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SONOLUMINESCENCE AND THE DYNAMICAL CASIMIR EFFECT

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It has been suggested by various authors that the 'dynamical Casimir effect' might prove responsible for the production of visible-light photons in the bubble collapse which occurs in sonoluminescence. Previously, I have argued against this point of view based on energetic considerations, in the adiabatic approximation. Those arguments have recently been strengthened by the demonstration of the equivalence between van der Waals and Casimir energies. In this note I concentrate on the other extreme possibility, that of the validity of the 'sudden approximation' where in effect the bubble instantaneously ceases to exist. Previous estimates which seemed to support the relevance of the Casimir effect are shown to be unconvincing because they require macroscopic changes on excessively small time scales, involving the entire volume of the bubble at maximum radius.

1 Introduction

The production mechanism of the intense flashes of light which occur at the end of bubble collapse in sonoluminescence remains mysterious.[1] A particularly intriguing possibility, put forth by Schwinger, was that the Casimir effect in some dynamical manifestation was responsible.[2,3,4,5,6] This idea was extended first by Eberlein,[7] and later by Carlson, Liberati, and others.[8,9]

Let us start by reviewing the relevant numbers for sonoluminscent light emission. Typically, a bubble of air in water is held in the node of an acoustic standing wave with overpressure of about 1 atmosphere, of a frequency 20 kHz. The bubble goes from a maximum radius of $\sim 4\times10^{-3}$ cm to a minimum radius of $\sim 4\times10^{-4}$ cm with a time scale τ_c of 10^{-5} s. The flash of light, which occurs near minimum radius, has a time scale τ_f of less than 10^{-11} s, and is characterized by the emission of 10^6 optical photons, so about 10 MeV of light energy is emitted per flash.

It seems likely that the adiabatic approximation should be valid. If the flash scale is not orders of magnitude less than 10^{-11} s, that scale is long compared to the optical time scale, $\tau_o\sim10^{-15}$ s. In that case, we can immediately test the Casimir idea. The Casimir energy of a dielectric ball in vacuum is

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equivalent to that of a bubble in a dielectric medium, and has recently been definitively evaluated.[10,11] The Casimir energy of a dilute ball, of dielectric constant ϵ, $|\epsilon-1|\ll 1$, of radius R is

$$E=\frac{23}{1536\pi R}(\epsilon-1)^2, \tag{1}$$

which may be alternatively calculated by summing the van der Waals energies between the molecules that make up the medium.[12] This value is 10 orders of magnitude too small to be relevant, as well as of the wrong sign. This is hardly a surprising result, since the magnitude of the effect is what one would expect from dimensional considerations.

However, others have come to an opposite conclusion. In particular, Schwinger, [3] without relying on detailed calculations, asserted that the 'dielectric energy, relative to that of the vacuum' was

$$E_c=-\int\frac{(d\mathbf{r})(d\mathbf{k})}{(2\pi)^3}\frac{1}{2}k\left(1-\frac{1}{\epsilon(\mathbf{r})^{1/2}}\right). \tag{2}$$

Although he argued this was true for slow variation in the dielectric constant, he applied it to a hole of radius a with a dielectric medium, therefore with a discontinuous boundary:

$$E_c=\frac{R^3}{12\pi}K^4\left(1-\frac{1}{\epsilon^{1/2}}\right). \tag{3}$$

Here K represents an ultraviolet cutoff, which Schwinger took to be $K\sim 2\times10^5$ cm^{-1}, which gives a sufficient energy, $E_c\sim 6$ MeV, to be relevant.

This conclusion is supported by the work of Carlson et al.,[8] who obtain the identical result. Why is there a discrepancy of the conclusion of these authors with the result given in Eq. (1)? The answer is simple. The term that Schwinger[3] and Carlson et al.[8] keep is indeed present as a quartically divergent term if one simply sums normal modes. But this is a intrinsic contribution to the self-energy of the dielectric medium. It was quite properly subtracted off at the outset in the first paper on the Casimir energy of a dielectric ball,[13] as it was in Schwinger's own detailed papers on the Casimir effect.[14] A detailed analysis of this issue is given in Ref. [12]. As Barton has noted, such divergent volume and surface terms 'would be combined with other contributions to the bulk and to the surface energies of the material, and play no further role if one uses the measured values.' [11] In other words, they serve to renormalize the phenomenological parameters of the model.

Further support for the irrelevance of the bulk energy comes from the above-noted identity between the dilute Casimir energy and the van der Waals energy.[10,11,12] This would seem *prima facie* evidence that the finite remainder is unambiguously determined. Note that the summed van der Waals energy must go like $(\epsilon-1)^2$, not the $\epsilon-1$ behavior that Eq. (2) displays.

2 Acceleration and Temperature

It seems plausible that the dynamical Casimir effect is closely allied with the so-called Unruh effect,[15] wherein an accelerated observer, with acceleration a, sees a bath of photons with temperature T,

$$T=\frac{a}{2\pi}. \tag{4}$$

Indeed, the observed radiation in sonoluminescence is consistent with the tail of a blackbody spectrum, with temperature ~20,000 K.[a] That is, kT is about 1 eV. Let us, rather naively, apply this to the collapsing bubble, where $a=d^2R/dt^2\sim R/\tau_f^2$, where τ_f is some relevant time scale for the flash. We then have

$$kT\sim\frac{R}{(c\tau_f)^2}\hbar c, \tag{5}$$

or

$$1\,\mathrm{eV}\sim\frac{10^{-3}\mathrm{cm}\,2\times10^{-5}\mathrm{eV\text{-}cm}}{\tau_f^2(3\times10^{10}\mathrm{cm\,s^{-1}})^2}\sim\frac{10^{-29}\mathrm{eV}}{\tau_f^2(\mathrm{s}^2)}. \tag{6}$$

That is, $\tau_f\sim10^{-15}$ s, which seems implausibly short; it implies a characteristic velocity $R/\tau_f\sim10^{12}$ cm/s $\gg c$. It is far shorter than the upper limit to the flash duration, 10^{-11} s. Indeed, if we use the latter in the Unruh formula (4) we get a temperature about 1 milli Kelvin! This conclusion seems consistent with that of Eberlein,[7] who indeed stressed the connection with the Unruh effect, but whose numbers required superluminal velocities.

However, we must remain open to the possibility that discontinuities, as in a shock, could allow changes on such short time scales without requiring superluminal speeds. Indeed, Liberati et al.,[9] following Schwinger's earlier suggestion,[2,4] indeed assume an extremely short time scale, so that rather than the adiabatic approximation discussed above being valid, a sudden approximation is more appropriate. We therefore turn to an analysis of that situation.

3 Instantaneous collapse and photon production

The picture offered by Liberati et al.[9] is that of the abrupt disappearance of the bubble at $t=0$, as shown in Fig. 1. On the face of it, this picture seems

[a] The temperature may be even higher. If so, τ_f is correspondingly reduced.

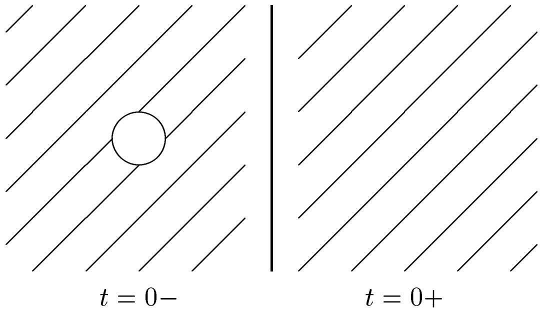

Figure 1: The sudden collapse of an otherwise static bubble.

preposterous—the bubble simply disappears and water is created out of nothing. It is no surprise that a large energy release would occur in such a case. Further, the static Casimir effect calculations employed in Ref. [9] are invalid in this instantaneously changing model. Therefore, rather than computing Bogoliubov coefficients from the overlap of states belonging to two static configurations, let us follow the original methodology of Schwinger,[2,4] which is essentially equivalent.

As in Schwinger's papers, let us confine our attention to the electric (TM) modes. They are governed by the time-dependent Green's function satisfying

$$(\partial_0 \epsilon(x) \partial_0 - \nabla^2) G(x, x') = \delta(x - x'). \tag{7}$$

The photon production is given by the effective two-photon source

$$\delta(JJ) = i\delta G^{-1} = i\partial_0 \delta\epsilon(x) \partial_0. \tag{8}$$

The effectiveness for producing a photon in the momentum element centered about $\mathbf{k}$ is

$$J_k = \sqrt{\frac{(d\mathbf{k})}{(2\pi)^3} \frac{1}{2\omega}} \int (dx) e^{-i(\mathbf{k}\cdot\mathbf{r} - i\omega t)} J(x), \quad \omega = |\mathbf{k}|. \tag{9}$$

Let us follow Schwinger and consider one complete cycle of disappearance and re-appearance of the bubble, which we assume disappears for a time τ_c: For a bubble centered at the origin, the dielectric constant as a function of time within the volume of the bubble is then taken to be

$$r < R: \quad \epsilon(r) = 1 + (\epsilon' - 1)\eta(\tau_c/2 - |t|). \tag{10}$$

Here ϵ' is the dielectric constant of all space when the bubble is gone. The dielectric constant of the region outside the volume occupied by the bubble is

$$r > R: \quad \epsilon(r) = \epsilon + (\epsilon' - \epsilon)\eta(\tau_c/2 - |t|). \tag{11}$$

Here ϵ is the dielectric constant outside the bubble when the bubble is present. Occurring here is the unit step function,

$$\eta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases} \tag{12}$$

Clearly, this model is based on the assumption that the disappearance time is short compared to the complete cycle time of bubble collapse and re-expansion.

In the spirit of a first approximation, let us suppose all the dielectric constants are nearly unity, that is, that we are dealing with dilute media. Let us further assume, appropriate to the instantaneous approximation, that the medium is a gas, which is capable of instantaneously filling the bubble. Then because the deviation of the dielectric constant from unity is proportional to the matter number density N,

$$\epsilon - 1 = 4\pi N\alpha, \tag{13}$$

where α is the *constant* molecular polarizability, matter conservation implies

$$(\epsilon' - 1)V = (\epsilon - 1)(V - v), \tag{14}$$

where V is the volume of all space, and v is the volume of the bubble. Thus the change of the dielectric constant inside the bubble, and outside, respectively, is

$$\begin{aligned} \delta\epsilon_{\text{in}} &= (\epsilon' - 1)\eta(\tau_c/2 - |t|), \\ \delta\epsilon_{\text{out}} &= (\epsilon' - \epsilon)\eta(\tau_c/2 - |t|) = -(\epsilon' - 1)\frac{v}{V - v}\eta(\tau_c/2 - |t|). \end{aligned} \tag{15}$$

The latter term here appears to be very small, and was therefore disregarded in Ref. [2,4,9]. However, we will see that the inclusion of this term could be significant.

From Eqs. (8) and (9), the two-photon production amplitude is proportional to ($v \ll V$)

$$\begin{aligned} J_k J_{k'} &= \sqrt{\frac{(d\mathbf{k})}{(2\pi)^3}\frac{(d\mathbf{k}')}{(2\pi)^3}\frac{1}{2\omega 2\omega'}} \int (d\mathbf{r}') \int_{-\tau/2}^{\tau/2} dt\, e^{-i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r} + i(\omega+\omega')t}(-i\omega\omega') \\ &\quad \times (\epsilon' - 1)\left[\eta(a - r) - \frac{v}{V}\eta(r - a)\right] \\ &\propto (\epsilon' - 1)\int_{-\tau/2}^{\tau/2} dt\, e^{i(\omega+\omega')t}(-i\omega\omega')\left[\int_{\text{in}} (d\mathbf{r})e^{-i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}} \right. \\ &\qquad \left. - \frac{v}{V}\int_{\text{out}} (d\mathbf{r})e^{-i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}}\right]. \end{aligned} \tag{16}$$

The probability of emitting two photons is proportional to the square of this amplitude. For sufficiently short wavelengths, $\lambda \ll R$, the square of the quantity in square brackets in Eq. (16) is the product of $(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')$ and v, that is, if the exterior contribution is negligible,

$$|J_k J_{k'}|^2 \propto (\epsilon'-1)^2\omega^2\sin^2\omega\tau_c\,\delta(\mathbf{k}+\mathbf{k}')v. \tag{17}$$

This is the same result found by Schwinger,[2,4] and by Liberati et al.[9] However, if as is plausible, the effective exterior volume V is not much bigger that the volume of the bubble v, a larger contribution results. Indeed, a careful discretized version of the momentum integrals in Eq. (16) gives in general for the factor multiplying the delta function in Eq. (17) $v(1+v/V)^2$. The interference is *constructive*, not destructive as I erroneously claimed in my Leipzig talk, and negligible as $V \to \infty$. Taking the latter limit (but remembering that there might be up to a factor of 4 enhancement), and, appropriate for $\tau_c/\tau_o \gg 1$, replacing $\sin^2\omega\tau_c \to 1/2$, we obtain the probability of emitting a pair with momenta $\mathbf{k}$ and $-\mathbf{k}$ just as given by Schwinger [4] (this now includes the equal contribution from the magnetic modes):

$$P_{\gamma\gamma} = v\frac{(d\mathbf{k})}{(2\pi)^3}\left(\frac{\epsilon-1}{4}\right)^2, \quad |\epsilon-1| \ll 1. \tag{18}$$

[For $|\epsilon-1|$ not small, Schwinger [4] generalized this to

$$P_{\gamma\gamma} = 2v\frac{(d\mathbf{k})}{(2\pi)^3}\ln\frac{\epsilon^{1/4}+\epsilon^{-1/4}}{2}. \tag{19}$$

The numerical effect of this correction is not significant for a first estimate.] The total number of photon pairs emitted is then, if dispersion is ignored,

$$N = \left(\frac{4\pi}{3}\right)^2\left(\frac{R}{\Lambda}\right)^3\left(\frac{\epsilon-1}{4}\right)^2, \tag{20}$$

where the cutoff wavelength is given by $K = 2\pi/\Lambda$. Such a divergent result should be regarded as suspect.[b] It was Eberlein's laudable goal [7] to put this type of argument on a sounder footing. Nevertheless, if we put in plausible numbers, $\sqrt{\epsilon} = 4/3$, $R = 4\times10^{-3}$ cm, and, as in Schwinger's earlier estimate, $\Lambda = 3\times10^{-5}$ cm, we obtain the required $N \sim 10^6$ photons per flash.

The problem with this estimate is one of time and length scales—for the instantaneous approximation to be valid, the flash time τ_f must be much less

[b]Although it is not clear how this is to be related to the divergent energy (3), Schwinger obtained both in Ref. [4] as the imaginary and real parts, respectively, of a complex action.

than the period of optical photons, $\tau_o \sim 10^{-15}$ s. This is consistent with the discussion in §2, and acknowledged by Liberati et al.[9] On the other hand, the collapse time $\tau_c \sim 10^{-5}$ s is vastly longer than τ_f, and is therefore totally irrelevant to the photon production mechanism. The flash occurs near minimum radius, and thus the appropriate value of R in Eq. (20) would seem to be at least an order of magnitude smaller, $R \sim 10^{-4}$ cm. This would lead to $N < 10^3$ photon pairs, totally insufficient.

4 Conclusions

We conclude by stating that the Casimir model fo sonoluminescence remains 'unproven.' The static Casimir effect can be applied only in the adiabatic approximation, where it seems clearly irrelevant. The instantaneous approximation grafted onto static configurations seems logically deficient, and again numerically irrelevant unless implausible parameters are adopted. What is still needed is a dynamical calculation of the Casimir effect. The burden of proof is on the proponents of this mechanism.

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References

1. B. P. Barber, R. A. Hiller, R. Löfstedt, S. J. Putterman, and K. Weniger, *Phys. Rep.* **281**, 65 (1997).
2. J. Schwinger, *Proc. Natl. Acad. Sci. USA* **90**, 958 (1993).
3. J. Schwinger, *Proc. Natl. Acad. Sci. USA* **90**, 2105 (1993).
4. J. Schwinger, *Proc. Natl. Acad. Sci. USA* **90**, 4505 (1993).
5. J. Schwinger, *Proc. Natl. Acad. Sci. USA* **90**, 7285 (1993).
6. J. Schwinger, *Proc. Natl. Acad. Sci. USA* **91**, 6473 (1994).
7. C. Eberlein, *Phys. Rev. A* **53**, 2772 (1996); *Phys. Rev. Lett.* **76**, 3842 (1996). An earlier, simplified, one-dimensional calculation, which explicitly used the adiabatic approximation, was given by E. Sassaroli, Y. N. Srivastava, and A. Widom, *Phys. Rev. A* **50**, 1027 (1994).
8. C. E. Carlson, C. Molina-París, J. Pérez-Mercader, and M. Visser, Phys. Lett. **B395**, 76 (1997); Phys. Rev. D **56**, 1262 (1997); C. Molina-París and M. Visser, Phys. Rev. D **56**, 6629 (1997).

9. S. Liberati, M. Visser, F. Belgiorno, and D. W. Sciama. "Sonoluminescence: Bogolubov Coefficients for the QED Vacuum of a Collapsing Bubble," quant-ph/9805023; S. Liberati, F. Belgiorno, M. Visser, and D. W. Sciama, "Sonoluminescence as a Quantum Vacuum Effect," quant-ph/9805031.
10. I. Brevik and V. N. Marachevsky, "Casimir Surface Force on a Dilute Dielectric Ball," I. Brevik, V. N. Marachevsky, and K. A. Milton, hep-th/9810062.
11. G. Barton, "Perturbative Check on the Casimir Energy on a Nondispersive Ball."
12. K. A. Milton and Y. J. Ng, *Phys Rev.* E **57**, 5504 (1998).
13. K. A. Milton, *Ann. Phys.* (N.Y.) **127**, 49 (1980).
14. J. Schwinger, L. L. DeRaad, Jr., and K. A. Milton, *Ann. Phys.* (N.Y.) **115**, 1 (1978); J. Schwinger, *Lett. Math. Phys.* **24**, 227 (1992); *Proc. Natl. Acad. Sci. USA* **89**, 4091, 11118 (1992).
15. P. C. W. Davies, *J. Phys. A: Gen. Phys.* **8**, 365 (1975); W. G. Unruh, *Phys. Rev.* D **14**, 870 (1976); N. D. Birrell and P. C. W. Davies, *Quantum Field Theory in Curved Space* (Cambridge University Press, Cambridge, England, 1982). See also G. T. Moore, *J. Math. Phys.* **11**, 2679 (1970); S. A. Fulling and P. C. W. Davies, *Proc. Roy. Soc. London* A **348**, 393 (1976); S. A. Fulling, *Aspects of Quantum Field Theory in Curved Space-Time* (Cambridge University Press, Cambridge, 1989).
16. S. W. Hawking, *Nature* **248**, 30 (1974); *Commun. Math. Phys.* **43**, 199 (1975).