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Dark Energy as Evidence for Extra Dimensions

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Abstract

It is argued that fluctuations of quantum fields in four-dimensional space do not give rise to dark energy, but are rather a negligible contribution to dark matter. By (relativistic) dark matter we mean that the relation between pressure and energy density is $p = \frac{1}{3}u$, while dark energy is characterized by $p = -u$. A possible source of dark energy are the fluctuations in quantum fields, including quantum gravity, inhabiting extra compactified dimensions. These fluctuations have been computed for some simple geometries, such as S^2, S^4, and S^6. If the extra dimensions are too small, they would give rise to a dark energy larger than that observed, whereas if they are too large they would be in conflict with experimental tests of Newton's law. This notion suggests that the size of the extra dimensions is of order 100 μm. If the limit on the size of extra dimensions becomes lower than this bound, extra dimensions probably do not exist, and another source for cosmological dark energy will have to be found.

1 Introduction

It has been appreciated for many years that there is an apparently fundamental conflict between quantum field theory and the smallness of the cosmological constant [1]. This is because the zero-point energy of the quantum fields (including gravity) in the universe should give rise to an observable cosmological vacuum energy density,

$$u_{\text{cosmo}} \sim \frac{1}{L_{\text{Pl}}^4}, \tag{1}$$

where the Planck length is

$$L_{\text{Pl}} = \sqrt{G_N} = 1.6 \times 10^{-33}\,\text{cm}. \tag{2}$$

(We use natural units with $\hbar = c = 1$. The conversion factor is $\hbar c \simeq 2 \times 10^{-14}\,\text{GeV cm}$.) This means that the cosmic vacuum energy density would be

$$u_{\text{cosmo}} \sim 10^{118}\ \text{GeV}\,\text{cm}^{-3}, \tag{3}$$

which is 123 orders of magnitude larger than the critical mass density required to close the universe:

$$\rho_c = \frac{3H_0^2}{8\pi G_N} = 1.05 \times 10^{-5} h_0^2\,\text{GeV}\,\text{cm}^{-3}, \tag{4}$$

in terms of the dimensionless Hubble constant, $h_0 = H_0/100\ \text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$. From relativistic covariance the cosmological vacuum energy density must be the 00 component of the expectation value of the energy-momentum tensor, which we can identify with the cosmological constant:

$$\langle T^{\mu\nu}\rangle = -u g^{\mu\nu} = -\frac{\Lambda}{8\pi G} g^{\mu\nu}. \tag{5}$$

[We use the metric with signature $(-1, 1, 1, 1)$.] Of course this is absurd with u given by Eq. (3), which would have caused the universe to expand to zero density long ago.

For most of the past century, it was the prejudice of theoreticians that the cosmological constant was exactly zero, although no one could give a convincing argument. Recently, however, with the new data gathered on the brightness-redshift relation for very distant type Ia supernovæ [2], corroborated by the balloon observations of the anisotropy in the cosmic microwave background [3], it seems clear that the cosmological constant is near the critical value, or

$$\Omega_\Lambda = \Lambda/8\pi G\rho_c \simeq 0.6 - 0.7. \tag{6}$$

It is very hard to understand how the cosmological constant can be nonzero but small.

2 Quantum Fluctuations

We here present a plausible scenario for understanding this puzzle. It seems quite clear that vacuum fluctuations in the gravitational and matter fields in flat Minkowski space give a zero cosmological constant. For example, we can consider fluctuations of conformal matter in $R \times S^3$, which give rise to the following forms of the energy and free energy at high and low temperature [4]

$$\begin{aligned} &F \sim -\frac{1}{a} a_4 (2\pi a T)^4, \quad E \sim \frac{1}{a} 3 a_4 (2\pi a T)^4, \quad aT \gg 1, \\ &F = E = \frac{a_0}{a}, \quad aT \ll 1, \end{aligned} \tag{7}$$

where a is the radius of S^3. Here, for example, $a_4 = 1/48$ and $a_0 = 3/16$ for $\mathcal{N} = 4$ SUSY. [It should be noted that at present the universe is in the high temperature regime; since inflation $aT \sim 10^{29}$.] Either regime corresponds to a relation typical of radiation,

$$p = -\frac{\partial}{\partial V} F = \frac{1}{3} u, \quad u = \frac{E}{V}, \quad V = 2\pi^2 a^3. \tag{8}$$

Other vacuum fluctuation phenomena are also unlikely to contribute to the cosmological constant. For example, fluctuations of quark and gluon fields inside a hadronic bag of radius R give a zero-point energy of roughly [5]

$$E_{\text{ZPE}} \sim \frac{0.7}{R}. \tag{9}$$

Johnson's model of the QCD vacuum [6] as consisting of a sea of virtual bags might suggest then a corresponding energy density

$$u_{\text{QCD}} \sim \frac{E_{\text{ZPE}}}{\frac{4\pi}{3} R^3} \sim 10^{39} \text{GeV}\,\text{cm}^{-3} \tag{10}$$

if $R = 0.5\,\text{fm}$, some 44 orders of magnitude too large. Yet this is surely an unreasonable inference: Rather E_{ZPE} is absorbed into a renormalization of QCD parameters, as a contribution, for example, to the masses of the observed hadrons. Thus, we expect quite confidently that there is no QCD vacuum gravitational effect.[1]

[1] After mentioning the QCD zero-point contribution to the cosmological constant, Weinberg in 1989 [1] pointed out that the classical vacuum energy resulting from spontaneous symmetry breaking need not give rise to an effective cosmological constant at the present era.

3 Extra Dimensions

On the other hand, since the work of Kaluza and Klein it has been an exciting possibility that there exist extra dimensions beyond those of Minkowski space-time. Why do we not experience those dimensions? The simplest possibility seems to be that those extra dimensions are curled up in a space $\mathcal{S}$ of size a, smaller than some observable limit.

Of course, in recent years, the idea of extra dimensions has become much more compelling. Superstring theory requires at least 10 dimensions, six of which must be compactified, and the putative M theory, supergravity, is an 11 dimensional theory. Perhaps, if only gravity experiences the extra dimensions, they could be of macroscopic size. Various scenarios have been suggested [7].

Macroscopic extra dimensions imply deviations from Newton's law at such a scale. Two years ago, millimeter scale deviations seemed plausible, and many theorists hoped that the higher-dimensional world was on the brink of discovery. Experiments were initiated [8]. Recently, the results of the first definitive experiment have appeared [9], which indicate no deviation from Newton's law down to 200 μm. This poses a serious constraint for model-builders.[2]

It seems to be commonly believed that submillimeter tests of gravity put no limits on the size of extra dimensions if $N > 2$. This is because of the relation of the size R of the extra dimensions in the ADD scheme to the fundamental $4 + N$ gravity scale M [12]:

$$R \sim \frac{1}{M}\left(\frac{M_{\rm Pl}}{M}\right)^{2/N}, \tag{11}$$

where $M_{\rm Pl} = 1/L_{\rm Pl} = 1.2 \times 10^{16}$ TeV is the usual Planck mass. Moreover, the supernova limits on ADD extra dimensions (due to production of Kaluza-Klein gravitons) become rapidly smaller with increase in N [13]:

$$N = 2: \quad R < 0.9 \times 10^{-4}\,\text{mm}, \tag{12}$$

$$N = 3: \quad R < 1.9 \times 10^{-7}\,\text{mm}. \tag{13}$$

[2]We might also mention short distance constraints on Yukawa-type corrections to the gravitational potential coming from Casimir measurements themselves. See Ref. [10]. Most recently, stringent limits have now appeared for Yukawa forces with ranges between 200 and 500 microns [11].

(Cosmological constraints are even stronger [14], but are less certain.) Thus direct tests of Newton's law are not competitive. However, as we will see, the resulting Casimir contribution to the cosmological constant would be enormous for such small compactified regions, and it would seem impossible to naturally resolve this problem.

The situation at first glance seems rather different with the RS scenario. In the original scheme, gravity is localized in the "Planck brane," while the standard-model particles are confined to the "TeV brane." As a consequence, it might appear that the quantum fluctuations of both brane and bulk fields are negligible [15]. It has been stated that the cosmological constant becomes exponentially small as the brane separation becomes large [16]. However, this is at the "classical level," without bulk fluctuations; explicit considerations show that quantum effects give rise to a large cosmological constant, of order of that given by Eq. (3), unless an appeal is made to fine tuning [17]. Moreover, if the scenario is extended so that the world brane contains compactified dimensions in which gravity lives [18], the constraints we deduce here directly apply.

Here we propose that a very tight constraint indeed emerges if we recognize that compact dimensions of size a necessarily possess a quantum vacuum or Casimir energy of order $u(z) \sim a^{-4}$. These can be calculated in simple cases. Appelquist and Chodos [19] found that the Casimir energy for the case of scalar field on a circle, $\mathcal{S} = S^1$, was

$$u_C = -\frac{3\zeta(5)}{64\pi^6 a^4} = -\frac{5.056 \times 10^{-5}}{a^4}, \tag{14}$$

which needs only to be multiplied by 5 for graviton fluctuations. The general case of scalars on $\mathcal{S} = S^N$, N odd, was considered by Candelas and Weinberg [20], who found that the Casimir energy was positive for $3 \leq N \leq 19$, with a maximum at $N = 13$ of $u_C = 1.374 \times 10^{-3}/a^4$. The even dimensional case was much more subtle, because it was divergent. Kantowski and Milton [21] showed that the coefficient of the logarithmic divergence was unique, and adopting the Planck length as the natural cutoff, found

$$S^N,\ N \text{ even}: \quad u_C^N = \frac{\alpha_N}{a^4} \ln \frac{a}{L_{\rm Pl}}, \tag{15}$$

but α_N was always negative for scalars. In a second paper [22] we extended the analysis to vectors, tensors, fermions, and to massive particles, among which cases positive values of the (divergent) Casimir energy could be found.

$\mathcal{S}$	Gravity	Scalar	Fermion	Vector
S^1	-2.53×10^{-4}	-5.06×10^{-5}	2.02×10^{-4}	—
S^1	2.37×10^{-4}	4.74×10^{-5}	-1.90×10^{-4}	—
S^2	1.70×10^{-2}	-8.04×10^{-5}	-7.94×10^{-4}	-8.04×10^{-5}
S^3	—	7.57×10^{-5}	1.95×10^{-4}	—
S^4	-0.489	-4.99×10^{-4}	-6.64×10^{-3}	1.21×10^{-2}
S^5	—	4.28×10^{-4}	-1.14×10^{-4}	—
S^6	5.10	-1.31×10^{-3}	-3.02×10^{-2}	4.90×10^{-2}
S^7	—	8.16×10^{-4}	5.96×10^{-5}	—

Table 1: The Casimir energy for $M^4 \times \mathcal{S}$ is tabulated for various field types in the compact geometry $\mathcal{S}$. We write $u = [\alpha \ln(a/L_{\rm Pl}) + \beta]a^{-4}$, and give α for even internal dimension and β for odd, where $\alpha = 0$. For S^1 the first entry denotes untwisted (periodic) while the second denotes twisted (antiperiodic) boundary conditions. The entries marked with dashes have not been calculated.

Some representative results for massless particles are shown in Table 1. In an unsuccessful attempt to find stable configurations, the analysis was extended to cases where the internal space was the product of spheres [23].

It is important to recognize that these Casimir energies correspond to a cosmological constant in our $3+1$ dimensional world, not in the extra compactified dimensions or "bulk." They constitute an effective source term in the 4-dimensional Einstein equations. Note that because the scale a makes no reference to four-dimensional space, the total free energy of the universe (of volume V) arising from this source is $F = Vu_c$, so as required for dark energy or a cosmological constant,

$$p = -\frac{\partial}{\partial V}F = -u_c, \quad T^{\mu\nu} = -u_c g^{\mu\nu}. \tag{16}$$

The goal, of course, in all these investigations was to include graviton fluctuations. However, it immediately became apparent that the results were gauge- and reparameterization-dependent unless the DeWitt-Vilkovisky formalism was adopted [24]. This was an extraordinarily difficult task. Among the early papers in which the unique effective action is given in simple cases we cite Ref. [25]. Only in 2000 did the general analysis for gravity appear, with results for a few special geometries [26]. Cho and Kantowski obtain the unique divergent part of the effective action for $\mathcal{S} = S^2$, S^4, and S^6, as polynomials in Λa^2. (Unfortunately, once again, they are unable to find any stable configurations.)

The results are also shown in Table 1, for $\Lambda a^2 \sim G/a^2 \ll 1$. It will be noted that graviton fluctuations dominate matter fluctuations, except in the case of a large number of matter fields in a small number of dimensions. Of course, it would be very interesting to know the graviton fluctuation results for odd-dimensional spaces, but that seems to be a more difficult calculation; it is far easier to compute the divergent part than the finite part, which is all there is in odd-dimensional spaces.

These generic results may be applied to recent popular scenarios. For example, in the ADD scheme only gravity propagates in the bulk, while the RS approach has other bulk fields in a single extra dimension.

Let us now perform some simple estimates of the cosmological constant in these models. The data suggest a positive cosmological constant, so we can exclude those cases where the Casimir energy is negative. For the odd N cases, where the Casimir energy is finite, let us write

$$\mathcal{S} = S^N, \quad N\,\text{odd}: \quad u_C^N = \frac{\beta_N}{a^4}, \tag{17}$$

so merely requiring that this be less than the critical density ρ_c implies ($\beta > 0$)

$$a > \beta^{1/4} h_0^{-1/2} 67\,\mu\text{m} \approx \beta^{1/4} 80\,\mu\text{m}, \tag{18}$$

taking $h_0 = 0.7$ (with about a 10–20% uncertainty). As seen in Table 2 these lower limits (for a single species) are still an order of magnitude below the experimental upper limit.

Much tighter constraints appear if we use the divergent results for even dimensions. We have the inequality ($\alpha > 0$)

$$a > [\alpha \ln(a/L_{\rm Pl})]^{1/4} 80\,\mu\text{m}, \tag{19}$$

where we can approximate $(\ln a/L_{\rm Pl})^{1/4} \approx 2.9$. Again results are shown in Table 2, which rules out all but one of the gravity cases (S^2) given by Cho and Kantowski. For matter fluctuations only, excluded are $N > 14$ for a single vector field and $N > 6$ for a single tensor field. (Fermions always have a negative Casimir energy in even dimensions.) Of course, it is possible to achieve cancellations by including various matter fields and gravity.

$\mathcal{S}$	Gravity	Scalar	Fermion	Vector
S^1 (u)	*	*	9.5 μm	—
S^1 (t)	9.9 μm	6.6 μm	*	—
S^2	84 μm	*	*	*
S^3	—	7.5 μm	9.5 μm	—
S^4	*	*	*	77 μm
S^5	—	11.5 μm	*	—
S^6	350 μm	*	*	110 μm
S^7	—	13.5 μm	7.0 μm	—

Table 2: The lower limit to the radius of the compact dimensions deduced from the requirement that the Casimir energy not exceed the critical density. The numbers shown are for a single species of the field type indicated. The dashes indicate cases where the Casimir energy has not been calculated, while asterisks indicate (phenomenologically excluded) cases where the Casimir energy is negative.

In general the Casimir energy is obtained by summing over the species of field which propagate in the extra dimensions,

$$u_{\rm tot} = \frac{1}{a^4} \sum_i \left[\alpha_i \ln(a/L_{\rm Pl}) + \beta_i\right] \approx \frac{\beta_{\rm eff}}{a^4}, \tag{20}$$

which leads to a lower limit according to Eq. (18). Presumably, if exact supersymmetry held in the extra dimensions (including supersymmetric boundary conditions), the Casimir energy would vanish, but this would seem to be difficult to achieve with *large* extra dimensions (1 mm corresponds to 2×10^{-4} eV.)

That there is a correlation between the currently favored value of the cosmological constant and submillimeter-sized extra dimensions has been noted qualitatively before [27].

4 Conclusions

We have proposed the following scenario to explain the predominance of dark energy in the universe.

- Quantum fluctuations of gravity/matter fields in extra dimensions give rise to a dark energy, or cosmological constant, $\propto 1/a^4$ where a is the size of the extra dimensions.

- The dark energy will be too large unless $a > 10 - 300\ \mu$m.
- Laboratory (Cavendish) tests of Newton's law require $a < 200\ \mu$m.
- Thus, we may be on the verge of discovery of extra dimensions, or
- Extra dimensions do not exist and dark energy has another origin.

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