Response to "Comment on 'Temperature dependence of the Casimir effect' "

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There has been recent criticism of our paper [Phys. Rev. E **71**, 056101 (2005)] saying it is in conflict with the third law of thermodynamics and in contradiction with experiment. We show that these claims are unwarranted.

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Recently there has been a comment [1] that has strongly criticized our work [2] on the temperature dependence of the Casimir force between a spherical lens and a plate, both coated with real metals (Au for example). This follows a long paper by some of the same authors [3] which also criticizes our work as being in conflict with fundamental thermodynamical requirements, particularly the third law of thermodynamics or the Nernst heat theorem. Furthermore, they claim that their recent experimental results are completely consistent with their theoretical approach, and in contrast completely rule out our theory. The purpose of this note is to respond to these criticisms, and emphasize that the theoretical claims are invalid, while the experimental situation is still too indecisive to draw definitive conclusions.

First, let us make a statement about the physical nature of the controversy. The conventional approach, dating back to Lifshitz [4], and reinforced by Schwinger et al. [5], describes an ideal metal by a formal $\varepsilon \to \infty$ limit, where ε is the permittivity of the material. Mathematically, the limit is taken in such a way that both the TE and TM zero modes (that is, the $m = 0$ term in the Matsubara sum) contribute. Recently, however, it has been recognized [6] that this is not correct, and that the TE zero mode cannot contribute either for finite ε or for a real metal. Consequently, in the ideal metal limit, this recognition gives rise to a linear temperature term in the pressure at low temperature, and a nonzero value of the entropy at zero temperature. However, real metals are not described by this ideal limit, and even though only one zero mode, the TM one, can be present, as one sees by either thermodynamic or electrodynamic considerations, the contradiction with the Nernst theorem is not present for actual conductors.

In contrast, the authors of [1, 3] propose two alternatives to describe the reflection coefficients that enter into the Lifshitz theory, the impedance approach and the plasma model. In the former the transverse momentum dependence in the surface impedance is simply disregarded (the so-called Leontovich approach), while in the second alternative the plasma dispersion relation with no relaxation is used for the dielectric constant. Neither of these omissions is in accordance with the properties of real materials, as we will detail in the following.

We start by remarking that the Lifshitz formula for the force between two parallel dielectric slabs is essentially geometrical, being determined entirely by multiple reflections at the interfaces between the parallel media (for example, see Sec. 3 of Ref. [7]). Now there is no doubt that one can use surface impedances to describe the reflection coefficients appearing in the theory, for these are merely the linear relations between transverse electric and magnetic fields at the surface of the metal. It is well known that use of either bulk permittivity or surface impedance are equivalent in describing the reflectivity properties, *provided that one incorporates transverse momentum,* $k_\perp$*.* The boundary conditions implied by Maxwell's equations provide the precise connection between these alternative descriptions [8]. In principle both bulk permittivity and surface impedance should depend on $k_\perp$, although theory and optical data suggest that the dependence of permittivity on $k_\perp$ is rather small. This would then imply a substantial dependence

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in the surface impedance. Indeed, in Ref. [9] we show that the exact impedance also leads to zero contribution of the TE zero mode.

The authors of Ref. [1, 3] ignore the transverse momentum dependence at zero frequency, because they believe that the "mass shell" condition $(\omega/c)^2 - k_\perp > 0$ must be satisfied. However, this relation can only be valid for real photons. In computing the temperature dependence, one is evaluating Green's functions periodic in imaginary time, with period $\beta = 1/kT$ [10]. The Matsubara frequencies, $\zeta_m = 2\pi m/\beta$, being imaginary, evidently break the mass-shell condition. Physically, evanescent waves or virtual photons are responsible for the thermal Green's functions. The inclusion of spatial dispersion, as required by Maxwell's equations, has been considered in detail in recent publications [11, 12], and the effects of the exclusion of the TE zero mode at both high and low temperatures are manifested.

The authors of Ref. [1, 3] also state that the plasma model is equally good at describing the optical properties of real metals. The plasma model is a special case of the Drude model, with the relaxation parameter neglected. In fact, optical data clearly show that this limit is inadequate: The Drude model describes the permittivity of real metals up to $\zeta \approx 2 \times 10^{15}$ rad/s, while the plasma model fails for frequencies below 5×10^{13} rad/s. It is precisely the low-frequency part of the spectrum that is relevant for the discussion of the disputed zero-mode contribution. Those authors argue that the neglect of relaxation is appropriate because the zero temperature limit of the relaxation parameter for an ideal metal (the Bloch-Grüneisen law) is zero. To this we reply that the relaxation parameter for real materials cannot vanish at zero temperature because of the scattering by impurities, and further that all present and proposed experiments are carried out at room temperature, so the temperature dependence of this parameter should be irrelevant. Moreover, it has recently been demonstrated [11] that at sufficiently low temperature the residual value of the relaxation parameter does not play a role, as the frequency characteristic of the anomalous skin effect becomes dominant. In contrast to the ad hoc procedure advocated in Ref. [1, 3], which in addition to using physically inadequate models, employ an unjustified extrapolation from the infrared region (that is, the plasma model, which disregards relaxation, is extrapolated from that region down to zero frequency), we use real optical data at room temperature for the bulk permittivity. Elsewhere it has been emphasized that a dielectric function valid over a wide frequency range and not merely the infrared must be employed, and indeed that frequencies very small compared to the characteristic frequency play a dominant role in the temperature dependence [11]. Of course, there is no data at zero frequency, so we must impose physical requirements (thermodynamics and Maxwell's equations) to exclude the TE zero mode.

As far as we can understand there is no disagreement about the validity of the Lifshitz formula for the Casimir force as long as the dielectric constant is finite. For this latter situation there is no $m = 0$ or zero frequency TE (transverse electric) mode, in agreement with Maxwell's equations of electrodynamics, and inclusion of such a term would violate the third law of thermodynamics [11].

The controversy arises in the limit of a perfect metal. It is well known that in this limit the mathematics of the limiting process can be ambiguous due to its singular nature. So the limit of a perfect metal can in one way be taken by letting $\varepsilon \to \infty$ when regarding the contribution from the $m = 0$ term. Since this term for all finite ε is zero, its limiting value is also zero. However, the other way is to take the limit $m \to 0$ while $\varepsilon = \infty$ [5]. In the latter case a non-zero contribution arises. The opinion of Bezerra et al. [1] is that the latter procedure is the proper one while we have the opposite opinion.

So the controversy left is for perfect metals of infinite extension. One of our arguments is that this special case is a limiting case of more realistic models with relaxation. As mentioned earlier, this latter limit can come in conflict with the third law of thermodynamics. But even in this limit the possible violation is ambiguous or not obvious. On one hand the entropy remains zero for temperature $T = 0$ in the limit $\varepsilon \to \infty$. On the other hand taking the limit $T \to 0$ with $\varepsilon = \infty$ will result in entropy different from zero. The latter is connected to the increasing and diverging slope of the entropy function as $\varepsilon \to \infty$ close to $T = 0$.

However, for realistic models for metals with relaxation (or finite conductivity) this controversy is not present. As we have found earlier the entropy then goes smoothly towards zero as $T \to 0$. Bezerra et al. [1] correctly point to the fact that the so-called MIM (modified ideal metal) violates the third law of thermodynamics. (This we stated explicitly in Ref. [13].) This violation is then in the sense discussed above, i.e., it is somewhat ambiguous depending upon how the limiting process is performed. It is also correct that according to our approach real metals approach MIM for increasing separation as then lower frequencies become more important. (MIM is the limit where the dielectric constant $\varepsilon \to \infty$ with the TE zero-mode equal to zero.) But the crucial difference between real metals and MI.M is that the former includes relaxation by which there will be no violation of the third law of thermodynamics (at least for finite separation of metal plates and for infinite separation where there is no force or interaction energy left). Moreover, numerical studies have shown that the MIM model disagrees strongly at short distances with values for the Casimir force calculated for real metals even for zero temperature.

Bezerra et al. [1] further claim that our unstated assumption is that perfect crystals with no defects or impurities do not or cannot exist, and, therefore, that thermodynamics can be violated for them. At best this is a misinterpretation of our arguments. There is no violation of the third law in this connection as quantization of lattice vibrations on a perfect lattice yields no problem; in fact it can be performed exactly. A reason for this is that the standard lattice has a finite dielectric constant and there is no ambiguity. However, if the perfect lattice turns into a perfect metal (with no relaxation) we can again see the problem discussed above. Then there will be no well defined thermal equilibrium as any steady current can stay on forever. And we do not think that Nernst had this latter very special and unrealizable situation in mind when he formulated his theorem.

Thus, we believe that the results obtained by excluding the TE zero mode are consistent with the third law of thermodynamics. The entropy at zero temperature vanishes, except in the limit of a perfect conductor of infinite extent, for which thermal equilibrium can never occur. There is a region in which the Casimir entropy is negative, but this is a rather familiar phenomenon, reflecting the fact that we are describing only part of the complete physical system. (Although Svetovoy and Esquivel [11] and Sernelius [12] agree with our conclusion that the entropy vanishes at zero temperature for real metals with no TE zero mode, and thus there is no contradiction with the Nernst theorem, the former authors believe that there is a thermodynamic difficulty with the negative entropy region.)

Two other recent papers also lend support to our point of view. Jancovici and Šamaj [14] and Buenzli and Martin [15] have examined the Casimir force between ideal-conductor walls with emphasis on the high-temperature limit. Not surprisingly, ideal inert boundary conditions are shown to be inadequate, and fluctuations within the walls, modeled by the classical Debye-Hückel theory, determine the high temperature behavior. The linear in temperature behavior of the Casimir force is found to be reduced by a factor of two from the behavior predicted by an ideal metal. This is precisely the signal of the omission of the $m = 0$ TE mode. Thus, it is very hard to see how the corresponding modification of the low-temperature behavior can be avoided.

Thus, we claim that there is now overwhelming theoretical evidence that the TE zero-mode must be omitted in the Lifshitz formula describing the interaction between real metal surfaces. This makes the purported exclusion of this theory by recent experiments, as forcefully stated in Refs. [1, 3], most difficult to understand. We believe that the resolution of this conundrum lies in an insufficient appreciation of the backgrounds making Casimir measurements so difficult. We and others [16] have mentioned the difficulty in accurately determining the absolute sphere-plate separation. This may be especially so since the roughness of the surfaces is much larger than the precision stated in the determination of the separation. Nonlocality in the electric properties of the surfaces, as for example seen in the calculation of Ref. [14], means that the location of the surface cannot be specified to better than some effective shielding length. Interferometric methods of determining distance may also be somewhat uncertain because of penetration of the surface by light. Also accurate determination of a small difference between experimental values at room temperature and purely theoretical values at $T = 0$ gives rise to further difficulties. In short, fitting data precisely to a preferred theory wherein various unknown experimental parameters must be determined does not constitute decisive evidence in favor of that theory; nor does the poorness of fit to an alternative theory when only one of many parameters is allowed to vary constitute decisive evidence against that theory.

In our view, the issue of temperature dependence cannot be settled until experiments are able to detect a variation of the Casimir force with temperature. Clearly, such experiments are difficult. The precise experiments at very short distances (~ 100 nm), where the Casimir forces are largest, are not the best place to look for temperature variation, for the temperature dependence is relatively small there. Rather, experiments should be conducted at the micrometer scale, where the effects, if our theory is correct, are at the 10% level. (The Lamoreaux experiment [17], conducted at that separation scale, was probably not that accurate.)

To this end, we present some new calculations of the force between parallel plates based on our theory, compared with that of Ref. [3]. The numerical computations are done on the basis of Eq. (4.18) in our paper [13], and follow the recent paper of Bentsen et al. [18]. The maximum value of the quantity $y = \sqrt{k_\perp^2 + \zeta_m^2}a$, a being the plate separation, is chosen to be $y_{\max} = 30$. In all calculations we impose the tolerance for the integrals to be 10^{-12}. The overall tolerance in the calculated sum over Matsubara numbers is taken to be 10^{-8}. This last tolerance determines the highest Matsubara frequencies occurring in the m sum. These values are shown in the tables.

As for permittivities, we are using the (new) data received from Astrid Lambrecht. These data extend from about 1.5×10^{11} rad/s to about 1.5×10^{18} rad/s. An important virtue of our tables is that they show which frequency domains we actually use. The largest frequency region naturally occurs when the temperature is low. Thus for $T = 1$ K, the temperature that we associate with $T = 0$ on physical grounds, we use the region from about 8×10^{11} rad/s to about 2×10^{16} rad/s. That means, we are, even at this low temperature, working with frequencies that lie entirely within the region of Lambrecht's data. This implies that we do not have to involve the Drude relation at all, for the finite frequencies. There is no analytical approximation involved in our formalism, for any finite frequency. The only exception is the zero frequency case. We then need the Drude relation to assure that there is no contribution to the force from the zero frequency TE mode. This zero frequency contribution to the Casimir force is found analytically, not numerically.

For comparison, we show in Table I the results given in Ref. [3] for the Leontovich impedance and plasma models. These results are for zero temperature. In their approach the temperature dependence is negligible. It will be noted that their two models do not agree, even though the authors seem to imply that either model is equally good. Our view

Separation nm	Impedance Method Ref. [3]	Plasma Method Ref. [3]
160	1144	1114.9
200	509.3	501.8
250	224.7	223.1
400	38.90	38.98
500	16.70	16.76
700	4.605	4.628

TABLE I: Casimir pressure between parallel plates in the two models discussed in Ref. [3]. Pressures are given in mPa.

Separation nm	Pressure mPa	Highest frequency rad/s	Number of terms in sum
160	1144	2.112×10^{16}	25674
200	508.2	1.713×10^{16}	20824
250	223.7	1.388×10^{16}	16869
400	38.61	8.875×10^{15}	10789
500	16.56	7.168×10^{15}	8714
700	4.556	5.187×10^{15}	6305
1000	1.143	3.674×10^{15}	4466

TABLE II: Our results for the Casimir pressure between gold plates, when $T = 1$ K. The first nonvanishing Matsubara frequency (corresponding to $m = 1$) is 8.226×10^{11} rad/s. The highest frequencies are dependent on the plate separation a, as shown, and are determined by our chosen tolerance 10^{-8} for the sum. The last column gives the number of terms in the sum.

is that the difference between these two models may be taken as a rough gauge of the accuracy of their predictions. Our results, with details about the frequency range used and the number of terms in the Matsubara sum, are given in Tables II–IV for $T = 1$ K (sufficiently close to $T = 0$ K, but a temperature at which our numerical technique is stable), $T = 300$ K, and $T = 350$ K. The latter is given in the hope that the temperature variation over a 50 K range near room temperature may soon become accessible to experiment.

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Separation nm	Pressure mPa	Highest frequency rad/s	Number of terms in sum
160	1127	2.122×10^{16}	86
200	497.8	1.727×10^{16}	70
250	217.6	1.407×10^{16}	57
400	36.70	8.884×10^{15}	36
500	15.49	7.403×10^{15}	30
700	4.127	5.429×10^{15}	22
1000	0.9852	3.702×10^{15}	15

TABLE III: Same as in Table II, but at $T = 300$ K. The first nonvanishing Matsubara frequency is 2.468×10^{14} rad/s. The highest frequencies are determined by the same tolerance 10^{-8} for the m sum as before.

Separation nm	Pressure mPa	Highest frequency rad/s	Number of terms in sum
160	1124	2.131×10^{16}	74
200	495.7	1.727×10^{16}	60
250	216.4	1.411×10^{16}	49
400	36.35	8.925×10^{15}	31
500	15.30	7.198×10^{15}	25
700	4.052	5.470×10^{15}	19
1000	0.9590	3.743×10^{15}	13

TABLE IV: Same as in Table III, but at $T = 350$ K. The first nonvanishing Matsubara frequency is 2.879×10^{14} rad/s. The highest frequencies are determined by the same tolerance 10^{-8} for the m sum as before.

Department of Energy.

[1] V. B. Bezerra, R. S. Decca, E. Fischbach, B. Geyer, G. L. Klimchitskaya, D. E. Krause, D. López, V. M. Mostepanenko, and C. Romero, arXiv:quant-ph/0503134, to be published in Phys. Rev. E.

[2] I. Brevik, J. B. Aarseth, J. S. Høye and K. A. Milton, Phys. Rev. E **71**, 056101 (2005), arXiv:quant-ph/0410231.

[3] R. S. Decca, D. López, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko, quant-ph/0503105v1, Ann. Phys. (N.Y.) **318**, 37 (2005).

[4] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **29**, 94 (1956) [Soviet Phys. JETP **2**, 73 (1956)]; I. D. Dzyaloshinskii, E. M. Lifshitz, and L. P. Pitaevskii, Usp. Fiz. Nauk **73**, 381 (1961) [Soviet Phys. Usp. **4**, 153 (1961)]; E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics, Part 2* (Pergamon Press, Oxford, 1980), Sec. 81.

[5] J. Schwinger, L. L. DeRaad, Jr., and K. A. Milton, Ann. Phys. (N.Y.) **115**, 1 (1978).

[6] M. Boström and B. E. Sernelius, Phys. Rev. Lett. **84**, 4757 (2000).

[7] K. A. Milton, J. Phys. A **37**, R209 (2004) [arXiv:hep-th/0406024].

[8] J. Schwinger, L. L. DeRaad, Jr., K. A. Milton, and W.-y. Tsai, *Classical Electrodynamics* (Perseus/Westview, New York, 1998), Chap. 41.

[9] I. Brevik, J. B. Aarseth, J. S. Høye and K. A. Milton, in K. A. Milton, ed., *Proceedings of the 6th Workshop on Quantum Field Theory under the Influence of External Conditions (QFEXT03), Norman, Oklahoma, 15–19 Sep 2003* (Rinton Press, Princeton, 2004), arXiv:quant-ph/0311094.

[10] P. C. Martin and J. Schwinger, Phys. Rev. **115** (1959) 1342.

[11] V. B. Svetovoy and R. Esquivel, Phys. Rev. E **72**, 036113 (2005), quant-ph/0508068.

[12] Bo E. Sernelius, Phys. Rev. B **71**, 235114 (2005).

[13] J. S. Høye, I. Brevik, J. B. Aarseth and K. A. Milton, Phys. Rev. E **67**, 056116 (2003) [arXiv:quant-ph/0212125].

[14] B. Jancovici and L. Šamaj, Europhys. Lett. **72**, 35 (2005), arXiv:cond-mat/0506363.

[15] P. R. Buenzli and Ph. D. Martin, Europhys. Lett. **72**, 42 (2005).

[16] D. Iannuzzi, I. Gelfand, M. Lisanti, and F. Capasso, in K. A. Milton, ed., *Proceedings of the 6th Workshop on Quantum Field Theory under the Influence of External Conditions (QFEXT03), Norman, Oklahoma, 15–19 Sep 2003* (Rinton Press, Princeton, 2004).

[17] S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997).

[18] V. S. Bentsen, R. Herikstad, S. Skriudalen, I. Brevik and J. S. Høye, arXiv:quant-ph/0505136, to appear in J. Phys. A.