned} \tag{9}$$

where $e^{2\phi} \equiv \sqrt{\epsilon/\mu}$ and the covariant derivatives are taken using the optical metric (see Ehlers [16] for details when $J^a = 0$). The form of the inhomogeneous equation is slightly modified and hence slightly more complicated, i.e., the $e^{2\phi}$ term is present in Eq. (9); however, the advantage is that there are no constitutive equations (2) to deal with, i.e., $\bar{F}^{ab}$ is just F_{ab} raised with the optical metric. Solutions constructed in the optical spacetime via Eq. (9) directly translate to solutions in physical spacetime via Eq. (6). Because the vacuum Maxwell equations are conformally invariant, any metric conformally related to Gordon's can be used to generate H^{ab}, see [16]. In the next section we show that light waves travel along null geodesics at the speed c in the optical spacetime (and hence in any conformally related spacetime), whereas the corresponding waves travel at speed c/n in physical spacetime.

III. GEOMETRICAL OPTICS APPROXIMATION

In this section we follow Sachs [15] and Ehlers [17] but use vectors rather than spinors to derive the transport equations for the amplitude and polarization of a geometrical optics wave. Readers not interested in the tensor calculus details can skip this section. We assume that the electromagnetic wave is planar on a scale large compared with the wavelength, but small compared with the curvature radius of spacetime. We write the covariant (and metric independent) field tensor as

$$F_{ab} = \Re\left\{ e^{iS/\lambda\!\!\!^{-}} \left(A_{ab} + \frac{\lambda\!\!\!^{-}}{i} B_{ab} + O(\lambda\!\!\!^{-2}) \right) \right\}, \tag{10}$$

where $\lambda\!\!\!^{-}$ is a wavelength related parameter, $S(x^a)$ is the so-called eikonal function and is real, and $\Re\{\cdot\}$ stands for the real part. The A_{ab} term represents the geometrical optics (GO) approximation and the B_{ab} term is its first order correction in both the physical and optical spacetimes. Defining the unitless (also metric independent) wave vector $k_a = \partial_a S$ and inserting Eq. (10) into the vacuum Maxwell equations ($J^a = 0$ in Eq. (9)) we obtain to order $\lambda\!\!\!^{-\,-1}$

$$\begin{aligned} A_{[ab}k_{c]} &= 0, \\ \bar{A}^{ab}k_b &= 0, \end{aligned} \tag{11}$$

and to order $\lambda\!\!\!^{-\,0}$

$$\begin{aligned} \partial_{[a}A_{bc]} + k_{[a}B_{bc]} &= 0, \\ \bar{\nabla}_b \bar{A}^{ab} + \bar{B}^{ab}k_b + 2\bar{A}^{ab}\phi_{,b} &= 0. \end{aligned} \tag{12}$$

All barred contravariant quantities throughout are obtained by raising indices with the optical metric, e.g., $\bar{A}^{ab} = \bar{g}^{ac}\bar{g}^{bd}A_{cd}$; unbarred are obtained by raising with the physical metric g^{ab}. Equations (11) tell us that $\bar{k}^a \equiv \bar{g}^{ab}k_b$ is tangent to null geodesics of the optical metric

$$\begin{aligned} \bar{k}^c k_c &= 0, \\ \bar{k}^b \bar{\nabla}_b \bar{k}^a &= 0, \end{aligned} \tag{13}$$

and that A_{ab} is of the form:

$$A_{ab} = -2k_{[a}\mathcal{E}_{b]}, \tag{14}$$

where $\mathcal{E}_a$ is spacelike and constrained by $\mathcal{E}_a\bar{k}^a = 0$ but has the remaining freedom of definition $\mathcal{E}_a \to \mathcal{E}_a + f(x)k_a$. It is $\bar{\mathcal{E}}^a$ that determines the amplitude and polarization of the GO wave seen by an observer and it is Eqs. (13) that establishes the speed of propagation as c.

Equation (12) tells us that

$$B_{ab} = 2(\mathcal{E}_{[a,b]} - k_{[a}\mathcal{D}_{b]}), \tag{15}$$

with a remaining freedom $\mathcal{D}_a \to \mathcal{D}_a + g(x)k_a$ and gives as the propagation equation for $\bar{\mathcal{E}}^a$

$$\dot{\bar{\mathcal{E}}}^a + \bar{\mathcal{E}}^a\theta + \bar{\mathcal{E}}^a\dot{\phi} = \frac{\bar{k}^a}{2}(\bar{\nabla}_b\bar{\mathcal{E}}^b + k_b\bar{\mathcal{D}}^b + 2\phi_{,b}\bar{\mathcal{E}}^b). \tag{16}$$

The affine parameter derivative symbolized by ' $\dot{}$ ' is the invariant derivative $\bar{k}^b\bar{\nabla}_b$ along the null geodesics generated by the vector field $\bar{k}^a$. If we now split $\bar{\mathcal{E}}^a$ into a scalar amplitude and a unit polarization vector, i.e.,

$$\bar{\mathcal{E}}^a = \mathcal{E}\bar{e}^a,\ \mathcal{E} \geq 0,\ \bar{e}^a\bar{e}^*_a = 1, \tag{17}$$

where $*$ means complex conjugate, the transport equation for the amplitude $\mathcal{E}$ becomes

$$\dot{\mathcal{E}} + \mathcal{E}\theta + \mathcal{E}\dot{\phi} = 0, \tag{18}$$

where θ is the expansion rate of the null rays defined by the vector field $\bar{k}^a$. It is defined by the divergence of $\bar{k}^a$

$$\theta \equiv \frac{1}{2}\bar{\nabla}_a\bar{k}^a = \frac{\dot{\sqrt{A}}}{\sqrt{A}}, \tag{19}$$

and is related to the fractional rate of change of the observer independent area A of a small beam of neighboring rays [15]. Given A, we are able to integrate Eq. (18)

$$\mathcal{E}\left(\frac{\epsilon}{\mu}\right)^{1/4}\sqrt{A} = \mathcal{E}_e\left(\frac{\epsilon_e}{\mu_e}\right)^{1/4}\sqrt{A_e}\ , \tag{20}$$

where the subscript e means evaluate at (or close to) the emitter.

For the calculation at hand we need the amplitude $\mathcal{E}$ only, however, if we were interested in the wave's polarization a suitable choice for $f(x)$ makes the right hand side of Eq. (16) vanish and also makes $\dot{\bar{e}}^a = 0$, i.e., a particular choice for a polarization vector can be made that is parallelly transported along the null geodesics of the GO wave.

The GO approximation is just the $O(\lambda^0)$ term in Eq. (10) i.e.,

$$\bar{F}^{ab} = -2\mathcal{E}\,\Re\{e^{iS/\lambda}\bar{k}^{[a}\bar{e}^{b]}\}. \tag{21}$$

The frequency and wavelength seen by observers comoving with the fluid can be computed using the fact that the phase of the wave changes by 2π when the observer ages by one period of the wave τ, or respectively steps a spatial distance of one wavelength λ in the direction of the wave, $\hat{k}^a$,

$$
\begin{aligned}
2\pi &= -(c\tau\, u^a)\partial_a\left(\frac{S}{\lambdabar}\right) = -\frac{c\tau}{\lambdabar}(u^a k_a), \\
2\pi &= (\lambda\, \hat{k}^a)\partial_a\left(\frac{S}{\lambdabar}\right) = -\frac{\lambda}{\lambdabar} n\,(u^a k_a).
\end{aligned} \tag{22}
$$

The natural choice of the constant parameter $\lambdabar$ is the rationalized wavelength $(\lambda_e/2\pi)$ at the emitter. This requires the eikonal satisfy $n_e(k_a u^a)_e = -1$.

The energy and momentum of this wave as seen by a comoving observer in physical spacetime $(u^a g_{ab} u^b = -1)$ is contained in the Poynting 4-vector

$$
\mathcal{S}^a = -c\,T^a_{\;b}u^b = \frac{c}{4\pi}\left[H^{ac}F_{cb} - \frac{1}{4}\delta^a_b H^{dc}F_{cd}\right]u^b, \tag{23}
$$

where all quantities in Eq. (23) are evaluated in the physical spacetime. When evaluated using Eqs. (6) and (21)

$$
\mathcal{S}^a = \frac{c}{4\pi\mu}\left[\bar{F}^{ac}F_{cb} - \frac{1}{4}\delta^a_b \bar{F}^{dc}F_{cd}\right]u^b = -\frac{c}{8\pi\mu}\mathcal{E}^2\bar{k}^a(k_b u^b). \tag{24}
$$

In the last equality the oscillations have been averaged over. The energy density and 3-d Poynting vector seen by observer u^a are respectively

$$
\begin{aligned}
U &= -\frac{1}{c}\mathcal{S}^a u_a = \frac{1}{8\pi\mu}\mathcal{E}^2(\bar{k}^a u_a)(k_b u^b) = \frac{n^2}{8\pi\mu}\mathcal{E}^2(k_a u^a)^2, \\
\mathcal{S}^a_\perp &= \mathcal{S}^a - c\,U u^a = -\frac{c}{8\pi\mu}\mathcal{E}^2(k_b u^b)\left[\bar{k}^a + n^2(k_c u^c)u^a\right],
\end{aligned} \tag{25}
$$

with magnitude

$$
\mathcal{S}_\perp \equiv \sqrt{S^a_\perp \mathcal{S}_{\perp a}} = \frac{cn}{8\pi\mu}\mathcal{E}^2(k_a u^a)^2 = \mathcal{S}_{\perp e}\frac{A_e}{A}\frac{\tau_e^2}{\tau^2}. \tag{26}
$$

In the last equality we have eliminated the amplitude $\mathcal{E}$ using Eq. (20) and continue using a subscript e to represent quantities evaluated near the emitter. Equation (26) simply says that the energy flux varies inversely with the beam's area and inversely with the square of the period, even in the presence of an index of refraction. We will use this expression in the next section to compute the luminosity distance-redshift relation for FLRW cosmologies that are filled with a transparent optical material.

IV. OPTICAL METRIC FOR ROBERTSON-WALKER (RW) SPACETIMES

The familiar spatially homogeneous and isotropic Robertson-Walker (RW) metric can be written as

$$ds^2 = -c^2\,dt^2 + R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\right\}, \tag{27}$$

where $k = 1, 0, -1$ for a closed, flat or open universe, respectively. The cosmic fluid associated with the RW metric is at rest in the co-moving spatial coordinates (r, θ, ϕ) and hence has a 4-velocity $u^a = \delta^a_t/c$. We assume that the cosmic fluid has associated with it a homogeneous and isotropic refraction index which depends only on the cosmological time t, i.e., $\sqrt{\epsilon\mu} = n(t)$. From Eq. (3) only the $\bar{g}_{tt}$ component of the optical metric is seen to differ from the physical metric, i.e.,

$$\bar{g}_{tt} = -\frac{c^2}{n^2(t)}\,. \tag{28}$$

The radial null geodesics of the optical metric are found by fixing (θ, ϕ) and integrating

$$\bar{ds}^2 = -\frac{c^2}{n^2(t)}dt^2 + R^2(t)\frac{dr^2}{1-kr^2} = 0. \tag{29}$$

Two such geodesics traveling between the origin and a fixed comoving point r satisfy

$$\int_{t_e}^{t_o}\frac{c\,dt}{n(t)R(t)} = \int_{t_e+\Delta t_e}^{t_o+\Delta t_o}\frac{c\,dt}{n(t)R(t)} = \int_0^r\frac{dr}{\sqrt{1-kr^2}} = \mathrm{sinn}^{-1}[r], \tag{30}$$

where we have defined

$$\mathrm{sinn}[r] \equiv \begin{cases} \sin[r] & k = +1, \\ r & k = 0, \\ \sinh[r] & k = -1. \end{cases} \tag{31}$$

In the above equation, (t_e, t_o) and $(t_e + \Delta t_e, t_o + \Delta t_o)$ represent the emitting and receiving times of the two respective null signals. We see from Eq. (30) that the differences in emission and observation times are related by

$$\frac{\Delta t_o}{n(t_o)R(t_o)} = \frac{\Delta t_e}{n(t_e)R(t_e)}, \tag{32}$$

and hence the redshift z_n is given by

$$1 + z_n = \frac{\Delta t_o}{\Delta t_e} = \frac{n(t_o)R(t_o)}{n(t_e)R(t_e)} = \frac{n(t_o)}{n(t_e)}(1+z). \tag{33}$$

We have used z_n as a measure of the observed frequency change but have also kept the usual z which measures the wavelength change. The tangent to the radial null geodesic ($\bar{k}^a k_a = 0$)

can be found directly from Eq. (29) by a variation

$$\bar{k}^a = \alpha \left(\frac{n}{c\,R}, \frac{\sqrt{1-kr^2}}{R^2}, 0, 0 \right), \tag{34}$$

with covariant components

$$k_a = \alpha \left(-\frac{c}{nR}, \frac{1}{\sqrt{1-kr^2}}, 0, 0 \right). \tag{35}$$

The constant α is arbitrary and equivalent to the freedom of choosing an affine parameter along a null geodesics. In the physical metric the light ray has a timelike tangent vector, i.e.,

$$\bar{k}^a g_{ab} \bar{k}^b = n^2(1-n^2)(k_a u^a)^2 = \alpha^2 \left(\frac{1-n^2}{R^2} \right) < 0. \tag{36}$$

The eikonal S of the GO approximation Eq. (21) can easily be found for this covariant vector field k_a assuming the spherical wave originates from an emitter located at $r = 0$

$$S(t,r) = \alpha \left(-\int_{t_e}^{t} \frac{c\,dt}{n(t)R(t)} + \mathrm{sinn}^{-1}[r] \right). \tag{37}$$

To relate α and the constant $\lambda\!\!\!^{-}$ of Eq. (21) to comoving wave length, we use Eqs. (22) and (35)

$$2\pi = -\frac{c\tau}{\lambda\!\!\!^{-}}(u^a k_a) = \frac{\alpha\, c\tau(t)}{\lambda\!\!\!^{-} n(t) R(t)} = \frac{\lambda}{\lambda_e} \frac{R(t_e)}{R(t)}. \tag{38}$$

The last equality results from choosing $\alpha = R(t_e)$ and $\lambda\!\!\!^{-} = \lambda(t_e)/2\pi$ and confirms our interpretation of the conventional $(1+z) = R/R_e$ as the wavelength redshift, see Eq. (22).

We are interested in computing the apparent-size and luminosity distances for RW models with an index of refraction $n(t)$ and must be careful in doing so. The optical metric gives the correct wave trajectories, but because it does not measure distances or times correctly, densities and rates will be incorrect. Because angles, areas, and redshifts are easier to calculate than energy fluxes we start with the apparent size distance-redshift $d_A(z_n)$. We will then compute the luminosity distance $d_L(z_n)$ by using the 3-d Poynting vector of Eq. (25). We use the optical metric in the form given in Eq. (28) with Eq. (27) because the coordinates (t, r, θ, ϕ) have direct physical interpretations in the RW metric itself. For an example, in the local rest frame of the source (observer), the proper time interval is Δt_e (respectively Δt_o) instead of $\Delta t_e/n(t_e)$ (respectively $\Delta t_o/n(t_o)$), and hence the observed shift in periods, $\Delta t_o/\Delta t_e$, is correctly given by Eq. (33). From Eq. (27) the apparent size distance (also called the angular size distance) of a source at coordinates (t_e, r) is just

$$d_A = r\,R(t_e), \tag{39}$$

as seen by an observer at $(0, t_o)$ where the three coordinates (r, t_e, t_o) are constrained by Eq. (30). To give $d_A(z_n)$ we must use Eqs. (30) and (33) to eliminate (r, t_e) in terms of (z_n, t_o). We start by using Eq. (33) to change variables in the remaining integral of Eq. (30) from t to z_n. The following steps are familiar except for the presence of the index of refraction $n(x)$ and the two redshift variables (z_n, z). The dynamical equations of Einstein are used to change from t to R and then to $(1+z) = R_o/R$.

$$\begin{aligned} \mathrm{sinn}^{-1}(r) &= \int_{t_e}^{t_o} \frac{cdt}{n(t)R(t)} = \int_{R_e}^{R_0} \frac{cdR}{n(R)R\,(dR/dt)} \\ &= \frac{c}{R_0 H_0} \int_0^{z(z_n)} \frac{dz}{n(z)h(z)}, \end{aligned} \tag{40}$$

where

$$h(z) \equiv \sqrt{[\Omega_\Lambda + \Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4]}, \tag{41}$$

and where

$$\Omega_k \equiv -\frac{c^2 k}{H_0^2 R_0^2} = 1 - (\Omega_\Lambda + \Omega_m + \Omega_r). \tag{42}$$

The wavelength redshift $z(z_n)$ as a function of the frequency redshift z_n is found by eliminating t_e in Eq. (33). We refer to solutions to Einstein's equations with a RW symmetry as Friedmann-Lemaître-Robertson-Walker or simply FLRW cosmologies. The three Ω constants represent, as usual, current relative amounts of non-interacting gravity sources: vacuum, pressureless matter, and radiation energies. From Eq. (39) we conclude that the apparent size distance-redshift relation for a FLRW cosmology with an index of refraction is

$$d_A(z_n) = \frac{1}{(1+z(z_n))} \frac{c}{H_0} \frac{1}{\sqrt{|\Omega_k|}} \mathrm{sinn}\left[\sqrt{|\Omega_k|} \int_0^{z(z_n)} \frac{dz}{n(z)h(z)}\right]. \tag{43}$$

To derive the luminosity-redshift relation $d_L(z_n)$ knowing $d_A(z_n)$ one ordinarily uses Etherington's [18] result

$$d_L(z) = (1+z)^2 d_A(z). \tag{44}$$

If this result were correct with an index of refraction present, one would need to know which redshift to use, frequency z_n or wavelength z. To know what to choose we evaluate the magnitude of the Poynting vector Eq. (26) and arrive at the correct replacement for Eq. (44). To find the needed area A we evaluate the expansion θ of Eq. (19) using Eqs. (34), (27), (28), and (5). We find a simple result

$$\theta = \frac{\dot{(rR)}}{(rR)}, \tag{45}$$

and hence the beam area $A \propto rR$ from which we have the needed Poynting vector magnitude, Eq. (26),

$$\mathcal{S}_\perp = \mathcal{S}_{\perp e}\frac{4\pi(rR)_e^2}{4\pi(rR)^2}\frac{\tau_e^2}{\tau^2} = \frac{L_e}{4\pi(rR)^2(1+z_n)^2} = \frac{L_e}{4\pi d_L^2}. \tag{46}$$

The latter identity defines the luminosity distance d_L in terms of the total power radiated at the emitter L_e and the flux received, i.e., the Poynting vector at the observer,

$$d_L = rR(1+z_n) = rR_e(1+z)(1+z_n), \tag{47}$$

which agrees with the Etherington result Eq. (44) only if we use one frequency redshift factor $(1+z_n)$ from Eq. (33) and one wavelength redshift $(1+z)$. Equation (46) also confirms a conserved photon number interpretation of the radiation even in the presence of a time dependent index of refraction (which has the potential of taking energy out of the radiation field). It is equivalent to having a fixed number of photons emitted in a time Δt_e each having energy $h\nu_e$ and all being collected over an area $4\pi(rR)^2$ in a time Δt_o but with redshifted energy $h\nu_o$.

V. AN EFFECTIVE INDEX OF REFRACTION INDUCED BY THE SACHS-WOLFE EFFECT

Up to now, we have been considering a refractive index modeled after the one generated by induced electromagnetic polarizations in inter and intra galactic media, dark or otherwise. Lensing has long been interpreted as a gravitationally induced refraction effect, and here we suggest that to 1^{st} order, inhomogeneities in the flat FLRW models are equivalent to effective indices of refraction.

Sachs & Wolfe [25] were two of the first to consider the effect of perturbations of the homogeneous and isotropic models on optical observations. In that classic paper, the authors used perturbations in the flat, i.e., $k=0$, FLRW spacetime to study the angular fluctuations in the CMB. They used a conformally flat version of the metric

$$ds^2 = R^2(\eta)\left[\eta_{ab} + h_{ab}\right] dx^a dx^b \tag{48}$$

with dimensionless coordinates and worked in a comoving gauge to derive the equations governing the evolution of the metric perturbation h_{ab} and perturbations of the energy-momentum tensor δT_{ab}. Here the conformal time coordinate of the flat Minkowski metric

η_{ab} is $x^0 = \eta$ and for the pressureless case is familiarly related to the comoving FLRW time coordinate t by $\eta = (3\,tH_0/2)^{1/3}$. The three Euclidean spatial coordinates are labeled by letters of the Greek alphabet. They then considered the deviations of null geodesics from the unperturbed case and derived the now famous temperature fluctuations in the micro-wave background caused by h_{ab}, see Eq. (42) of Ref. [25]. Among the scalar, vector, and tensor perturbation modes in dust ($p = \delta p = 0$, see Eq. (22) of Ref. [25]), the scalar density perturbations, i.e., the relatively decreasing $A(x^\gamma)$ mode and relatively increasing $B(x^\gamma)$ mode (responsible for the famous Sachs-Wolfe effect) give

$$h_{\alpha\beta} = -\frac{1}{\eta^3}A_{,\alpha\beta} + \delta_{\alpha\beta}B + \frac{\eta^2}{10}B_{,\alpha\beta}\,, \tag{49}$$

and $h_{0a} = 0$. The arbitrarily specified form of the scalar modes are related to the density perturbation $\delta\rho$ through Poisson's equation

$$\delta\rho = \frac{H_0^2}{32\pi G}\nabla^2\left(\frac{6A}{\eta^9} - \frac{3B}{5\eta^4}\right), \tag{50}$$

(See Eq. (22) of Ref. [25]). The time component of the perturbation h_{00} vanishes because of a comoving gauge choice ($u^a \propto \delta^a_0$) and $h_{0\alpha}$ is only present for the rotational perturbations. To connect this metric to Gordon's optical metric we must make the following non-gauge change of coordinates

$$\begin{aligned} x^\alpha &= \bar{x}^\alpha + \frac{1}{2\bar{\eta}^3}A_{,\alpha}(\bar{x}) - \frac{\bar{\eta}^2}{20}B_{,\alpha}(\bar{x}), \\ \eta &= \bar{\eta}\left(1 - \frac{3}{2\bar{\eta}^5}A(\bar{x}) - \frac{1}{10}B(\bar{x})\right), \end{aligned} \tag{51}$$

and obtain to 1^{st} order

$$ds^2 = R^2(\bar{\eta})\left(1 - \frac{3}{\bar{\eta}^5}A(\bar{x}) + \frac{3}{10}B(\bar{x})\right)^2\left\{-\frac{d\bar{\eta}^2}{\left(1 - \frac{6}{\bar{\eta}^5}A(\bar{x}) + \frac{3}{5}B(\bar{x})\right)^2} + d\bar{r}^2 + \bar{r}^2(d\theta^2 + \sin^2\theta d\phi^2)\right\}. \tag{52}$$

This simply says that the Sachs-Wolfe metric is a conformally transformed Gordon metric corresponding to a k=0, FLRW metric with a spacetime index of refraction $n = 1 - 6A(\bar{x})/\bar{\eta}^5 + 3B(\bar{x})/5$. Since conformal transformations don't alter light cones (see [16]) we have arrived at the connection of null geodesics of Sachs-Wolfe's density perturbations with the light curves of an unperturbed FLRW spacetime possessing an index of refraction. In contrast to the homogeneous optical fluid discussed in Section IV, the comoving frame of the index of refraction in Eq. (52) is not the same as the comoving frame of the matter density in Eq. (48). However, they are related by the coordinate change of Eq. (51).

We have our doubts about extending the index of refraction comparison beyond linear perturbations, and make no claims as to that possibility. Such an extension would be quite interesting because old work [26, 27] on non-linear observational effects in Swiss Cheese cosmologies are again in the literature [2–5] also hoping to find sources of apparent acceleration other than a cosmological constant. Work on interpreting effects of local density perturbations on the Hubble curve are numerous [6–14], the results of which can be compared to the above in the 1^{st} order regime.

VI. FITTING SUPERNOVA DATA WITH A REFRACTION INDEX MODEL

In this section we use the index of refraction model of Section IV to fit the current supernova data [28–32]. We use the 178 supernova from the gold sample [2] with redshifts greater than $cz = 7000\,\mathrm{km/s}$, see Fig. 1. The Hubble constant we use is $H_0 = 65$ km/s/Mpc and since we are concerned with the matter dominated era, we exclude radiation ($\Omega_r = 0$). We compare the distance modulus versus redshift, $\mu(z)$, of the concordance model, $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$, $n = 1$ with two $n(z) > 1$ models. The first is a baryonic matter only model ($\Omega_m = 0.04$) and no cosmological constant ($\Omega_\Lambda = 0$) with $n(z) = 1 + 0.1z^2 - 0.045z^3$. The second model includes a dark matter contribution, $\Omega_m = 0.3$, no cosmological constant, and $n(z) = 1 + 0.15z^2 - 0.05z^3$. Also included is a now disfavored dark matter only model, $\Omega_\Lambda = 0$, $\Omega_m = 0.3$, $n = 1$. In the inset of Fig. 1, we use this case to compare with the two $n \neq 1$ models and with the concordance model. The critical redshift region is between $0.2 < z < 1.2$, where most of the supernova data is concentrated. Both $n \neq 1$ models fit the data much better in this region than models with the same Ω parameters but with no refraction. The two refraction indices are plotted in the insets of Fig. 2. As the reader can easily see the effects of a suitable index of refraction $n(z)$ can simulate the accelerating effects of a cosmological constant.

The Supernova data is currently considered the best evidence for the existence of dark energy because of the observed acceleration in the expansion of the universe (see Fig. 7 of

[2] http://braeburn.pha.jhu.edu/~ariess/R06/.

Ref. [29]). For the homogeneous FLRW models, the acceleration is directly related to density and pressure by

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3\frac{p}{c^2}). \tag{53}$$

A true observed acceleration, i.e., $\ddot{R} > 0$, requires $p < -\rho c^2/3$, and hence implies an unusual equation of state such as vacuum energy ($p = -\rho c^2$). What we show here is that an overlooked index of refraction can cause a misinterpretation of the Hubble curve, suggesting an acceleration. In Fig. 2 we plot $H(z)$ and $\dot{R}(z)/R_o = H(z)/(1+z)$ for the two $n \neq 1$ models. They can be compared with similar plots in Ref. [29]. The data points are plotted using flux averaging [33, 34] and uncorrelated redshift binning [35] algorithms. To apply these techniques to non-flat cases, we define

$$\begin{aligned} g(z_n) &\equiv \int_0^{z(z_n)} \frac{dz}{n(z)h(z)} \\ &= \frac{1}{\sqrt{|\Omega_k|}} \mathrm{sinn}^{-1} \left[\frac{\sqrt{|\Omega_k|}}{1+z_n} \frac{H_0}{c} 10^{\frac{\mu}{5}-5} \right], \end{aligned} \tag{54}$$

where μ is the distance modulus

$$\mu = 5 \log \frac{d_L}{1 \text{ Mpc}} + 25. \tag{55}$$

We furthermore defined

$$x_i = \frac{g(z_n^{i+1}) - g(z_n^i)}{z^{i+1}(z_n^{i+1}) - z^i(z_n^i)}, \tag{56}$$

which when averaged inside each bin gives us an estimate of the inverse of the product of $n(z)$ with $h(z) \equiv H(z)/H_0$ (compare with Eq. (5) of Ref. [35]). The presence of an index of refraction produces a degeneracy in determining the value of $H(z)$ and hence in $\ddot{R}(t)$. A suitably decreasing $n(t)$ and a non-accelerating $R(t)$ will mimic an accelerating universe.

We can see that our index of refraction models fit the data well. However, we need to remind the reader that the binned data plotted on the $H(z)$ and $\dot{R}(z)/R_0$ curves are model dependent. The binning process as designed in Ref. [35] requires use of $d_L(x_n)$, i.e., $g(z_n)$ in Eq. (54). Rather than using this technique to argue for an observed accelerating $H(z)$, we argue for an observed $n(z)$ with a non-accelerating $H(z)$. The point we make is that we can fit the $\mu(z)$ data with no Λ, and are able to get rid of the acceleration.

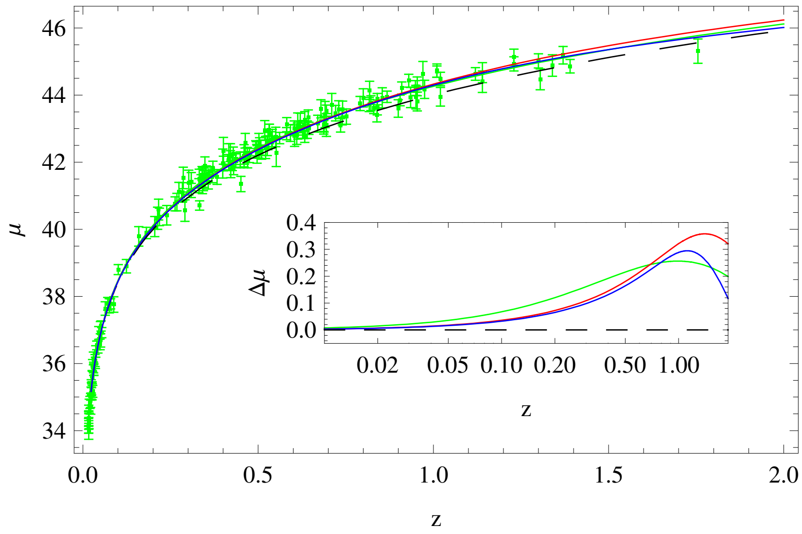

FIG. 1: Distance modulus μ versus redshift z. Dashed curve: $\Omega_\Lambda = 0, \Omega_m = 0.3, n = 1$; Green curve: $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$, $n = 1$; Red curve: $\Omega_\Lambda = 0$, $\Omega_m = 0.04$, $n(z) = 1 + 0.1z^2 - 0.045z^3$; Blue curve: $\Omega_\Lambda = 0$, $\Omega_m = 0.3$, $n(z) = 1 + 0.15z^2 - 0.05z^3$. $\Delta\mu(z)$ are given in the inset. The difference is taken with respect to the fiducial case where $\Omega_\Lambda = 0, \Omega_m = 0.3, n = 1$.

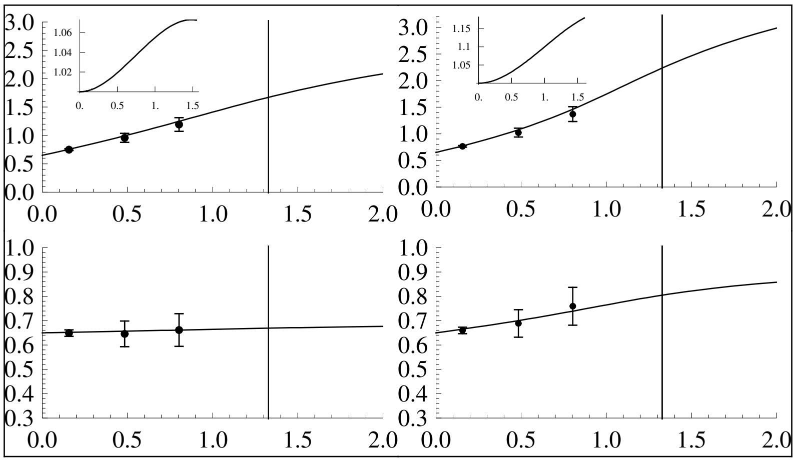

FIG. 2: $H(z)$ (upper panel) and $\dot{R}(z)/R_o$ (lower panel) curves for the two $n \neq 1$ models. Left column parameters: $\Omega_\Lambda = 0$, $\Omega_m = 0.04$, $n(z) = 1 + 0.1z^2 - 0.045z^3$; Right column parameters: $\Omega_\Lambda = 0$, $\Omega_m = 0.3$, $n(z) = 1 + 0.15z^2 - 0.05z^3$.

VII. FLATNESS OF THE UNIVERSE

The conclusion drawn from the latest WMAP data, when combined with the SNe Ia Hubble curve, that vacuum energy exists, depends crucially on many unconfirmed theoretical

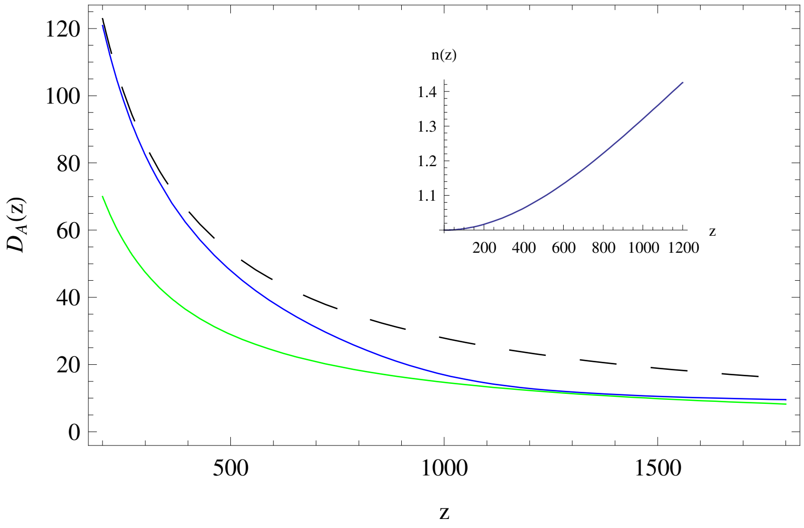

FIG. 3: Dashed curve: $\Omega_\Lambda = 0$, $\Omega_m = 0.3$, $n(z) = 1$; Blue Curve: $\Omega_\Lambda = 0$, $\Omega_m = 0.3$, $n(z) = 1 + 4.45 \times 10^{-7} z^2 - 1.25 \times 10^{-10} z^3$; Green curve: $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$, $n = 1$.

assumptions including the adiabatic power law assumption for the initial perturbation spectrum [36, 37]. Such observations have motivated efforts to find ways to produce a perceived acceleration other than by a real Λ [6–14]. This section is another such effort.

The angular position of the first acoustic peak is commonly believed to be the strongest piece of evidence for the flatness of the universe. The characteristic wavelengths of the acoustic oscillations at the last scattering surface depend very weakly on Λ, but their observed angular size as seen by us now depends significantly on a combination of Ω_m and Ω_Λ, see Eq. (43). Assuming our Universe is of the FLRW type with no refraction index, a first acoustic peak at $\sim 0.8^o$ is almost fit by a flat universe.[3] With a suitable index of refraction and no cosmological constant we can produce an angular diameter distance comparable to the angular diameter distance of a flat cosmology at any given redshift, independent of the Hubble parameter H_0. In Fig. 3 we have used a $\Omega_\Lambda = 0$, $\Omega_m = 0.3$ model with an index of refraction $n(z)$ shown in the inset. We chose this $n(z)$ because it produces similar distances

[3] Within the context of a power law ΛCDM model ($w = 1$), WMAP data alone does not rule out non-flat models. With a prior on the Hubble constant, $H_0 > 40 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$, or combined with other astronomical observations, such as SDSS LRG sample, HST constraint on the Hubble constant, or SNe data, WMAP data strongly favors a nearly flat universe with nonzero vacuum energy, see Table 12, Fig. 20, and Fig. 21 of [37]. For a more general model of dark energy, e.g., one with a time evolving equation of state parameter $w \neq 1$ instead of a cosmological constant Λ, significant spatial curvature is still allowed even when H_0 is not restricted to be small, see e.g. [38, 39].

over the large redshift range $z > 1000$. To conclude that WMAP implies flatness requires the acceptance of the accuracy of theoretical assumptions beyond the initial perturbation spectrum; e.g., even the accuracy of the optics of homogeneous FLRW models is now being questioned as was pointed out in Section V.

VIII. DISCUSSION

We have reviewed Gordon's optical metric theory [1] which incorporates an index of refraction into its geometry. We then used the optical spacetime to derive the transport equations for the amplitude and polarization of a geometrical optics wave. We applied it to the homogeneous, $\Lambda = 0$, FLRW models and estimated the refraction index needed to fit current SNe Ia and WMAP data. We found that an $n(z) \approx 1.07$ at redshift $z = 1.5$ in a baryon only model, or an $n(z) \approx 1.15$ at $z = 1.5$ in a dark matter model, could easily fit the supernova data (see Fig. 1, and Fig. 2), and that an $n(z)$ as big as 1.3 at the last scattering surface in a dark matter model would give the same angular diameter distance $d_A(z)$ as the concordance model (see Fig. 3).

The question is, where could such an index of refraction come from? If it had its origin in atomic dipole moments or charges in plasmas the densities would have to be much larger than they actually are. A critical mass density now is about $8 \times 10^{-30}\,\mathrm{g \cdot cm^{-3}}$, which translates to $1 \times 10^{-20}\,\mathrm{g \cdot cm^{-3}}$ at $z = 1100$. The density of air on the earth is about $1.2 \times 10^{-3}\,\mathrm{g \cdot cm^{-3}}$, some 10^{17} times denser than the universe at recombination and yet its index of refraction is only $n = 1.0003$. We conclude that there is little hope for a baryon-lepton origin for n. A long shot would be a colorless index of refraction for the mysterious dark matter.

Severe limits have already been estimated on direct interactions of the dark matter particles with photons caused by fractional charge [40, 41] ($q/e < 10^{-5} - 10^{-7}$ depending on the particle's mass) and by electric/magnetic dipole moments [42] (dipole moment $< 3 \times 10^{-16} e\,\mathrm{cm}$). General limits on photon interaction rates have even been estimated by requiring the associated collisional damping scale be small enough to allow structures larger than 100 kpc to form [43]. Proposing an n of unknown source is perhaps outlandish, but not much more than proposing a non-intuitive repulsive cosmological constant Λ to produce acceleration. Even though the latter has become fashionable, we wish to add a dark index of refraction theory to the lists of alternatives to think about.

In this paper we developed the general framework needed for using Gordon's optical metric in cosmological observations, but have applied it only to an index of refraction model which is homogeneous and isotropic. If the dark matter and it's assumed index of refraction were truly homogeneous we could have additionally proposed a redshift dependence for $n(z)$ modeled after a dilute dielectric gas or plasma. However, such a model would still contain unknowns equivalent to ionization densities and/or molecular polarizabilities. Instead we chose a phenomenological expression in the form of a cubic containing two parameters which we adjusted (i.e., $n(z) = 1 + \alpha z^2 + \beta z^3$). Such a simple starting point is prudent because we know the real universe is filled with low density voids, and high density condensations, as well as associated velocity perturbations all of which would modify the refraction index $n(x^a)$. If an index of refraction model such as the one proposed here has merit, future efforts can look into how such perturbations, including local variations in the magnetic field of the intergalactic medium, might impact distance-redshift. However, the optical metric theory Eq. (3) is still the applicable theory. We also leave to the future, further exploration of the equivalence of the optical effects of gravitational inhomogeneities (beyond the linear perturbation results of Sachs-Wolfe in Section V) and our index of refraction proposal. Complete equivalence would be quite interesting and useful in light of the current interest in Swiss Cheese optics [2–5, 26, 27]. Modifications in distance-redshift caused by random spacetime perturbations could then be interpreted as being caused by an effective index of refraction. The idea of an optical metric can also be applied to other massless particles which follow null geodesics in vacuum. If the presence of material causes interference of the propagating waves of the particles, an effective refraction index should exist. For an example, gravitons and some neutrinos are massless and local inhomogeneities, such as the Sachs-Wolfe perturbations discussed in Section V, would alter propagation of their waves [48].

The idea of solving cosmological problems via a changing light speed model is not new, see [44–46]. Ellis [47] has recently pointed out consistency constraints required of such theories. Our proposal is fundamentally different from those cited above in that it is based on a classical electrodynamics analogy (the cosmological fluid simply has an unexpected refraction index which reduces the propagation speed of electromagnetic waves). Because we are not proposing a change in the vacuum light speed c or the limiting causal speed, the proposal is not subject to Ellis's criticism.

Finally we note that since an accelerating universe is consistent with other observations, such as Baryon Acoustic Peaks detected in galaxy surveys [49–51] and the interesting $H(z)$ relation obtained from ages of passively evolving galaxies in [52], additional comparisons with refraction models are in order.

IX. ACKNOWLEDGMENTS

This work was supported in part by NSF grant AST-0707704, and US DOE Grant DE-FG02-07ER41517.

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[1] W. Gordon, Ann. Phys. 22, 421 (1923).

[2] V. Marra, E. W. Kolb, S. Matarrese, and A. Riotto, Phys. Rev. D 76, 123004 (2007).

[3] V. Marra, E. W. Kolb, and S. Matarrese, arXiv:0710.5505M.

[4] T. Mattsson, arXiv:astro-ph/0711.42264.

[5] N. Brouzakis, B. Tetradis, and E. Tzavara, arXiv:astro-ph/0703586v2.

[6] C. H. Chuang, J. A. Gu, and W. Y. P. Hwang, arXiv:astro-ph/0512651.

[7] H. Alnes, M. Amarzguioui, and O. Gron, Phys. Rev. D 73, 083519 (2006).

[8] H. Alnes and M. Amarzguioui, Phys. Rev. D 74, 103520 (2006).

[9] R. A. Vanderveld, E. E. Flanagan, and I. Wasserman, Phys. Rev. D 74, 023506 (2006).

[10] D. J. H. Chung and A. E. Romano, Phys. Rev. D 74, 103507 (2006).

[11] A. E. Romano, Phys. Rev. D 75, 043509 (2007).

[12] E. W. Kolb, S. Matarrese, and A. Riotto, New J. Phys. 8, 322 (2006).

[13] A. Conley et al. Astrophys. J. 664, L13 (2007).

[14] S. Sarkar, arXiv:astro-ph/0710.5307.

[15] R. Sachs, Proc. R. Soc. A 294, 195 (1961).

[16] J. Ehlers, in *Perspectives in Geometry and Relativity*, edited by B. Hoffmann (Indiana Univ., Indiana, 1966), p. 127.

[17] J. Ehlers, Z. Naturforsch. 22 a, 1328 (1967).

[18] I. M. H. Etherington, Philos. Mag. 15, 761 (1933).

[19] Riess et al. Astron. J. 116, 1009 (1998).

[20] Perlmutter et al. Astrophys. J. 517, 565 (1999).

[21] G. F. R. Ellis, in *General Relativity and Cosmology*, edited by R. K. Sachs (Academic, New York, 1971).

[22] J. B. Hartle, *Gravity An Introduction to Einstein's General Relativity* (Addison Wesley, San Francisco, 2003).

[23] E. M. Lifshitz, J. Phys. (Moscow) 10, 116 (1946).

[24] E. M. Lifshitz and I. M. Khalatnikov, Adv. Phys. 12, 185 (1963).

[25] R. K. Sachs and A. M. Wolfe, Astrophys. J. 147, 73 (1967).

[26] R. Kantowski, Astrophys. J. 155, 89 (1969).

[27] R. Kantowski, Astrophys. J. 507, 483 (1998).

[28] Riess et al. Astrophys. J. 607, 665 (2004).

[29] Riess et al. Astrophys. J. 659, 98 (2007), http://braeburn.pha.jhu.edu/~ariess/R06/.

[30] P. Astier, et al. Astron. Astrophys. 447, 31 (2006).

[31] Davis et al. arXiv:astro-ph/0701510.

[32] Wood-Vasey et al. arXiv:astro-ph/0701041.

[33] Y. Wang, Astrophys. J 536, 531 (2000).

[34] Y. Wang and P. Mukherjee, Astrophys. J. 606, 654 (2004).

[35] Y. Wang and M. Tegmark, Phys. Rev. D 71, 103513 (2005).

[36] D. N. Spergel et al. Astrophys. J. Suppl. Ser. 148, 175 (2003).

[37] D. N. Spergel et al. Astrophys. J. Suppl. Ser. 170, 377 (2007).

[38] K. Ichikawa, M. Kawasaki, T. Sekiguchi, and T. Takahashi, arXiv:astro-ph/0605481v2.

[39] K. Ichikawa and T. Takahashi, arXiv:astro-ph/0612739v2.

[40] M. Taoso, G. Bertone, and A. Masiero, arXiv:astro-ph/0711.4996v2.

[41] S. Davidson, S. Hannestad, and G. Raffelt, JHEP 0005, 3 (2000).

[42] K. Sigurdson, M. Doran, A. Kurylov, R. R. Caldwell, and M. Kamionkowski, Phys. Rev. D 70, 083501 (2004).

[43] C. Boehm and R. Schaeffer, Astron. Astrophys. 438, 419 (2005).

[44] A. Albrecht and J. Magueijo, Phys. Rev. D 59, 043516 (1999).

[45] J. D. Barrow, Phys. Rev. D 59, 043515 (1999).

[46] B. A. Bassett, S. Liberati, C. Molina-Paris, and M. Visser, Phys. Rev. D 62, 103518 (2000).

[47] G. F. R. Ellis, Gen. Relativ. Gravit. 39, 511 (2007).

[48] Y. B. Zeldovich and I. D. Novikov, *The Structure and Evolution of the Universe* (Univ. of Chicago, Chicago, 1983), Vol. 2, Chap. 12.

[49] M. Tegmark, et al. Phys. Rev. D 69, 103501 (2004).

[50] M. Tegmark, et al. Astrophys. J. 606, 702 (2004).

[51] D. J. Eisenstein, et al. Astrophys. J. 633, 560 (2005).

[52] J. Simon, L. Verde, and R. Jimenez, Phys. Rev. D 71, 123001 (2005).