September 17, 2008

PHYS 6333 (General Relativity)

Assignment #4, Due Wednesday evening September 24, 2008

1. Read: Section 7.1 of Hughston & Tod.
2. Read: Sections 10.1-.2. and 11.1-.3 of Rindler.
3. Read: Sections 8.1 and 9.1 of Hartle.
   (a) Calculate the connection symbols for the Schwarzschild metric in (t, r, θ, ϕ) coordinates. Use the notation $r_s = 2GM/c^2$. Expect to get the same 6 Γ's you got for Minkowski space in spherical polar coordinates, except 2 will now depend on $(1 - r_s/r)$. Three additional Γ's appear that vanish when $r_s = 0$.
   (b) Try to find $r(t)$ for a radial time-like geodesic. Assume the geodesic originates at r_0 when $t = 0$ and that $r_s < r_0$. Also assume the Geodesic starts with an initial radial velocity V_0

$$V_0 = \frac{\left.\frac{dr}{dt}\right|_{t=0}}{(1 - \frac{r_s}{r_0})} = \left\{ \frac{\left.\frac{dr}{ds}\right|_{t=0}}{\left.\frac{dt}{ds}\right|_{t=0}} \right\} \frac{1}{(1 - \frac{r_s}{r_0})}.$$

   Time-like geodesics satisfy the constraint

$$g_{ab}\frac{dx^a}{ds}\frac{dx^b}{ds} = \pm 1$$

   ($\pm$ depending on the signature of your metric) when they are parameterized by proper distance s. Radial means $\theta =$ constant and $\phi =$ constant. The best you can do at finding $r(t)$ is to give an elliptic integral for $t(r)$.

   For what values of $V_0 > V_{escape}$ will the partice escape to "∞"?

   If $V_0 < V_{escape}$ to what maximum r will the particel reach?