The Muon Anomalous Magnetic Moment: A Probe for the Standard Model and Beyond

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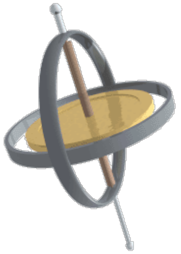
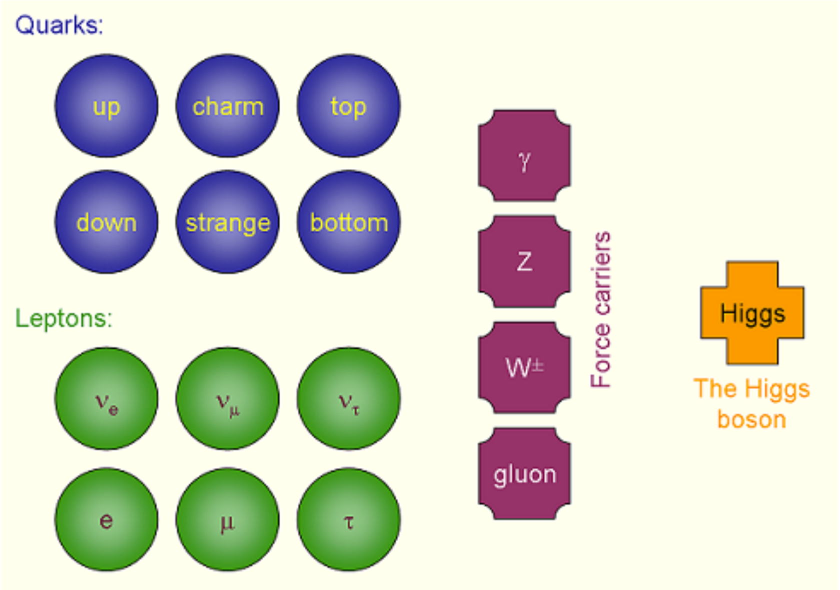
Outline

- Physics Motivation
- Properties of the Muon
- Brookhaven g-2 Experiment
- Future Fermilab g-2 Experiment
- Theory Status
- New Physics

Physics Motivation



The Standard Model (SM) of Particle Physics



- The lepton number (L) is a conserved quantity: Leptons L = 1 and antileptons L = -1.
- The SM has been successful in describing 3 of the 4 fundamental forces: electromagnetic, weak, and strong interactions.
- But there are some unresolved problems with the SM:
  - The force of gravity
  - Dark matter (~26%)
  - …

What is Missing?

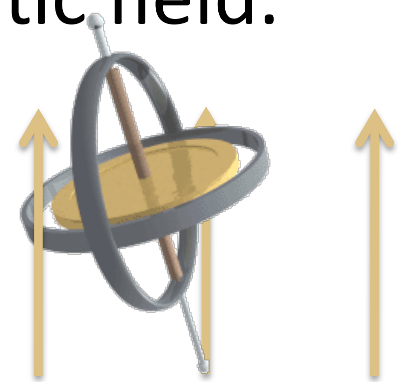
The main approaches in HEP are:

- Energy frontier: Search for new particles by studying high energy interactions at the LHC. Discovery of a particle compatible with the SM Higgs boson in 2012!
- Intensity frontier: Conduct precision tests of the SM.
  Measure a quantity that is sensitive to SM physics: the magnetic moment of charged leptons!

Magnetic Moment

Classically,

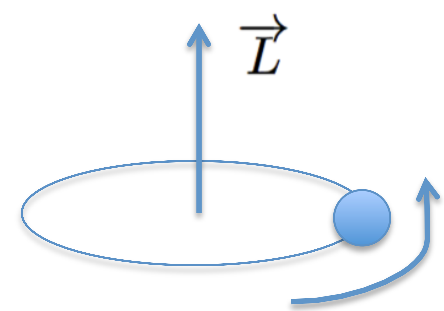
- Bar magnet will align with the magnetic field.
- Spinning magnet will precess around the magnetic field at the Larmor frequency:



$$\vec{\omega}_L = g\frac{q}{2mc}\vec{B}$$

- g: gyromagnetic ratio or g-factor.

$$\vec{\mu} = g\frac{q}{2mc}\vec{L} \qquad g = 1$$



Quantum mechanically,

- The magnetic moment is intrinsic for any charged particle with spin.
- The spin magnetic moment is

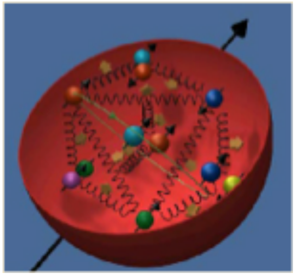
$$\vec{\mu} = g\frac{q}{2mc}\vec{S} \qquad\qquad \vec{S} = \frac{\hbar}{2}\vec{\sigma}$$

- g tells something fundamental about the particle and its interactions.
  - For elementary particles such as electrons: $g = 2$

  (Dirac, 1928)

$$\mathcal{H} = \frac{\left(\vec{P} - q\vec{A}/c\right)^2}{2m} - 2\frac{q\hbar}{2mc}\frac{\vec{\sigma}}{2}\cdot\vec{B} + qA^0 \qquad \mathcal{H}_{int} = -\vec{\mu}\cdot\vec{B} = -g\frac{q\hbar}{2mc}\frac{\vec{\sigma}}{2}\cdot\vec{B}$$

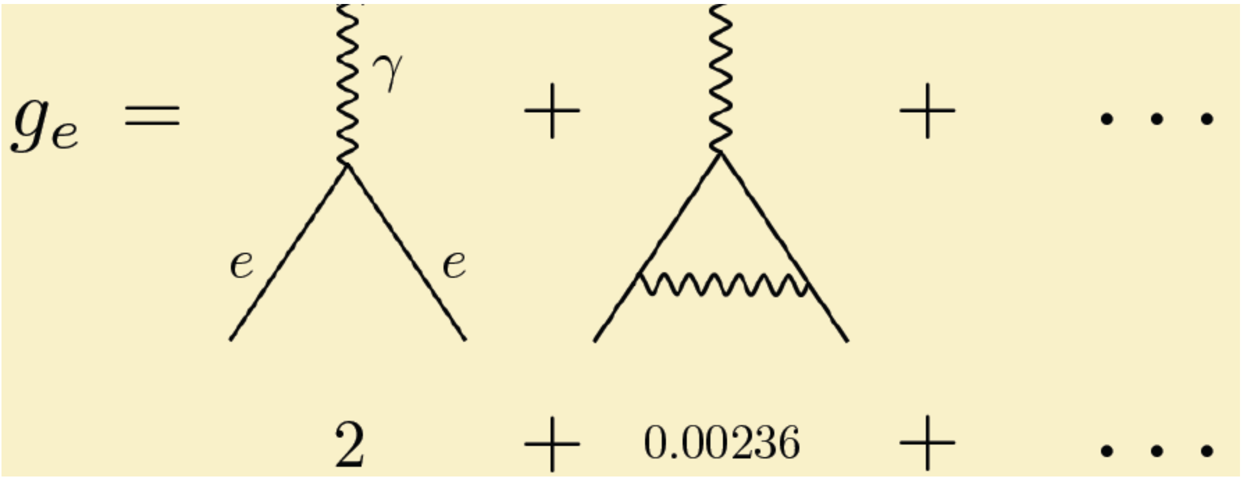
  - For composite particles: $g \neq 2$
    - Proton: $g = 5.6$ (1933)
    - Neutron: $g = -3.8$



- 1948: Kush and Foley measured $g_e = 2.00283(6)$
- What is the source of this 0.1% deviation?

  Empty space is not really empty.

- Soon after, Schwinger calculated first order QED correction α/π



- Beginning of QED and the SM!

Why Study Muons?

- Possibilities: 

- Sensitivity to new corrections scales as m_l^2.
- g_μ is more sensitive than g_e by $(m_\mu/m_e)^2 \approx 4 \times 10^4$!
- The muons are easy to produce, their lifetime is long enough to make an observation (2.2×10^{-6} s)
- Taus have a better sensitivity ($m_\tau \approx 1.8$ GeV/c^2), however they are very short lived (2.9×10^{-13} s).

Muons are the best candidates to look for new physics by precision measurement of the magnetic moment.

- Define the anomalous magnetic moment:

$$a_\mu = \frac{g_\mu - 2}{2}$$

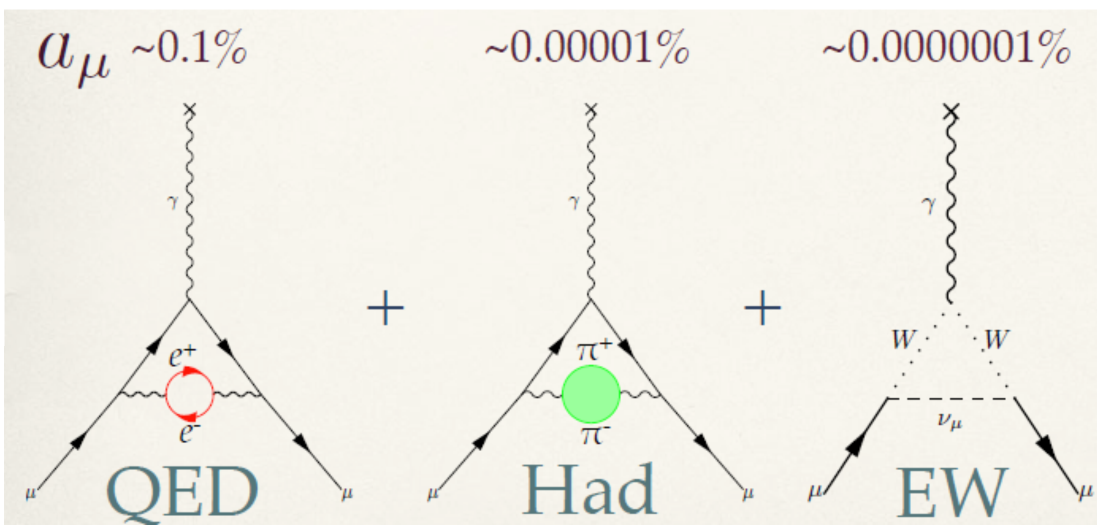
(The famous g-2 term!)

We want to measure this!

- Coupling of the muon spin to virtual fields: Radiative Corrections (RC).
- Empty space includes all quantum fluctuations: QED, hadronic, electroweak contributions, and ??

$$a_\mu^{SM} = a_\mu^{\text{QED}} + a_\mu^{\text{Hadronic}} + a_\mu^{\text{EW}}$$

$$\delta a_\ell \sim \frac{m_\ell^2}{M^2}$$



+

New corrections beyond the EW scale?
Beginning of physics Beyond Standard Model?

The Measured and Calculated Anomaly

- The Brookhaven g-2 experiment measured the anomaly to 0.54 ppm:

$$a_\mu^{\mathrm{E821}} = (116\,592\,08.0 \pm 6.3) \times 10^{-10}\,(0.54\ \mathrm{ppm})$$

- The SM evaluation stands at 0.39 ppm:

$$a_\mu^{SM} = (116\,591\,82.8 \pm 4.5) \times 10^{-10}\,(0.39\ \mathrm{ppm})$$

- There is a discrepancy

$$\Delta a_\mu\,(\mathrm{E821} - \mathrm{SM}) = (25.2 \pm 7.7) \times 10^{-10}$$

- A difference of over 3 standard deviations!
  Is it real or a statistical fluctuation?

Properties of the Muon

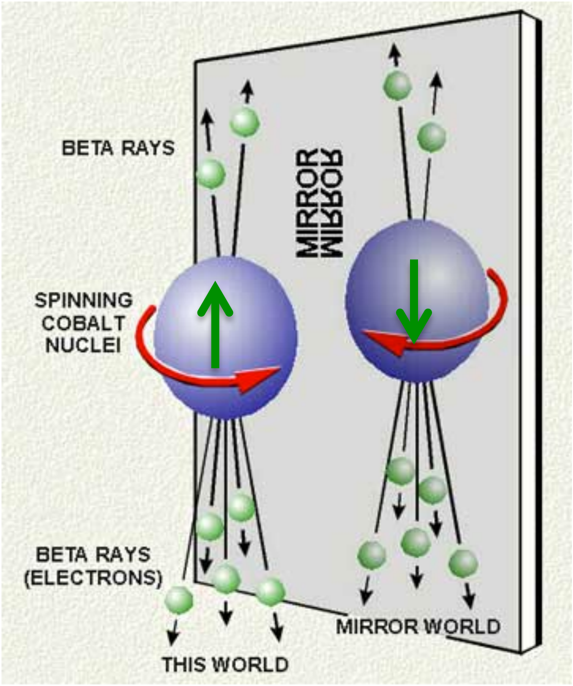
Parity Violation

- Parity refers to the mirror image of a physical process.
- Are mirror images of all physical processes possible? Yes, for strong and electromagnetic interactions.
- In 1956, Lee and Yang proposed a test carried out by Wu:

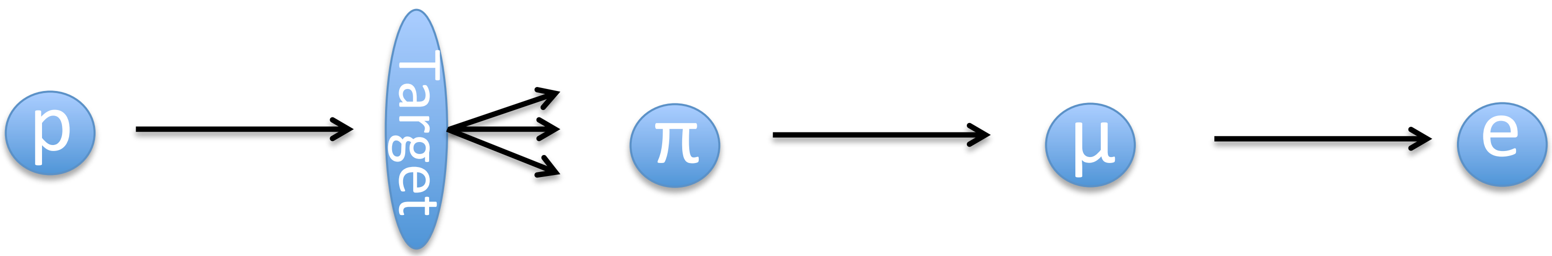
$$^{60}Co \rightarrow ^{60} Ni + e + \bar{\nu}_e$$

- More electrons in one direction.

  Parity is not conserved in weak interactions.

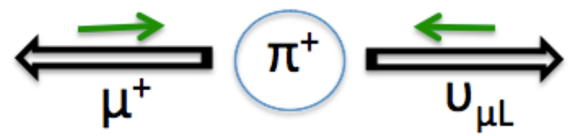


Producing Muons



- Pion decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$

  Obtain highly polarized muons.

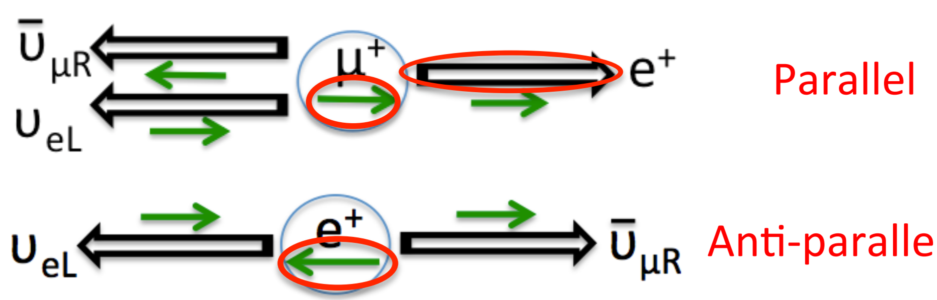


Rest frame

- Muon decay $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$

$$E_{e,max} \approx \frac{m_\mu c^2}{2} = 53 \text{ MeV}$$

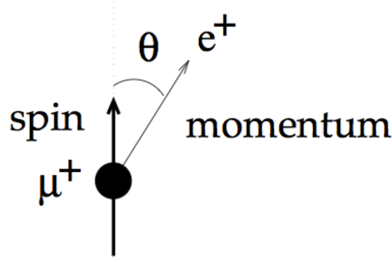
$$E_{e,min} = 0$$



Correlation between the muon spin and the positron momentum.

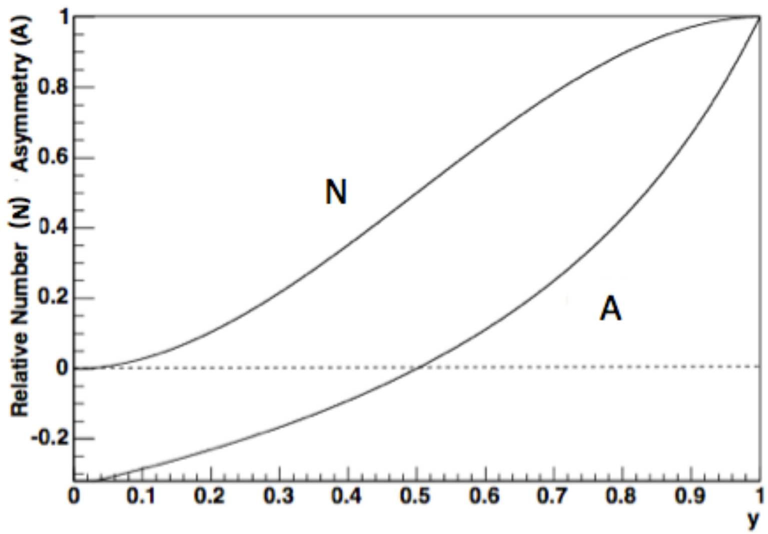
- Differential decay probability for a positron (dP^+) with energy $y = E/E_{e,max}$

$$dP^+ \propto N\left(E_e\right)\left(1 + A(E_e)\cos\theta\right) dy d\Omega$$



$N(E_e)$: Number of decay positrons.

$A(E_e)$: Asymmetry factor that reflects parity violation.

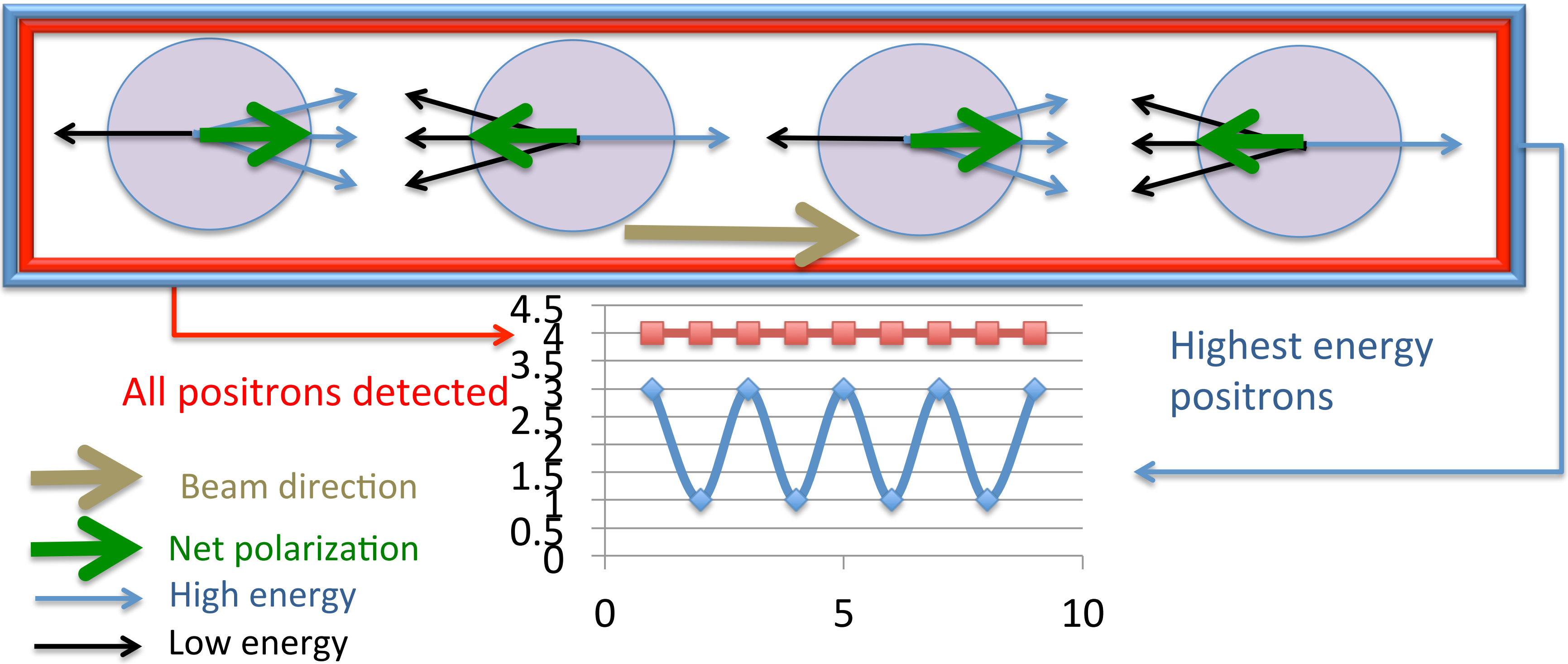


Muon rest frame

- More high energy positrons emitted at $\theta = 0$.
- Track highest energy positrons for the spin direction.

- If the muons' polarization is made to precess, and only the highest energy positrons are detected:
  - Maximum when the muon spin is parallel to the emitted positrons.
  - Minimum when the muon spin is anti-parallel.

  For a small enough period, the number of decays is constant:



- By detecting energetic positrons, we can determine the spin precession rate!
- Muons should all have the same polarization in order for this technique to work.

Muons in a Storage Ring

- Storage ring: Maintains charged particles from an accelerator at precise circular orbits.
  - Magnetic dipoles for bending.
  - Electric quadruples for focusing.
- Relativistic muons travel in circular orbits at the cyclotron frequency:

$$\overrightarrow{\omega_c} = \frac{q}{\gamma m_\mu c}\left\{\overrightarrow{B} + \frac{\gamma^2}{\gamma^2 - 1}\left(\overrightarrow{E} \times \overrightarrow{\beta}\right)\right\}$$

$$\overrightarrow{\beta} = \frac{\overrightarrow{v}}{c} \qquad \gamma = 1/\sqrt{1-\beta^2}$$

Spin Precessions

- Larmor precession: Interaction of the muon magnetic moment with the magnetic field.
- Thomas precession: Kinematic effect due to the transverse acceleration of muons.
- If muons are constrained to a circular orbit by a uniform magnetic field, both effects are present and described by the BMT equation (1959):

$$\vec{\omega}_s = \frac{q}{m_\mu c}\left\{\left(a_\mu + \frac{1}{\gamma}\right)\vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} + \left(a_\mu + \frac{1}{\gamma+1}\right)\vec{E}\times\vec{\beta}\right\}$$

- Choose a reference frame that rotates with the velocity vector. The precession is then:

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c$$

$$\vec{\omega}_a = \frac{q}{m_\mu c}\left\{a_\mu \vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)\cancelto{0}{\left(\vec{\beta}\cdot\vec{B}\right)}\vec{\beta} + \left(\cancelto{0}{a_\mu - \frac{1}{\gamma^2-1}}\right)\vec{E}\times\vec{\beta}\right\}$$

- If we only could cancel the 2nd and 3rd terms, the equation will become simpler:

$$\vec{\omega}_a = a_\mu \frac{q}{m_\mu c}\vec{B}$$

- a_μ represents the difference between the spin precession and the cyclotron frequency.

$$\vec{\omega_a} = \frac{q}{m_\mu c}\left\{a_\mu \vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} + \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{E}\times\vec{\beta}\right\}$$

CERN III g-2 experiment found a solution:

- Use a magnetic field normal to the beam:

$$\vec{\beta}\cdot\vec{B} = 0$$

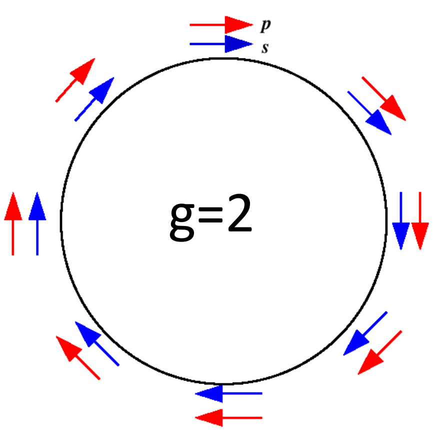
- Select muons at the “magic” momentum:

$$a_\mu - 1/\left(\gamma^2 - 1\right) = 0$$

Electric fields do not affect the beam!

$$a_\mu \approx 1.66\times 10^{-3}, \quad \gamma \approx 29.30$$

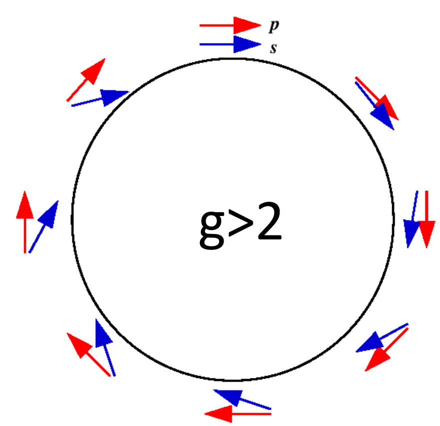
$$p \approx 3.09 \text{ GeV}$$



$$\vec{\omega_a} = \vec{\omega_s} - \vec{\omega_c}$$

$$a_\mu = \frac{g_\mu - 2}{2}$$

$$\vec{\omega_a} = a_\mu \frac{q}{m_\mu c}\vec{B}$$



CERN (1970s): 7.3 ppm

Brookhaven g-2 Experiment E821

Experimental Method

- Produce a proton beam by an accelerator complex.
- Send the proton beam to a nickel target to produce pions.
- Collect polarized muons from the pion decay:

$$\pi^+ \to \mu^+ + \nu_\mu$$

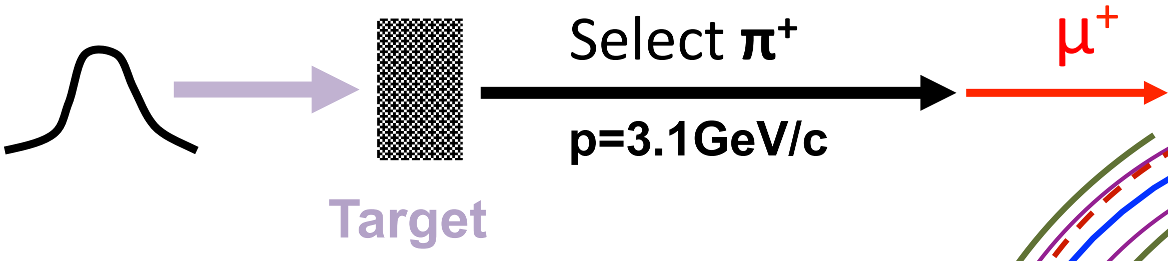
- Inject the muon beam into the g-2 storage ring.
- "Kick" the muon beam onto a stored orbit.
- Measure the arrival time and energy of the positrons from the decay:

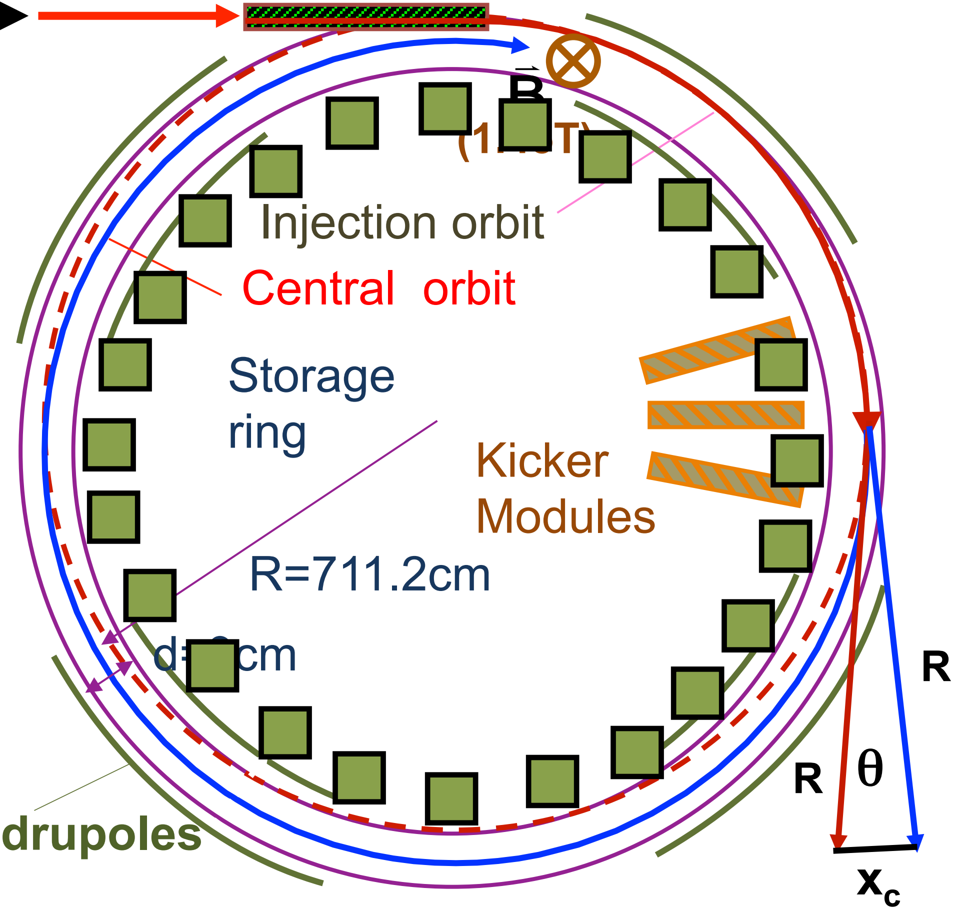
$$\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$$

25ns bunch of
5 X 10^{12} 25GeV
protons from
accelerator

$x_c \approx 77$ mm

$\theta \approx 10$ mrad

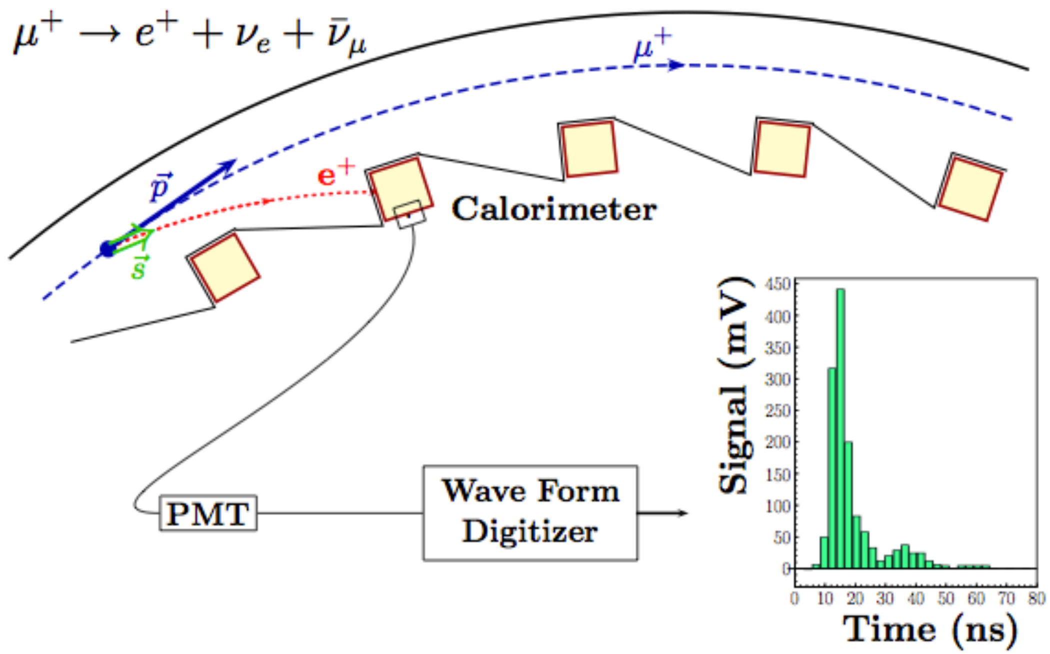




- **Proton beam**
- **Pion beam**
- **Polarized muons**
- **Inject muons**
- **Kick the muon beam**
- **Focus the beam**
- **Measure time and energy of e^+**

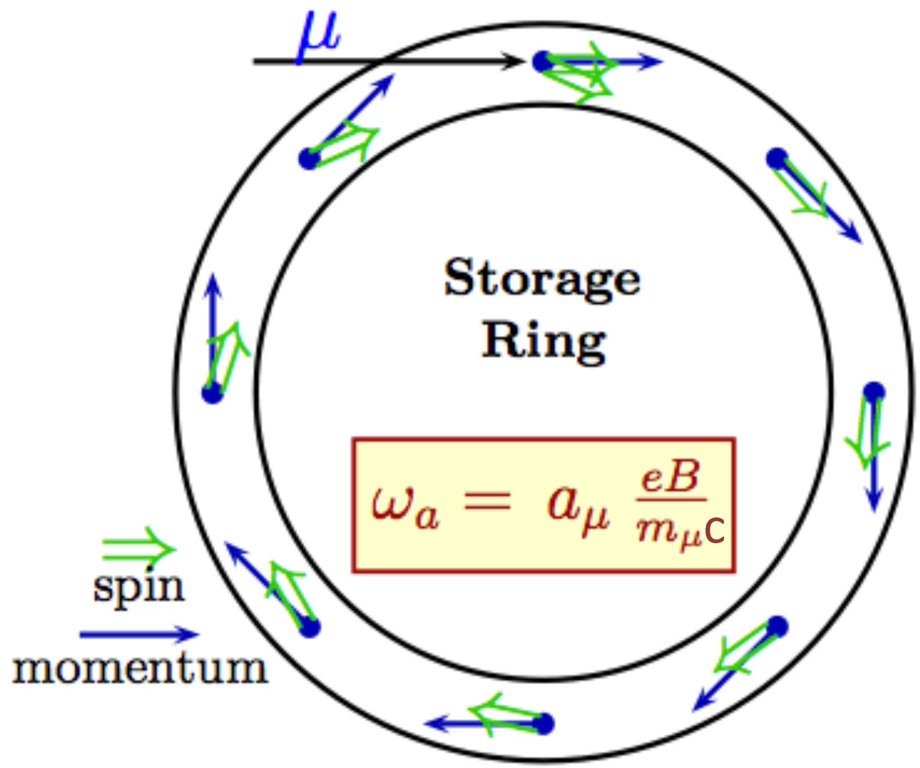
Credit: Steve Maxfield

Detection Apparatus



- Positron emitted in the spin direction.
- 24 symmetrically distributed calorimeters made out of plastic-scintillator material read out by photomultiplier tubes (PMTs).
- A wave form digitizer captures raw analog PMT signals and digitizes them.
- An extrapolation of the positron trajectory allows the determination of the decay position.

Muons at rest: 2.2 μs.
Muons in the ring: 64 μs.
Cyclotron period: 149 ns.
Angular change: 12°/orbit



- To determine a_μ
  - Measure ω_a
  - Measure B
  - Measure $\frac{e}{m_\mu c}$

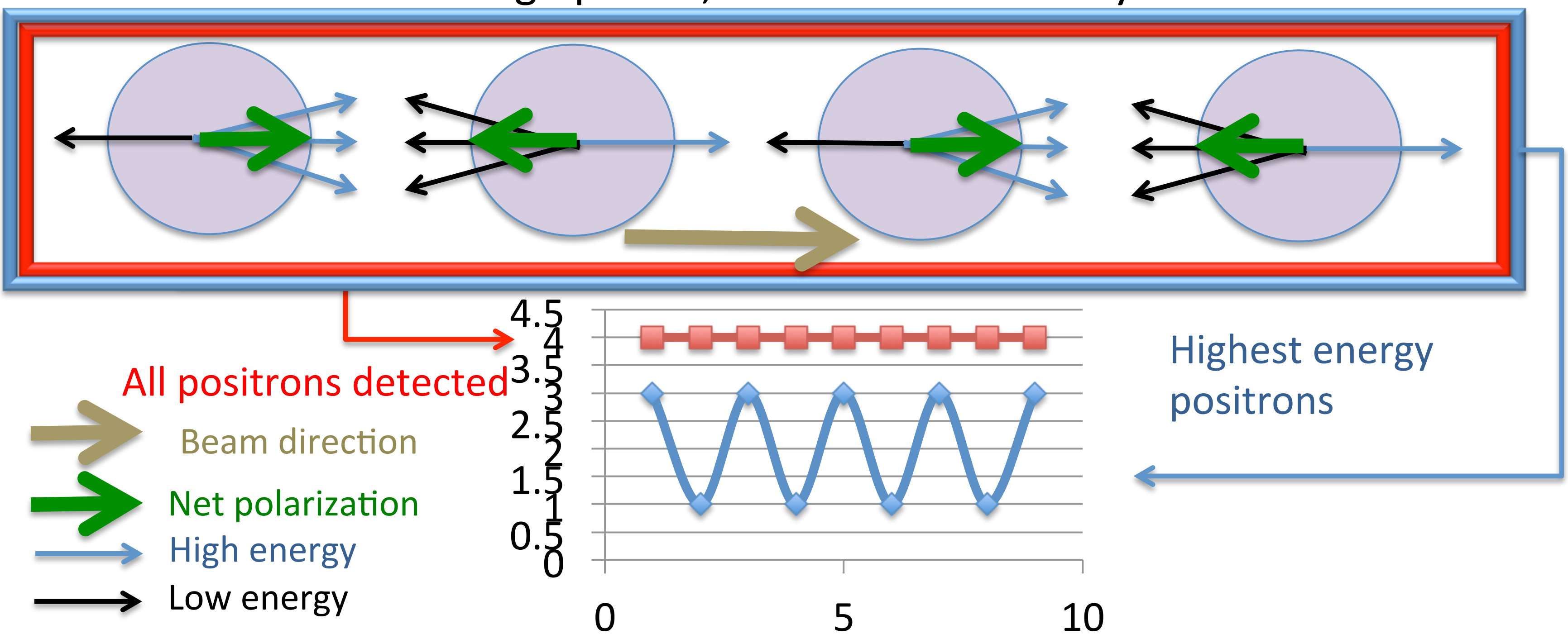
Measurement of ω_a

- All positrons detected: Number is proportional to $\exp(\frac{-t}{\gamma\tau_\mu})$ and no oscillation is present.
- Make a cut on a laboratory observable to see the oscillation:

Highest energy positrons!

- If the muons' polarization is made to precess, and only the highest energy positrons are detected:
  - When the number is maximum, the muon spin is parallel to the emitted positrons.
  - When the number is minimum, the muon spin is anti-parallel to the emitted positrons.

For a small enough period, the number of decays is constant:

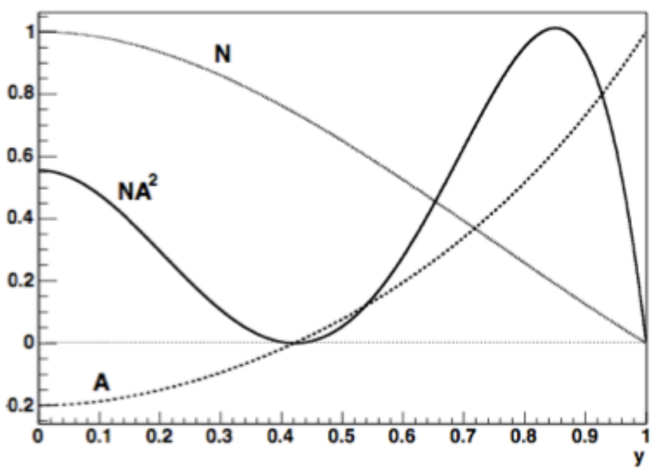


- By detecting positrons, we can determine the spin precession rate!
- Muons should all have the same polarization in order for this technique to work.

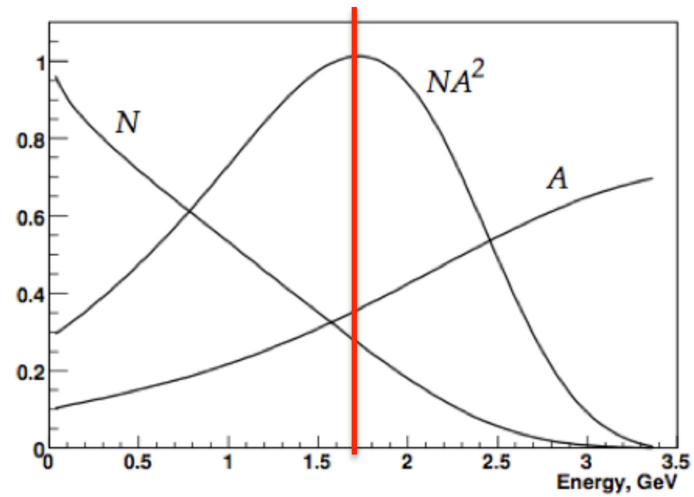
Measurement of ω_a

- All positrons detected: Number is proportional to $\exp(\frac{-t}{\gamma\tau_\mu})$ and no oscillation is present.
- Make a cut on a laboratory observable to see the oscillation:

  Highest energy positrons!
- Maximize the sensitivity by choosing an energy threshold that maximizes the figure of merit NA^2: $E_{th} \approx 1.8$ GeV.
- In the lab frame:



N, A, and NA^2 vs. fractional energy



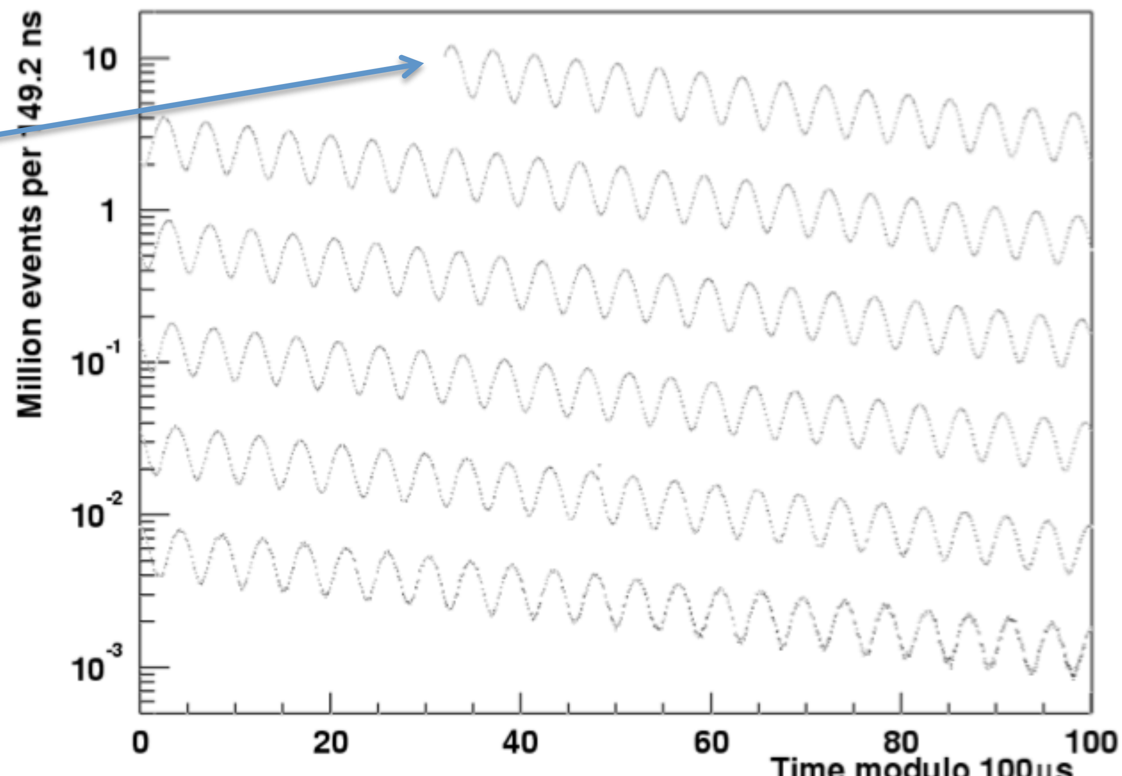
Integrated N, A, and NA^2 vs. energy (to the right of E_{th})

- The integrated number of positrons above E_{th} is an exponential decay modulated at frequency ω_a with a threshold dependent asymmetry:

$$N(t, E_{th}) = N_0(E_{th}) \exp\left(\frac{-t}{\gamma\tau_\mu}\right) [1 + A(E_{th}) \sin(\omega_a t + \phi(E_{th}))]$$

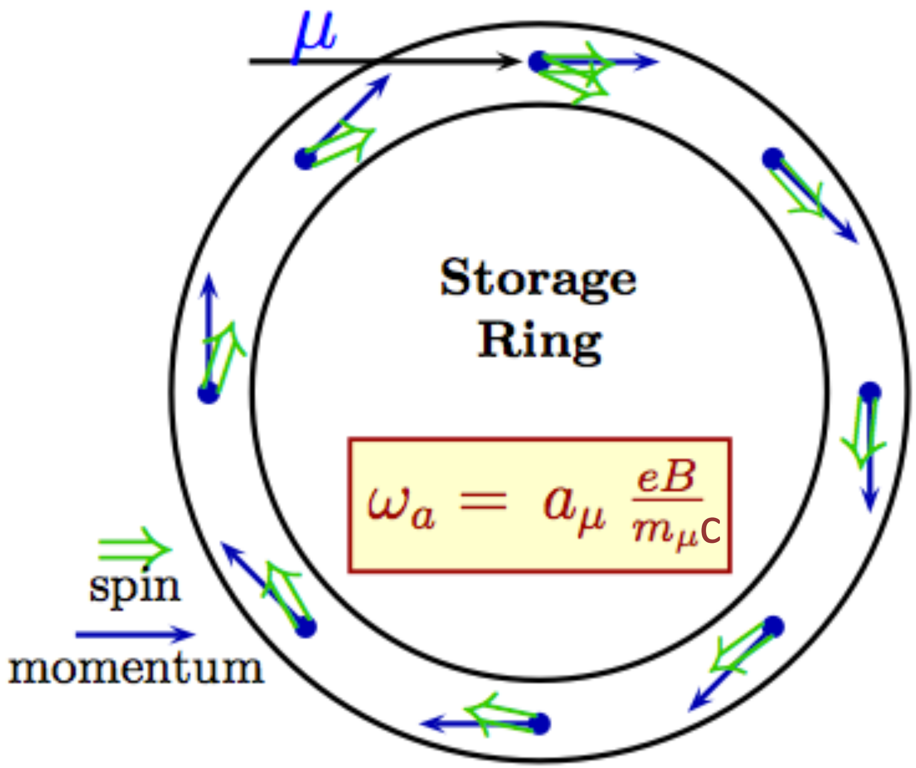
- Fit to:

Start at ~ 30 μs
to avoid the burst of particles at injection that saturates photomultiplier tubes.



τ_μ ~ 64 μs
At t ~ $9\tau_\mu$, N ~ 10^{-3}
Indeed N ~ $N_0 e^{-t/\tau_\mu}$ ~ 10 *10^{-4}

Muons at rest: 2.2 μs.
Muons in the ring: 64 μs.
Cyclotron period: 149 ns.
Angular change: 12°/orbit



- To determine a_μ
  - Measure ω_a
  - $\boxed{\text{– Measure B}}$
  - Measure $\frac{e}{m_\mu}$

Measurement of B

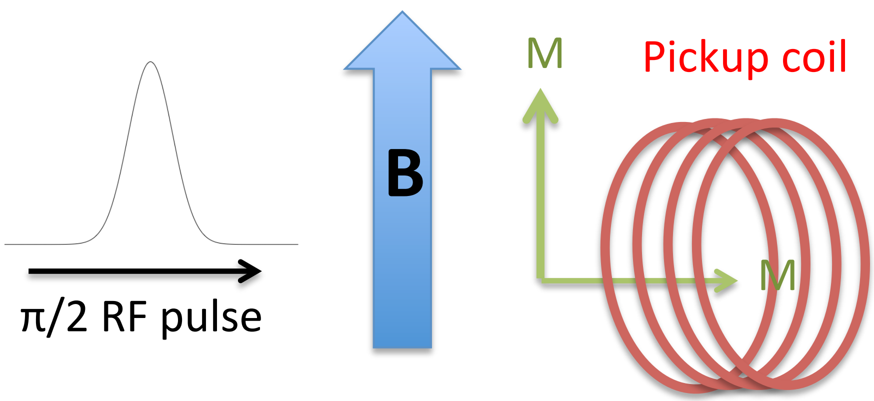
- The weighted magnetic field B is:

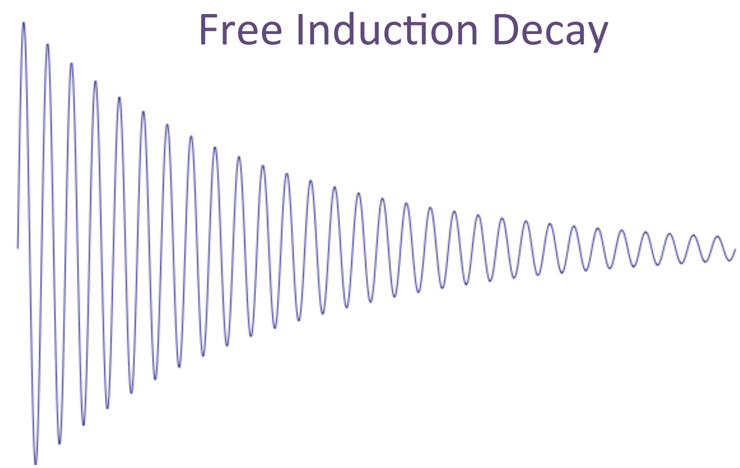
$$\langle B \rangle = \int M(r,\theta) B(r,\theta) r dr d\theta$$

M(r,θ) is the muon distribution.

B(r,θ) is the magnetic field.

- The goal is to achieve a uniform magnetic field to sub-ppm.
- Use high precision (ppb) Nuclear Magnetic Resonance (NMR) of protons in a water sample.





- Obtain the Larmor frequency of a free proton ω_p from the frequency of a proton in water.
- B in terms of the proton magnetic moment μ_p: $B = \frac{\hbar \boxed{\omega_p}}{2\mu_p}$

Muons at rest: 2.2 μs.
Muons in the ring: 64 μs.
Cyclotron period: 149 ns.
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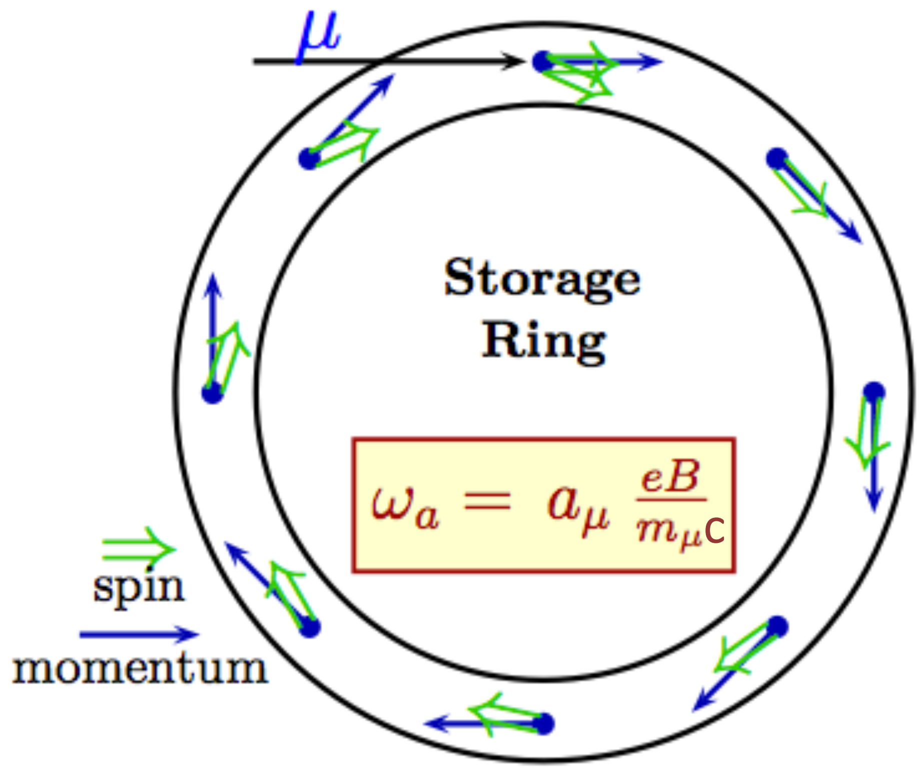


- To determine a_μ
  - Measure ω_a
  - Measure B
  - $\boxed{\text{Measure } \frac{e}{m_\mu c}}$

Measurement of $\frac{e}{m_\mu c}$

- Avoid uncertainty from muon charge to mass ratio by expressing a_μ in terms of dimensionless quantities:

$$\omega_a = a_\mu \frac{e}{m_\mu c} B$$

$$\left.\begin{array}{l} \mu_\mu = g_\mu \dfrac{e}{2 m_\mu c} \dfrac{\hbar}{2} \\ g_\mu = 2\left(1 + a_\mu\right) \end{array}\right\} \quad \frac{e}{m_\mu c} = \frac{4 \mu_\mu}{2\left(1 + a_\mu\right)\hbar}$$

$$B = \frac{\hbar \omega_p}{2 \mu_p}$$

$$\boxed{a_\mu = \frac{\mathcal{R}}{\lambda - \mathcal{R}} \qquad \mathcal{R} \equiv \frac{\omega_a}{\omega_p} \quad \lambda \equiv \frac{\mu_\mu}{\mu_p}}$$

- R is measured in this experiment: ω_a and ω_p are measured simultaneously and independently.
- λ is the muon-to-proton magnetic moment ratio determined by precision measurement of the muonium ($\mu^+ e^-$) hyperfine structure (Los Alamos).

$$\lambda_+ = \frac{\mu_{\mu^+}}{\mu_p} = 3.183\,345\,137\,(85)$$ 27 ppb precision!

- The use of λ_+ to determine a_{μ^-} assumes CPT invariance:

$$\Delta\mathcal{R} = \mathcal{R}_{\mu^-} - \mathcal{R}_{\mu^+} = (3.6 \pm 3.7) \times 10^{-9}$$

Corrections

$$\vec{\omega}_a = \frac{q}{m_\mu c}\left\{a_\mu \vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)\overset{\neq 0}{\left(\vec{\beta}\cdot\vec{B}\right)}\vec{\beta} + \overset{\neq 0}{\left(a_\mu - \frac{1}{\gamma^2-1}\right)}\vec{E}\times\vec{\beta}\right\}$$

- Electric quadrupoles cause the beam to oscillate around equilibrium in the transverse plane: betatron oscillation.
- Betatron motion perturbs the muons trajectory => Corrections required in:
  - Momentum direction for $\vec{\beta}\cdot\vec{B} = 0$: Pitch correction.
  - Momentum spread for $a_\mu - 1/\left(\gamma^2-1\right) = 0$: Electric field correction.

Systematics in ω_a

Systematic	2001 (ppm)
Pileup	0.08
Lost Muons	0.09
E-field and pitch	0.05
CBO	0.07
Gain Changes	0.12
Total	0.18

- Pileup: Misinterpreted low energy electrons.
- Lost muons: Motion without bound due to the coupling of perturbations in E or B fields to betatron oscillation.
- Coherent betatron oscillation (CBO): Inflector and storage ring mismatch.
- Gain changes: Calibration of the readout system.

Systematics in ω_p

- NMR probes are placed off the beam path.
- Protons' oscillation is measured in water.
- Extrapolation and calibration is required to get the free proton precession in the beam path.

Systematic	2001 (ppm)
Absolute calibration of standard probe	0.05
Calibration of the trolley probes	0.09
Trolley measurements of B	0.05
Interpolation with fixed probes	0.07
Uncertainty from muon distribution	0.03
Others	0.10
Total	0.17

Others: Higher multipoles in B expansion, changes in the trolley power supply voltage, and temperature effects on the trolley probes and electronics.

E821 Results

- E821 performed 4 μ^+ runs (1997-2000) and 1 μ^- run (2001).

Years	Electrons [millions]	$\omega_a/(2\pi)$ [Hz]	E/pitch [ppm]	$\omega_p/(2\pi)$	$\mathcal{R} = \omega_a/\omega_p$
1999 (μ^+)	950	229 072.8(3)	0.81(8)	61 791 256(25)	0.003 707 204 1(51)
2000 (μ^+)	4000	229 074.11(16)	0.76(3)	61 791 595(15)	0.003 707 205 0(25)
2001 (μ^-)	3600	229 073.59(16)	0.77(6)	61 791 400(11)	0.003 707 208 3(26)
Average					0.003 707 206 3(20)

Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
1999	μ^+	11 659 202(15)	1.3
2000	μ^+	11 659 204(9)	0.73
2001	μ^-	11 659 214(9)	0.72
Average		11 659 208.0(6.3)	0.54

$$a_\mu = \frac{\mathcal{R}}{\lambda - \mathcal{R}}$$

$$\mathcal{R} \equiv \frac{\omega_a}{\omega_p} \qquad \lambda \equiv \frac{\mu_\mu}{\mu_p}$$

- The anomalous magnetic moment is:

$$a_\mu^{\mathrm{E821}} = 11\,659\,208\,(5.4)_{stat}\,(3.3)_{sys}\,(6.3)_{tot} \times 10^{-10} \qquad (0.54 \text{ ppm})$$

Statistical uncertainty: 0.46 ppm

Systematic uncertainty: 0.28 ppm

- If an experiment runs twice as long and collects twice as much data, the statistical error is reduced by a factor of $\sqrt{2}$.

Run for an additional 8 years!

- Or build a new experiment to reduce the statistical and systematic uncertainties and check the validity of the result.

Future Fermilab g-2 Experiment E989

Goals

- Statistics: Increase muons by a factor of ~21.
- Systematics: Reduce by a factor of 3.

$$a_{\mu}^{E821} = 11659208(5.4)_{stat}(3.3)_{sys}(6.3)_{tot} \times 10^{-10}$$

$$(5.4)_{stat} \oplus (3.3)_{sys} \rightarrow (1.1)_{stat} \oplus (1.1)_{sys}$$

$$(6.3)_{tot} \rightarrow (1.6)_{tot}$$

$$(0.46\text{ppm})_{stat} \oplus (0.28\text{ppm})_{sys} \rightarrow (0.1\text{ppm})_{stat} \oplus (0.1\text{ppm})_{sys}$$

$$0.54\text{ppm} \rightarrow 0.14\text{ppm}$$

With the same central values and improved experimental error

5.3σ!

Improvements

- Same E821 storage ring in a better environment.
- Deliver more muons and a purer beam:
  - less than 2 years: ×21 BNL!
  - Pion decay line ~ 1.9 km => Pure muon beam!
- Improve detectors and electronics:
  - Use silicon PMTs to read signal from lead-fluoride crystal (compact, high resolution).
  - New trackers to reduce systematics.
- Refine the magnetic field uniformity and improve the calibration

Comparison of Systematics

For ω_a:

Category	BNL (2001) (ppm)	E989 Goal (ppm)
Pileup	0.08	0.04
Lost Muons	0.09	0.02
E-field and pitch	0.05	0.03
CBO	0.07	< 0.03
Gain Changes	0.12	0.02
Total systematic error on ω_a	0.18	0.07

For ω_p:

Category	BNL (2001) (ppm)	E989 Goal (ppm)
Absolute calibration of standard probe	0.05	0.035
Calibration of the trolley probes	0.09	0.03
Trolley measurements of B	0.05	0.03
Interpolation with fixed probes	0.07	0.03
Uncertainty from muon distribution	0.03	0.03
Time dependent external B fields	—	0.005
Others	0.10	0.03
Total systematic error on ω_p	0.17	0.070

Theory Calculation

$$a_\mu^{SM} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Hadronic}} + a_\mu^{\mathrm{EW}}$$

a_μ^{exp} = 0.00 $\boxed{11\ 659}$ $\boxed{2}$ $\boxed{08.9}$ (6.3)

a_μ^{QED} = 0.00 $\boxed{11\ 658}$ $\boxed{4}$ $\boxed{718}$ .951(.008) >99.99% a_μ

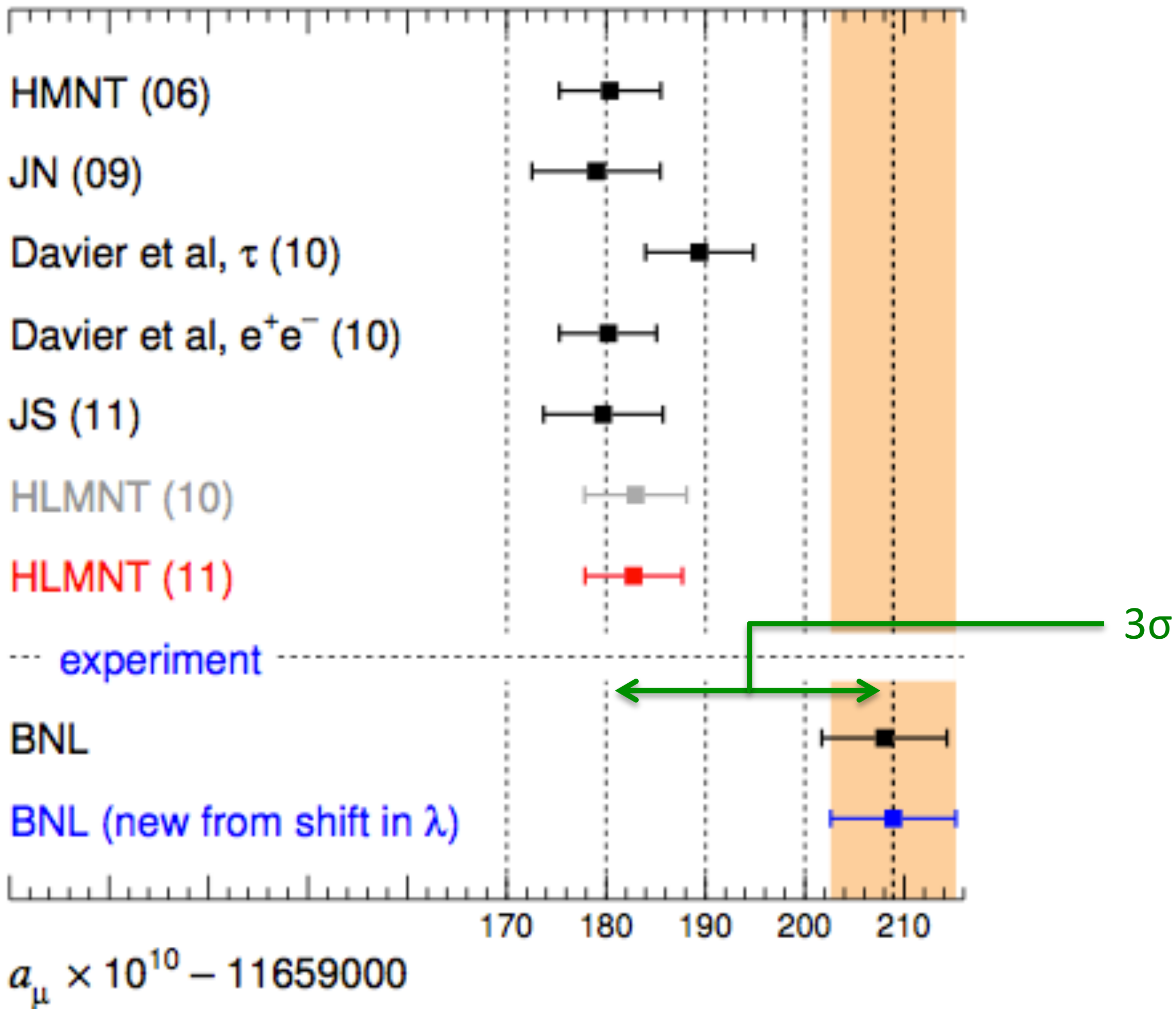
a_μ^{Had} = 0.00 $\boxed{00\ 000}$ $\boxed{6}$ $\boxed{93.0}$ (4.9) Highest error

a_μ^{EW} = 0.00 $\boxed{00\ 000}$ $\boxed{0}$ $\boxed{15.4}$ (0.2) 1.3 ppm > BNL 0.54

$m_\mu c^2$ ~ 106 MeV below pQCD region =>
Use low energy e^+e^- -> had data and models for Had.

Experiment vs. Theory

More than one SM calculation!
The difference is in the hadronic contribution evaluation.

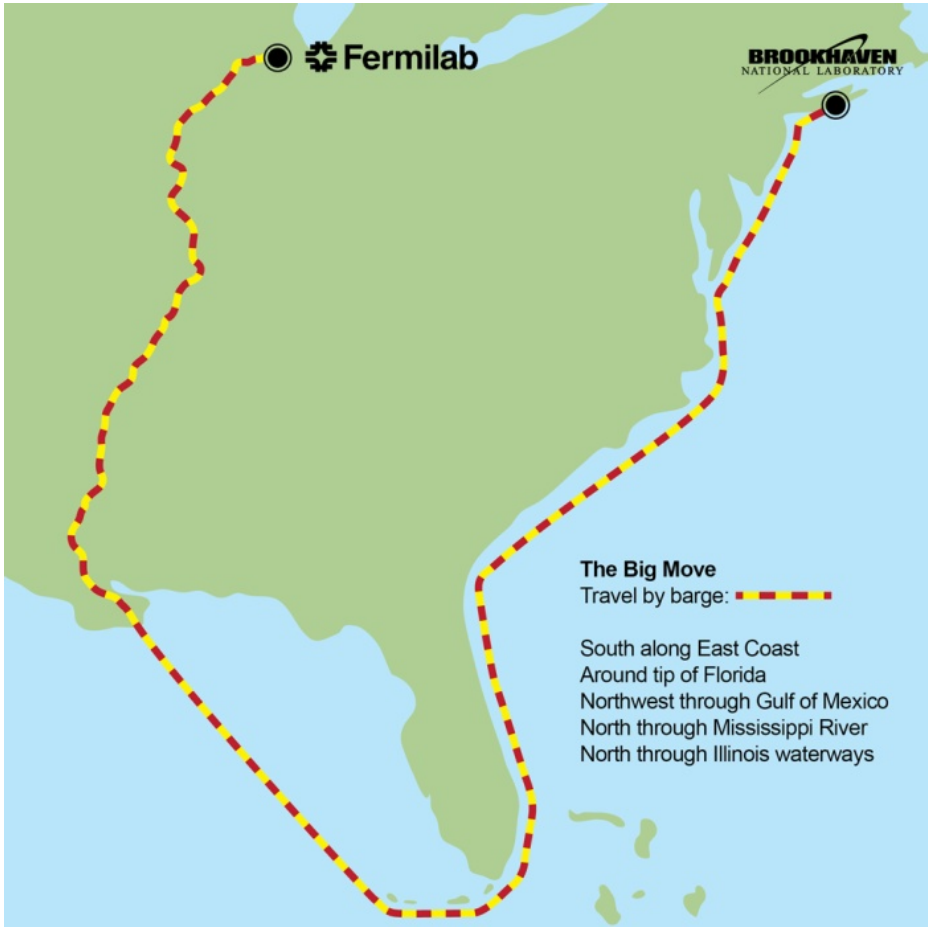


New Physics?

$$\Delta a_\mu \,(\mathrm{E821} - \mathrm{SM}) = (25.2 \pm 7.7) \times 10^{-10}$$

- Smaller than a_μ^{EW}!
- This value tightly constrains new physics models.
- It complements searches at the LHC.
- One possible candidate: supersymmetry (SUSY) postulates a space-time symmetry between fermions and bosons and different models predict a value for the anomaly.
- One of the few remaining tools to search for physics beyond the SM at the TeV scale.

Moving the ring …



2M to move, 30M to build

Lessons Learned

- All you measure is frequency.
- “The closer you look the more there is to see.”
  F. Jegerlehner
- Always backup your data!

Questions

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Backup

Fundamental Forces Strenght

Force	Strength
Strong	$\alpha_s \sim 1$ (1GeV scale)
EM	$\alpha \sim 10^{-2}$
Weak	$\alpha_W \sim 10^{-6}$
Gravity	$\alpha_g \sim 10^{-39}$

Magnetic Moment

$$\overrightarrow{\mu} = \int I da \hat{n}$$

$$\mu = \frac{q}{2\pi r}\frac{v}{c}\pi r^2$$

$$L = mvr$$

$$\frac{\mu}{L} = \frac{q}{2mc}$$

Weighted Average

$$\hat{\mu} = \frac{1}{\omega} \sum_{i=1}^{N} \omega_i x_i$$

$$\sigma_{\hat{\mu}} = \frac{1}{\sqrt{\omega}}$$

$$\omega_i = \frac{1}{\sigma_i^2} \qquad \omega = \sum_i \omega_i$$

Magnetic Moment

- Proton g = 5.5856912 pm 0.0000022
- Neutron g = -3.8260837 pm 0.0000018
- Fundamentally different than the electron!
- Electron:

$$g_e = 2\left(1 + \frac{1}{2}\frac{\alpha}{\pi} + \mathcal{O}\left(\left(\frac{\alpha}{\pi}\right)^2\right)\right)$$

$$\alpha^{-1}\,(\mathrm{Rb}) = 137.035\,999\,049(90)$$

$$a_\mu^{\mathrm{QED,LO}} = \frac{\alpha}{2\pi} \approx 1.16 \times 10^{-3}$$

Magnetic Moment from quarks

For p = uud:

$$\psi\left(\frac{1}{2},\frac{1}{2}\right)=\sqrt{\frac{2}{3}}\chi\left(1,1\right)\phi\left(\frac{1}{2},-\frac{1}{2}\right)-\sqrt{\frac{1}{3}}\chi\left(1,0\right)\phi\left(\frac{1}{2},\frac{1}{2}\right)$$

χ represents uu and ϕ d.

$$\gamma_p=\frac{2}{3}\left(2\gamma_u-\gamma_d\right)+\frac{1}{3}\gamma_d=\frac{4}{3}\gamma_u-\frac{1}{3}\gamma_d$$

Similarly for n = udd, the result is

$$\gamma_n=-\frac{2}{3}\gamma_p$$

which agrees with experiment -0.685.

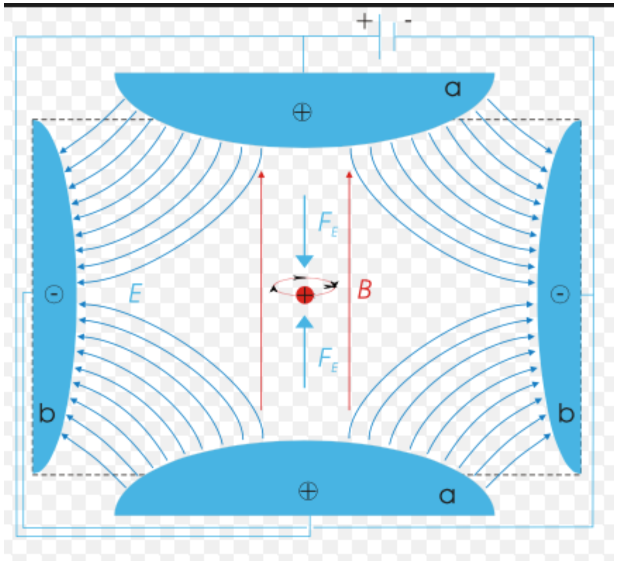
$$\gamma_{n,p}=\frac{g_{n,p}}{2}$$

Electron Magnetic Moment

- a_e^{exp} =1159652180.73(28) × 10^{-12} 0.3 ppt [Hanneke et al., PRL100 (2008) 120801]

Maintain single electron in a penning trap (magnetic field and quadrupole electric field), 3 frequencies: oscillation in z, cyclotron, spin. Measure frequencies to get anomaly.

- Find α by $a_e^{exp} = a_e^{SM}$!
- $a_e^{Had} = 1.628(20) \times 10^{-12}$
  $a_e^{EW} = 0.0297(5) \times 10^{-12}$
  [M.Passera INT2008]



Tau Magnetic moment

- $a_\tau^{exp} = -0.018 \pm 0.017$ (DELPHI at LEP 2004)

$$e^+e^- \to e^+e^-\tau^+\tau^-$$

SM vertex $\gamma\tau\tau$ to predict cross section, and compare with experiment to get a limit in a_τ.

- $a_\tau^{SM} = 117\,721\,(5) \times 10^{-8}$

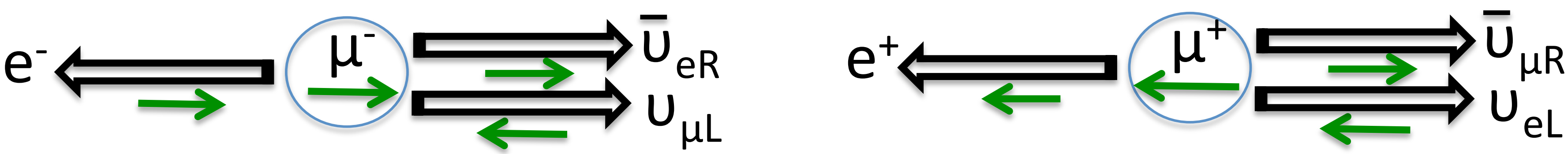
Pion and Muon Decays

- Arrow convention:

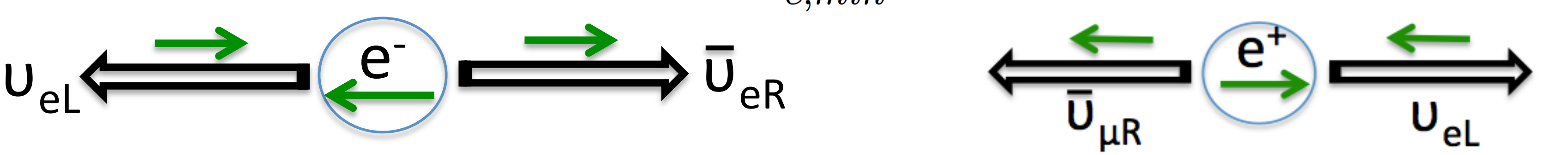
- Pion Decays:

- Mu Decays:

$$E_{e,max} \approx \frac{m_\mu c^2}{2} = 53 \text{ MeV}$$

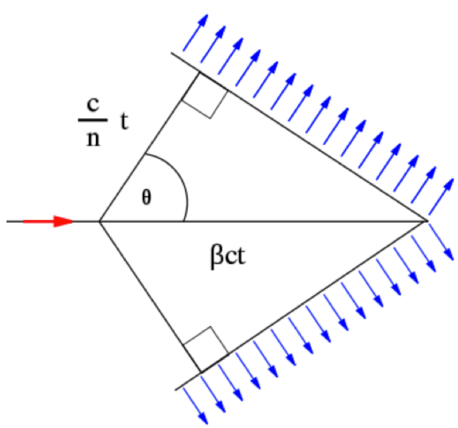


$$E_{e,min} = 0$$



Calorimeter

- Scintillator: (crystals) ionizing e excites atomic states that de-excite and give off light.
- Cherenkov light: when partilce's speed is larger than c in a medium. Light is emitted in a cone of fixed angle for a given velocity.
  - Separate particle species
  - Measure the velocity from the angle.

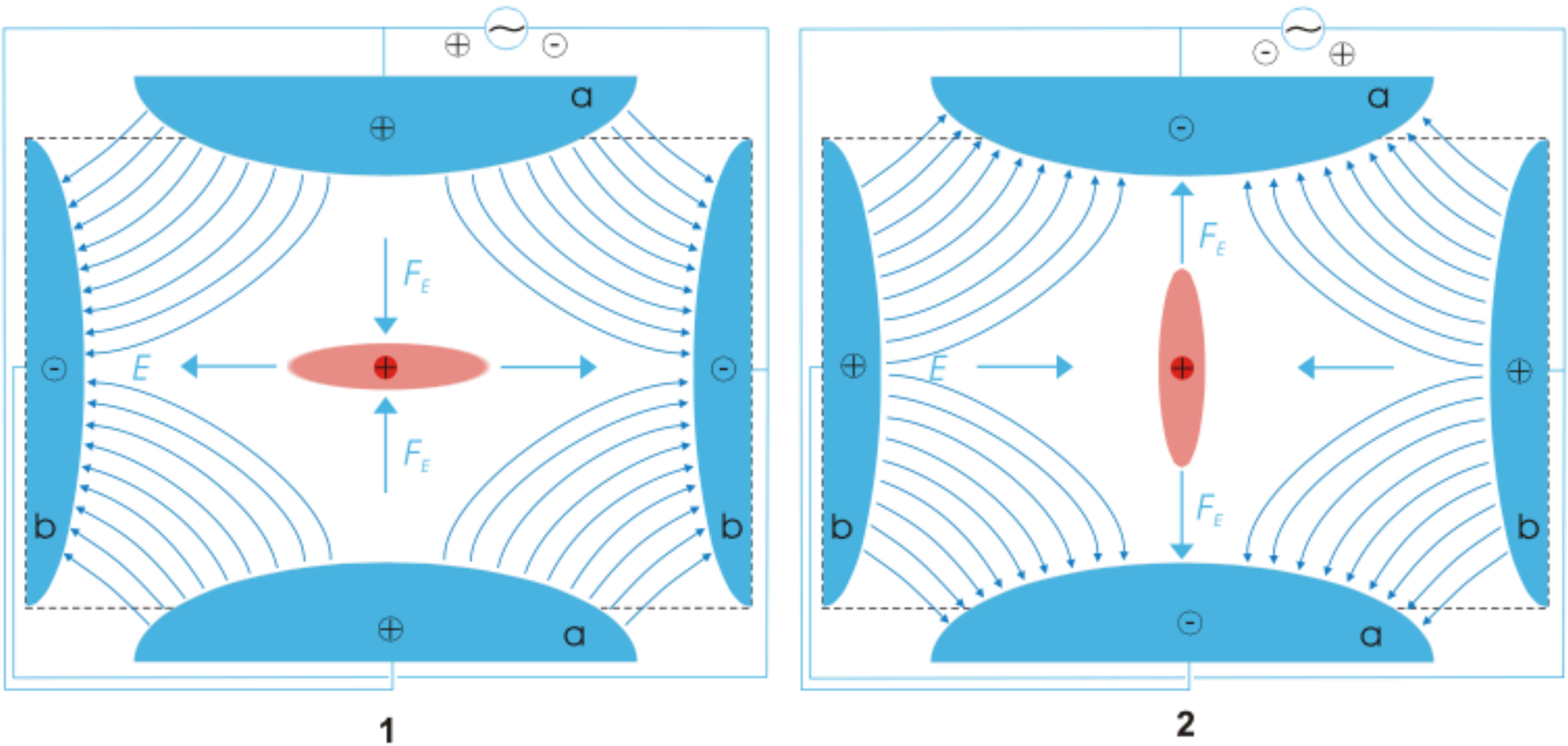


$$\cos\theta = \frac{1}{\beta n}$$

Proton Production

- Proton production: $H+e^- \rightarrow H^-$
- Accelerate H^-
- Pass through a carbon foil that strips off the extra electrons and permits the protons to pass through.

Quadrupole Field

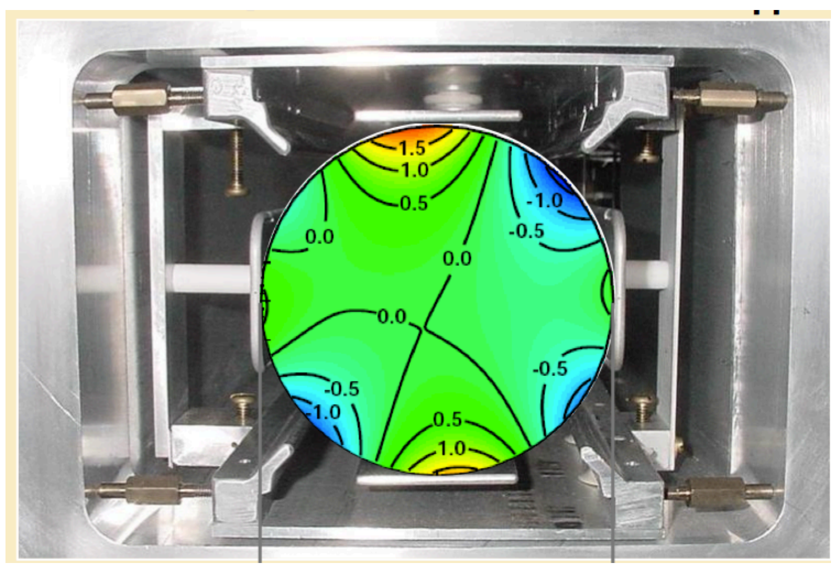


Measurement of B

$$\langle B \rangle = \int M(r,\theta) B(r,\theta) r dr d\theta$$

$$B(r,\theta) = \sum_{n=0}^{\infty} r^n \left[c_n \cos n\theta + s_n \sin n\theta\right]$$

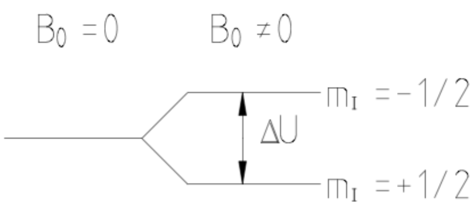
$$M(r,\theta) = \sum_{m=0}^{\infty} \left[\xi_m(r) \cos m\theta + \sigma_m(r) \sin m\theta\right]$$



Nuclear Magnetic Resonance

$$\vec{\mu} = \gamma \vec{S} \qquad \gamma = g\frac{q}{2mc}$$

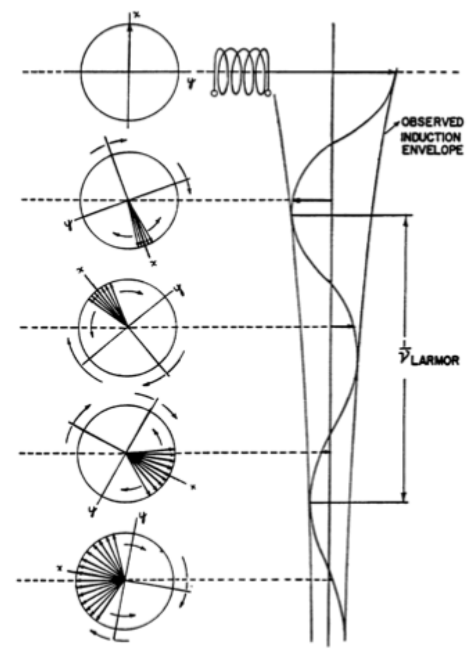
For spin 1/2:



$$H = -\vec{\mu} \cdot \vec{B}$$

$$E_{\pm} = \mp \frac{\gamma \hbar B_0}{2}$$

$$\Delta E = \hbar \omega = \gamma \hbar B_0$$



Send a pulse with frequency ω_{RF} and field B_1:

$$\vec{B}_{total} = B_0 \hat{z} + B_1 \cos(\omega_{RF})\, \hat{x} + B_1 \sin(\omega_{RF})\, \hat{y}$$

Rotates the magnetization vector by angle $\alpha = \omega_{RF} t = \gamma B_1 t$.

- T1 Spin-Lattice Relaxation: exponential recovery of longitudinal magnetization.
- T2 Spin-Spin Relaxation: random interactions and de-phasing leading to the loss of signal.

Muonium System

Energy level: $E \propto \alpha^2 mc^2$

Fine structure: Relativistic + Spin(e)-Orbit(mu) coupling

$$\Delta E_{fs} \propto \alpha^4 m_e c^2$$

Hyperfine splitting: Spin(mu)-Orbit(e) + Spin(e)-Spin(mu)

$$\Delta E_{hf} = \left(\frac{m_e}{m_\mu}\right) \alpha^4 m_e c^2 \frac{g_\mu}{4n^3} \frac{\pm 1}{\left(f+\frac{1}{2}\right)\left(l+\frac{1}{2}\right)}$$

$$f = l + s_e + s_\mu = j \pm \tfrac{1}{2}.$$

Given $l = 0$, $f = 0$ for the singlet (spins anti-parallel)

$f = 1$ for the triplet (spins parallel)

Lamb shift: quantization of the electromagnetic field

$$\Delta E_{Lamb} \propto \alpha^5 mc^2$$

LAMPF Experiment

$$\mathcal{H} = h\Delta\nu \vec{S_\mu}\cdot\vec{J} - \mu_B^\mu g_\mu^{e\mu}\vec{S_\mu}\cdot\vec{H} + \mu_B^e g_e^{e\mu}\vec{J}\cdot\vec{H}$$

$$\mu_\mu = \frac{g_\mu}{2}\mu_B^\mu \qquad \mu_B^\mu = \frac{e\hbar}{2m_\mu c}$$

$$\mu_e = \frac{g_e}{2}\mu_B^e \qquad \mu_B^e = \frac{e\hbar}{2m_e c}$$

$$g_e^{e\mu} = g_e\left(1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2}\frac{m_e}{m_\mu} + \frac{\alpha^3}{4\pi}\right)$$

$$g_\mu^{e\mu} = g_\mu\left(1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2}\frac{m_e}{m_\mu}\right)$$

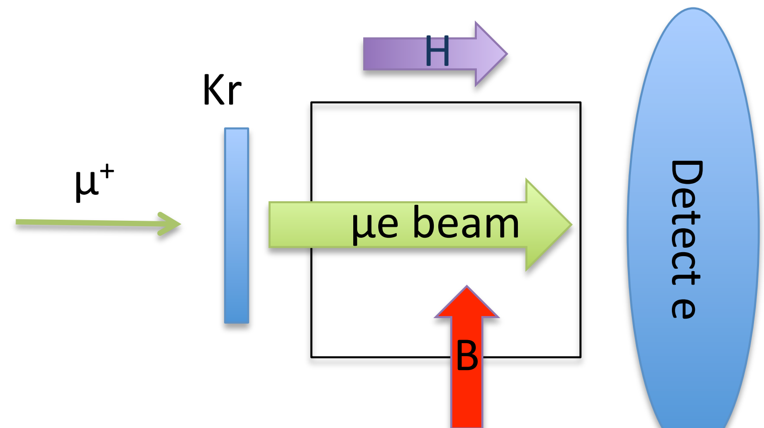
$$(M_J, M_\mu)$$
$$\nu_{12} : (1/2, 1/2) \leftrightarrow (1/2, -1/2)$$
$$\nu_{34} : (-1/2, -1/2) \leftrightarrow (-1/2, 1/2)$$

$$\nu_{12} = -\frac{\mu_B^\mu g_\mu^{e\mu}}{h} + \frac{\Delta\nu}{2}\left[(1+x) - \sqrt{1+x^2}\right]$$

$$\nu_{34} = +\frac{\mu_B^\mu g_\mu^{e\mu}}{h} + \frac{\Delta\nu}{2}\left[(1+x) - \sqrt{1+x^2}\right]$$

$$x = \left(g_e^{e\mu}\mu_B^e + g_\mu^{e\mu}\right)H/\left(h\Delta\nu\right)$$

$$\mu^+ + Kr \to \mu^+ e^- + Kr^+$$

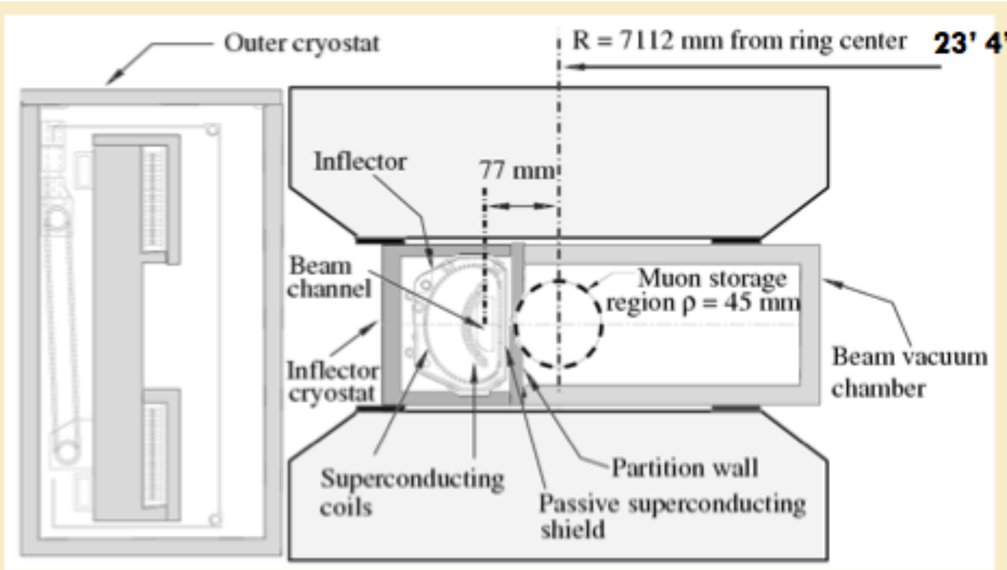


Drive transitions to flip spins

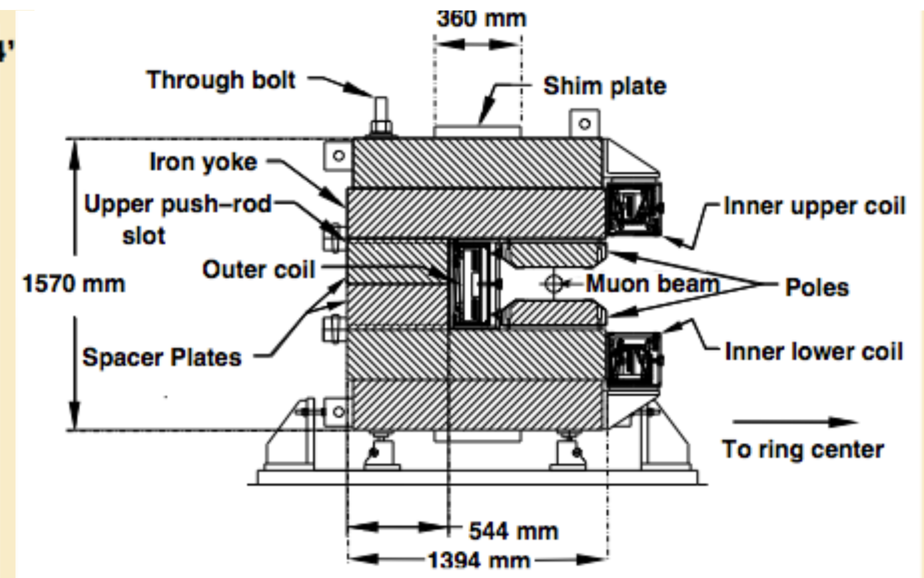
$$\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$$

Extract $\Delta\nu$ and $\frac{\mu_\mu}{\mu_p}$!

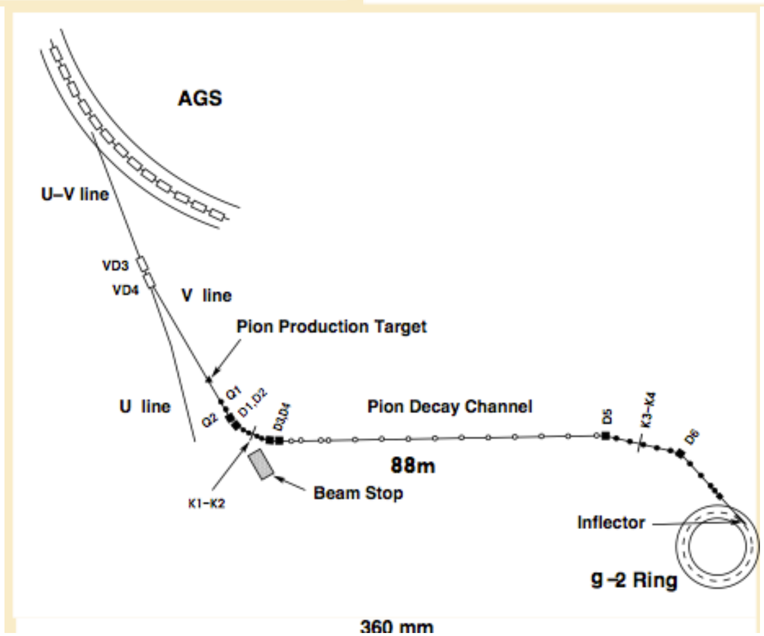
BNL



Outer cryostat
R = 7112 mm from ring center
23' 4'
Inflector
77 mm
Beam channel
Muon storage region ρ = 45 mm
Inflector cryostat
Beam vacuum chamber
Superconducting coils
Partition wall
Passive superconducting shield

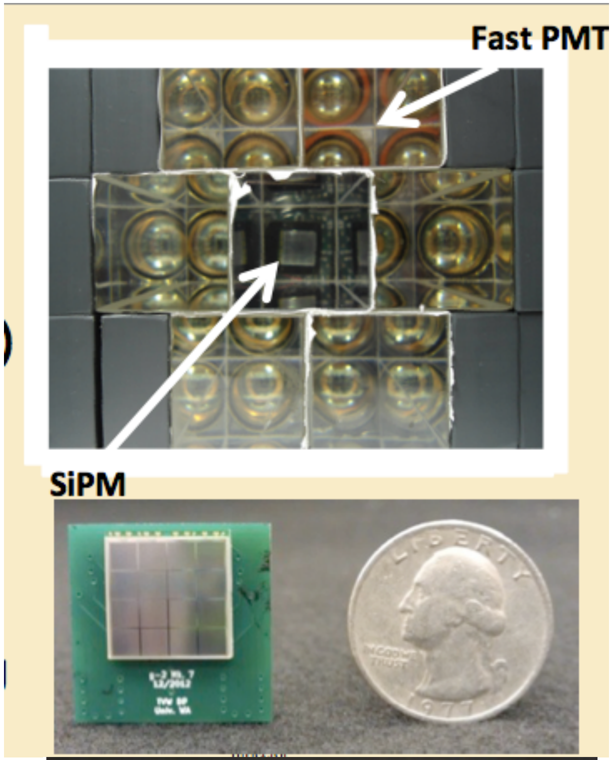


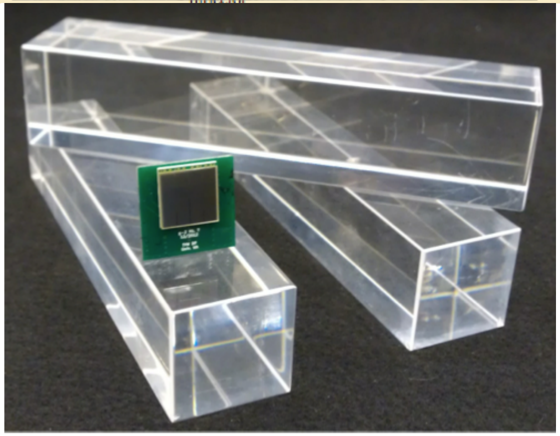
360 mm
Through bolt
Shim plate
Iron yoke
Upper push–rod slot
Inner upper coil
1570 mm
Outer coil
Muon beam
Poles
Spacer Plates
Inner lower coil
To ring center
544 mm
1394 mm

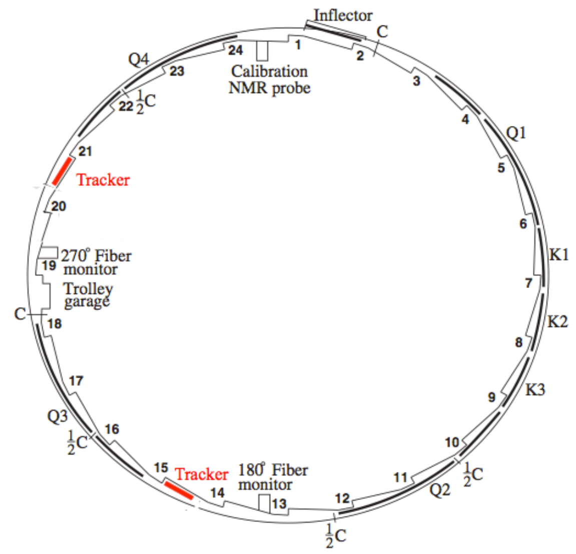


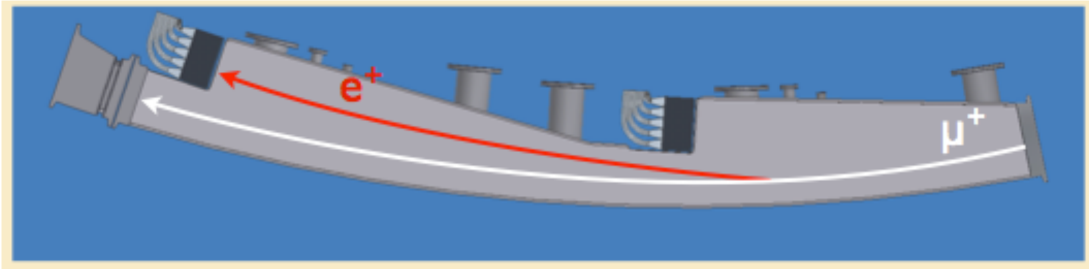
AGS
U–V line
VD3
VD4
V line
Pion Production Target
U line
Q1
Q2
D1,D2
D3,D4
Pion Decay Channel
D5
K3–K4
D6
88m
Beam Stop
K1–K2
Inflector
g-2 Ring
360 mm

Improvements





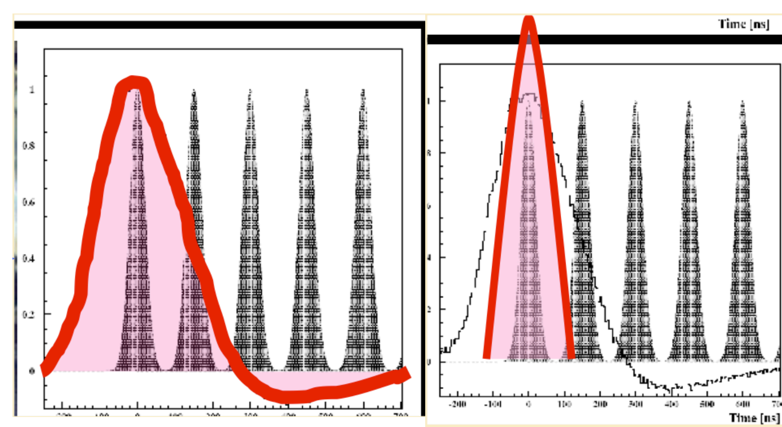




ω_a systematics

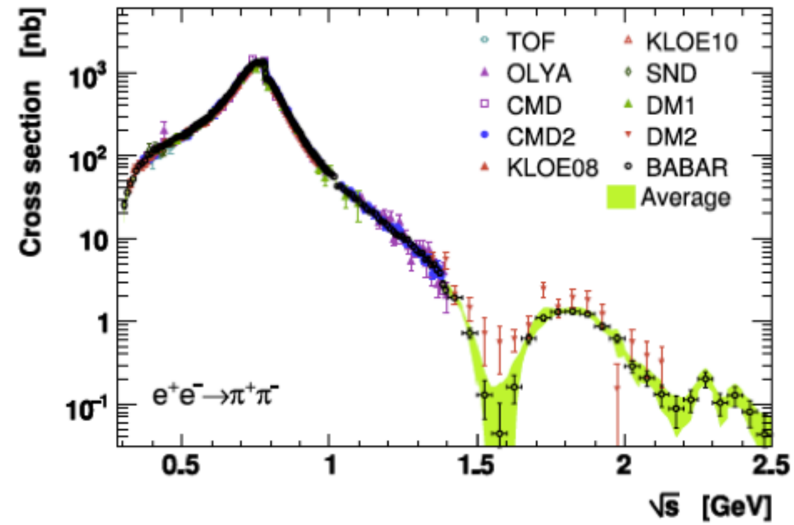
Category	BNL (2001) (ppm)	E989 Goal (ppm)
Pileup	0.08	0.04
Lost Muons	0.09	0.02
E-field and pitch	0.05	0.03
CBO	0.07	< 0.03
Gain Changes	0.12	0.02
Total systematic error on ω_a	0.18	0.07

- Segmented calorimeters
- New tracking, open inflector
- Improved kickers
- No hadronic flash, better calibration

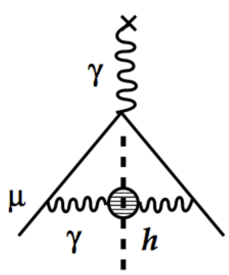


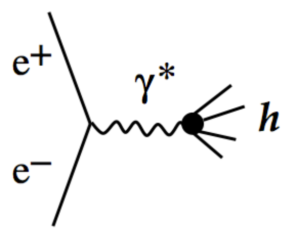
Hadronic Contribution

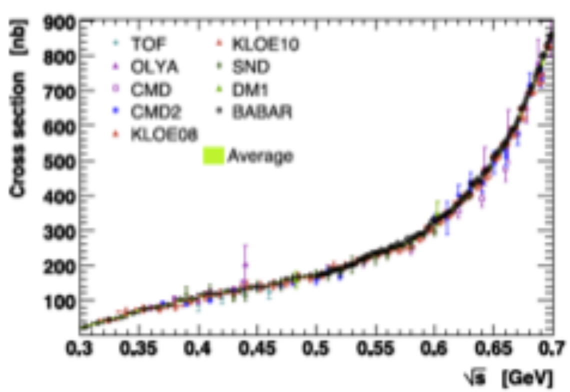
$$a_\mu^{\mathrm{had;LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s)$$

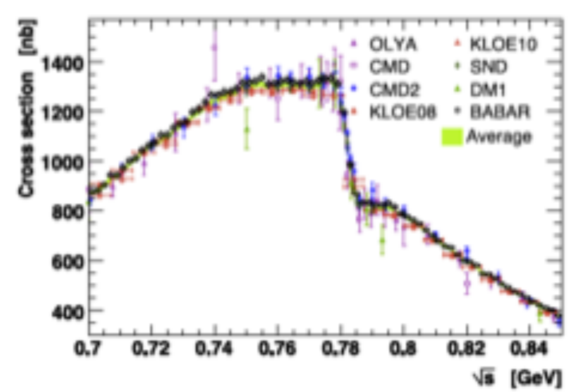


$$R \equiv \frac{\sigma_{\mathrm{tot}}(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$









SUSY

- SUSY with mass scale of several 100GeV's accounts for the discrepancy

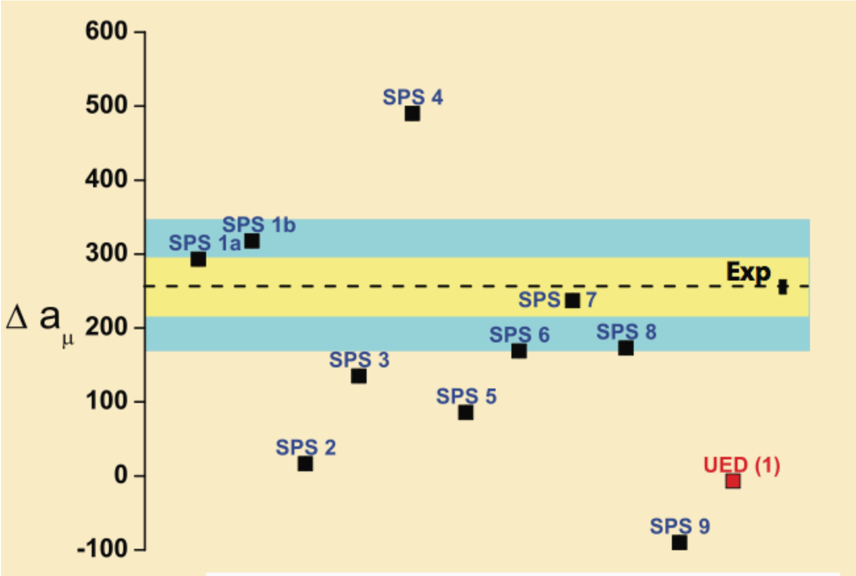
$$a_\mu(\mathrm{SUSY}) \simeq \mathrm{sgn}\,(\mu)\; 130 \times 10^{-11}\; \tan\beta\; \left(\frac{100\ \mathrm{GeV}}{\Lambda}\right)^2$$

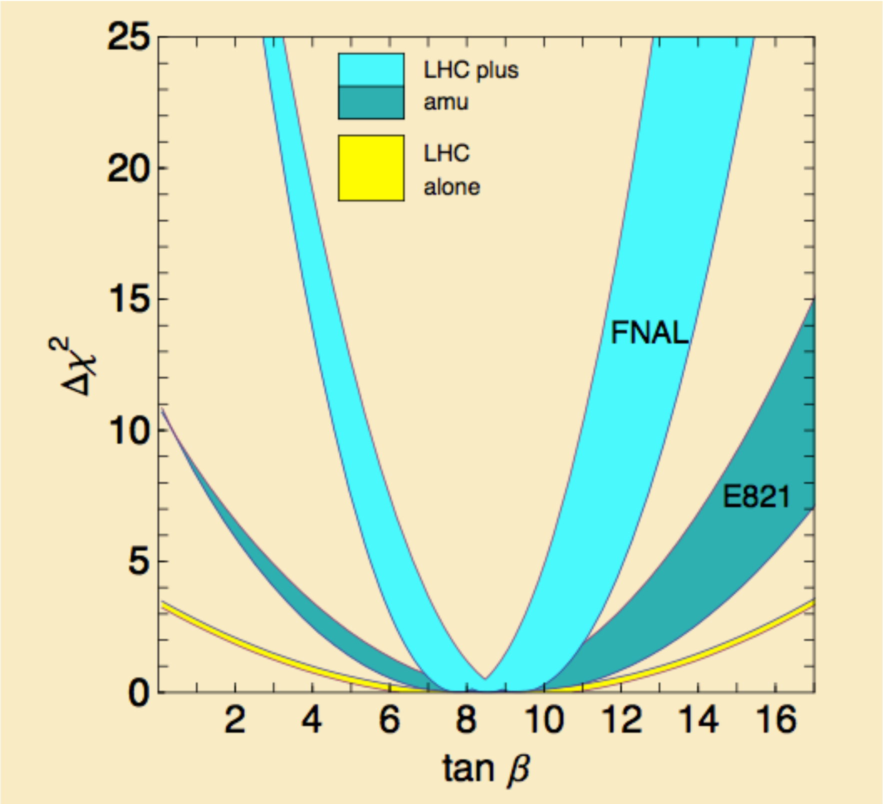
  - tan(β) is the ratio of the two vacuum expectation values of the 2 neutral higgses.
  - μ mass term mixing between the two higgses doublets that can be + or -.
  - Λ is the energy scale.

$$\tan\beta = \frac{v_u}{v_d}; \qquad v_u =< h_u^0 >, \;\; v_d =< h_d^0 >$$

$$\mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan\beta^2}{\tan\beta^2 - 1} - \frac{M_z^2}{2}$$

Complementary to LHC

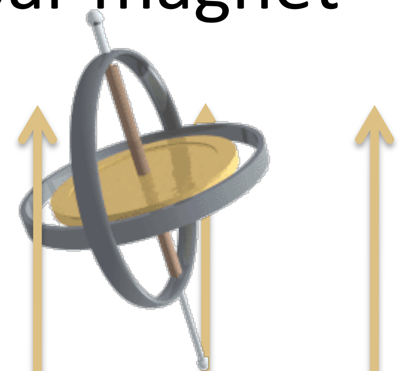




Magnetic Moment

Classically,

- A uniform magnetic field will rotate a bar magnet to align it with the field.
- But, if the bar magnet is spinning, to conserve angular momentum, the bar magnet will precess around the magnetic field at the Larmor frequency.



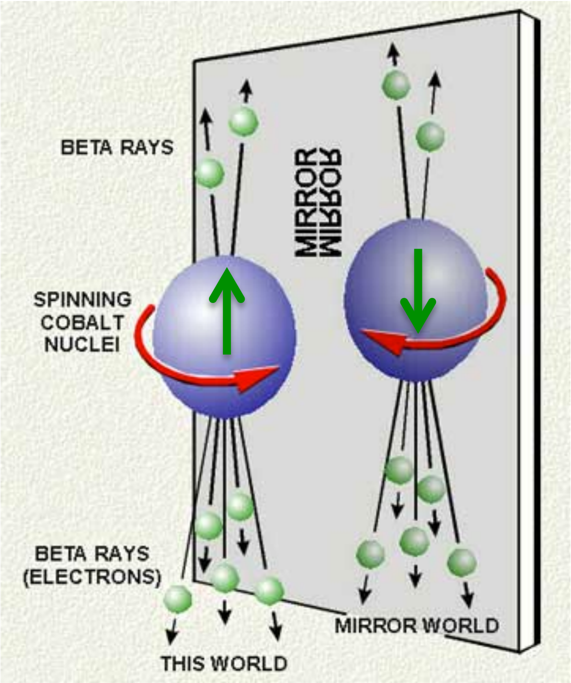
- The frequency of “gyration” is related to the magnetic field by the factor g called the gyromagnetic ratio or g-factor:

$$\vec{\mu} = g\frac{q}{2mc}\vec{L} \qquad\qquad g = 1$$

Parity Violation

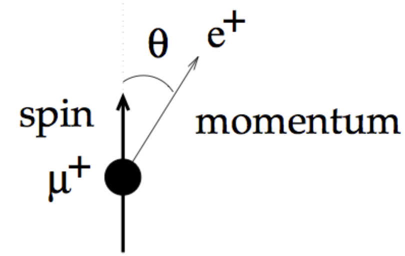
- Parity, represented by the operator P, refers to the mirror image of a physical process.
  For a vector $\overrightarrow{x}$, $P|\overrightarrow{x}\rangle = -|\overrightarrow{x}\rangle$.
- Are mirror images of all physical processes possible?
  Yes, for strong and electromagnetic interactions.
  But how about weak processes?
- In 1956, Lee and Yang proposed a test carried out by Wu on radioactive Cobalt 60 nuclei: Align the net spin along the z-axis and record the direction of emitted electrons.

$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e + \overline{\nu}_{\mu}$$



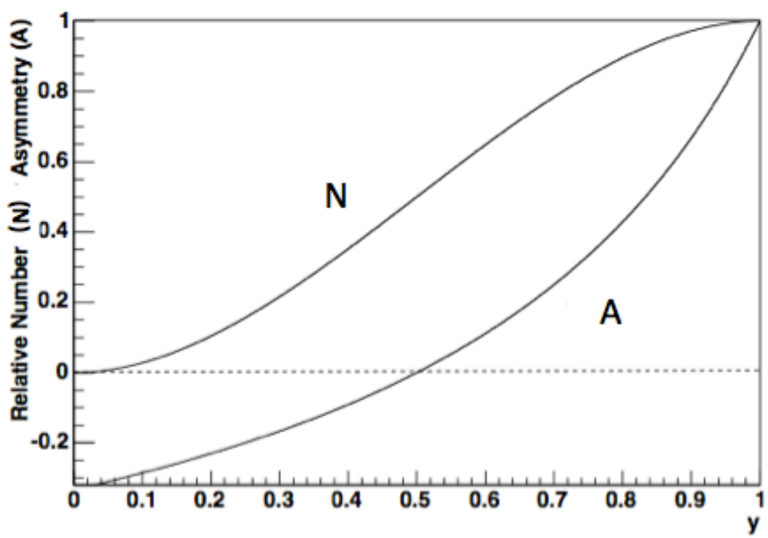
- Electrons came out antiparallel to the the nuclear spin.
- In the mirror image, electrons came out parallel.
  The mirror image does not occur!
- Parity is not conserved in weak interactions.

- The differential decay probability for a positron (dP⁺) to be emitted with a normalized energy $y = E/E_{e,max}$ at an angle θ is:



$$dP^{+} \propto N\left(E_e\right)\left(1 + A(E_e)\cos\theta\right) dy d\Omega$$

$N(E_e)$: The number of decay positrons per unit energy.
$A(E_e)$: The decay asymmetry factor that reflects parity violation.



Muon rest frame

- There are more high energy positrons emitted when their momenta are parallel to the muon spin ($\cos\theta = 1$).
- By only selecting the high energy positrons, the muon spin direction can be inferred.

Muons in a Magnetic Field

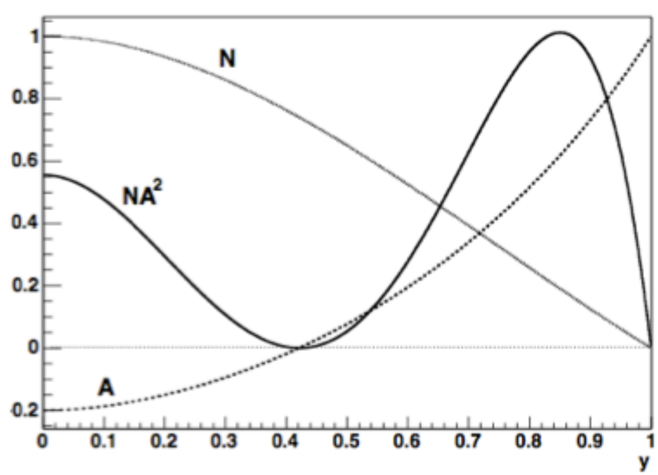
- Effect 1: The interaction of the muon magnetic moment with the magnetic field causes the spin to precess at the Larmor frequency.
- Effect 2: The transverse acceleration of relativistic muons causes the spin to precess at the Thomas frequency.
- If muons are constrained to a circular orbit by a uniform magnetic field, both effects are present and described by the BMT equation (1959):

$$\vec{\omega_s} = \frac{q}{m_\mu c}\left\{\left(a_\mu + \frac{1}{\gamma}\right)\vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} + \left(a_\mu + \frac{1}{\gamma+1}\right)\vec{E}\times\vec{\beta}\right\}$$

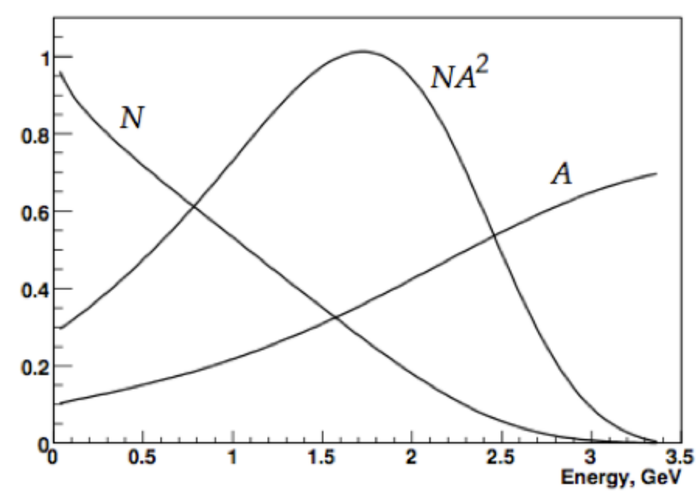
$$\vec{\beta} = \frac{\vec{v}}{c} \qquad \gamma = 1/\sqrt{1-\beta^2}$$

Measurement of ω_a

- If all decay positrons are detected, the number observed is proportional to $\exp(\frac{-t}{\gamma\tau_\mu})$ and no oscillation is present.
- Make a cut on a laboratory observable to see the oscillation: Highest energy positrons!
- Maximize the measurement sensitivity by choosing an energy threshold that maximizes the figure of merit NA^2: $E_{th} \approx 1.8$ GeV.
- In the lab frame:



N, A, and NA^2 vs. fractional energy



Integrated N, A, and NA^2 vs. energy (to the right of E_{th})

p =3.1 GeV pm 10%

Systematics in ω_a

Years	1999 (ppm)	2000 (ppm)	2001 (ppm)
Pileup	0.13	0.13	0.08
Accelerator background	0.10	0.01	†
Lost Muons	0.10	0.10	0.09
Timing Shifts	0.10	0.02	†
E-field and pitch	0.08	0.03	†
Fitting/Binning	0.07	0.06	†
CBO	0.05	0.21	0.07
Gain Changes	0.02	0.13	0.12
Total systematic error on ω_a	0.3	0.31	0.21

† Total of 0.11 ppm

- Pileup: Mis-interpreted low energy electrons.
- Accelerator background: Higher flux of pions from protons on target.
- Lost muons: Perturbations in E or B fields coupling to betatron oscillation at resonance lead to a motion without bound for muons.
- Timing shifts and gain: Pulse sent through the readout system to monitor time and energy resolutions.
- Coherent betatron oscillation (CBO): Mismatch between the inflector and storage ring at injection causes the beam to widen and narrow as it circulates the ring.

Systematics in ω_p

- NMR probes are placed off the beam path.
- Protons' oscillation is measured in water.
- Extrapolation and calibration is required to get the free proton precession in the beam path.

Years	1999 (ppm)	2000 (ppm)	2001 (ppm)
Absolute calibration of standard probe	0.05	0.05	0.05
Calibration of the trolley probes	0.20	0.15	0.09
Trolley measurements of B	0.10	0.10	0.05
Interpolation with fixed probes	0.15	0.10	0.07
Uncertainty from muon distribution	0.12	0.03	0.03
Inflector fringe field uncertainty	0.20	†	†
Others	0.15	0.10	0.10
Total systematic error on ω_p	0.4	0.24	0.17

† Inflector replaced.

Improvements

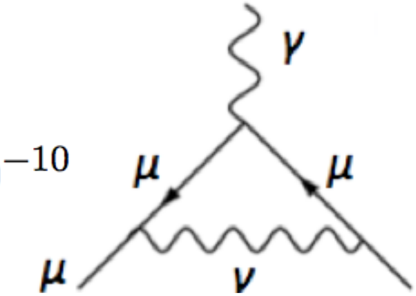
- Same E821 storage ring in a better environment: temperature and mechanical stability.
- Deliver more muons and a purer beam:
  - Accelerator complex will annually deliver 2.3×10^{20} 8 GeV protons on target => 1.8×10^{11} detected positrons above energy threshold in less than 2 years: ×21 BNL!
  - Pion decay line ~ 1.9 km (8.3τ thus $N \sim 2.5 \times 10^{-4} N_0$)=> Pure muon beam! No pions background, no hadronic flash.
- Improve detectors and electronics:
  - New calorimeters that use silicon PMTs with a high photo-detection efficiency to read signal from lead-fluoride crystal with a better energy resolution and a fast Cherenkov response.
  - New trackers to understand beam dynamics, limit pileup, and measure momentum independently.
  - Upgraded electronics and data acquisition to handle the increased data rate.
- Refine the magnetic field uniformity and improve the calibration: Magnetic field measurement improved by more precise position measurement and more frequent measurements.

Theory Calculation

- QED contribution:

$$a_\mu^{\mathrm{QED}} = 116\,584\,71.895\;(0.0009)(0.0019)(0.0007)(0.0077) \times 10^{-10}$$

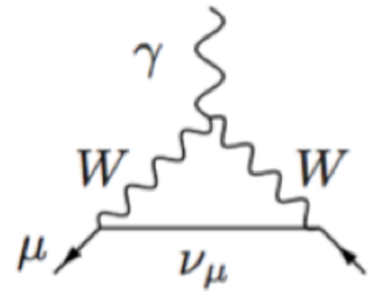
lepton mass ratios · 8^{th} order · 10^{th} order · α(Rb)

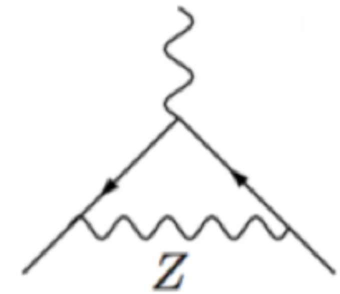


- ➢ 99.99% a_μ
- ➢ 12,672 diagrams

$$a_\mu^{\mathrm{EW}} = 15.36(0.1) \times 10^{-10}:$$

3^{rd} order





1.3 ppm and BNL 0.54 ppm: Prob

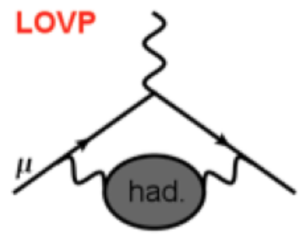
$$a_\mu^{\mathrm{Had}} = a_\mu^{\mathrm{Had,LOVP}} + a_\mu^{\mathrm{Had,HOVP}} + a_\mu^{\mathrm{LbL}}$$

$$a_\mu^{\mathrm{Had,LOVP}} = 694.9(4.3) \times 10^{-10}$$

$$a_\mu^{\mathrm{Had,HOVP}} = -9.84(0.07) \times 10^{-10}$$

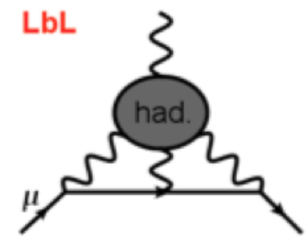
$$a_\mu^{\mathrm{Had,LbL}} = 10.5(2.6) \times 10^{-10}$$

LOVP



HOVP

LbL



$m_\mu c^2$ ~ 106 MeV below pQCD region

⇒ Use low energy e^+e^- -> had data and models

⇒ Highest uncertainty ~ 0.4ppm