THEORETICAL AND EXPERIMENTAL STATUS
OF MAGNETIC MONOPOLES

KIMBALL A. MILTON,* GEORGE R. KALBFLEISCH, and WEI LUO

Department of Physics and Astronomy, University of Oklahoma
Norman, OK 73019-0225, USA

LEONARD GAMBERG

Department of Physics and Astronomy, University of Pennsylvania
Philadelphia, PA 19104-6396, USA

The Tevatron has inspired new interest in the subject of magnetic monopoles. First there was the 1998 D0 limit on the virtual production of monopoles, based on the theory of Ginzberg and collaborators. In 2000 the first results from an experiment (Fermilab E882) searching for real magnetically charged particles bound to elements from the CDF and D0 detectors were reported. This also required new developments in theory. The status of the experimental limits on monopole masses will be discussed, as well as the limitation of the theory of magnetic charge at present.

1. Maxwell's Equations

The most obvious virtue of introducing magnetic charge is the symmetry thereby imparted to Maxwell's equations,

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 4\pi\rho_e, \quad \boldsymbol{\nabla} \cdot \mathbf{B} = 4\pi\rho_m, \tag{1a}$$

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} + \frac{4\pi}{c}\mathbf{j}_e, \quad -\boldsymbol{\nabla} \times \mathbf{E} = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B} + \frac{4\pi}{c}\mathbf{j}_m. \tag{1b}$$

These equations are invariant under a global *duality* transformation. If $\mathcal{E}$ denotes any electric quantity, such as $\mathbf{E}$, ρ_e, or $\mathbf{j}_e$, while $\mathcal{M}$ denotes any magnetic quantity, such as $\mathbf{B}$, ρ_m, or $\mathbf{j}_m$, the dual Maxwell equations are invariant under

$$\mathcal{E} \to \mathcal{E}\cos\theta + \mathcal{M}\sin\theta, \quad \mathcal{M} \to \mathcal{M}\cos\theta - \mathcal{E}\sin\theta, \tag{2}$$

where θ is a constant.

J. J. Thomson[1] (1904) observed the remarkable fact that a *static* system of an electric (e) and a magnetic (g) charge separated by a distance $\mathbf{R}$ possesses an

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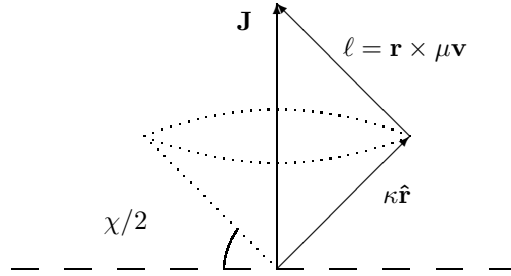*E-mail address: milton@mail.nhn.ou.edu

Fig. 1. The relative motion of two dyons is confined to the surface of a cone about the direction of the angular momentum.

angular momentum,

$$
\begin{aligned}
\mathbf{J} &= \int (d\mathbf{r}) \, \mathbf{r} \times \mathbf{G} = \int (d\mathbf{r}) \, \mathbf{r} \times \frac{\mathbf{E} \times \mathbf{B}}{4\pi c} \\
&= \frac{1}{4\pi c} \int (d\mathbf{r}) \, \mathbf{r} \times \left[\frac{e\mathbf{r}}{r^3} \times \frac{g(\mathbf{r} - \mathbf{R})}{(\mathbf{r} - \mathbf{R})^3} \right] = \frac{eg}{c} \hat{\mathbf{R}},
\end{aligned}
\tag{3}
$$

which follows from symmetry (the integral can only supply a numerical factor, which turns out to be 4π). The quantization of charge follows by applying semiclassical quantization of angular momentum:

$$
\mathbf{J} \cdot \hat{\mathbf{R}} = \frac{eg}{c} = n\frac{\hbar}{2}, \quad \text{or} \quad eg = \frac{n}{2}\hbar c, \quad n = 0, \pm 1, \pm 2, \ldots.
\tag{4}
$$

2. Classical Scattering

Actually, earlier in 1896, Poincaré[2] investigated the motion of an electron in the presence of a magnetic pole. Let's generalize to two dyons (a term coined by Schwinger in 1969) with charges e_1, g_1, and e_2, g_2, respectively. There are two charge combinations $q = e_1 e_2 + g_1 g_2$, $\kappa = -\frac{e_1 g_2 - e_2 g_1}{c}$. Then the classical equation of relative motion is (μ is the reduced mass and $\mathbf{v}$ is the relative velocity)

$$
\mu \frac{d^2}{dt^2} \mathbf{r} = q\frac{\mathbf{r}}{r^3} - \kappa \mathbf{v} \times \frac{\mathbf{r}}{r^3}.
\tag{5}
$$

The constants of the motion are the energy and the angular momentum,

$$
E = \frac{1}{2}\mu v^2 + \frac{q}{r}, \quad \mathbf{J} = \mathbf{r} \times \mu \mathbf{v} + \kappa \hat{\mathbf{r}}.
\tag{6}
$$

Note that Thomson's angular momentum is prefigured here.

Because $\mathbf{J} \cdot \hat{\mathbf{r}} = \kappa$, the motion is confined to a cone, as shown in Fig. 1. Here the angle of the cone is given by

$$
\cot \frac{\chi}{2} = \frac{l}{|\kappa|}, \quad l = \mu v_0 b,
\tag{7}
$$

where v_0 is the relative speed at infinity, and b is the impact parameter. The scattering angle θ is given by

$$\cos\frac{\theta}{2} = \cos\frac{\chi}{2}\left|\sin\left(\frac{\xi/2}{\cos\chi/2}\right)\right|, \quad \text{where} \tag{8a}$$

$$\frac{\xi}{2} = \begin{cases} \tan^{-1}\left(\frac{|\kappa|v_0}{q}\cot\frac{\chi}{2}\right), & q > 0, \\ \pi - \tan^{-1}\left(\frac{|\kappa|v_0}{|q|}\cot\frac{\chi}{2}\right), & q < 0. \end{cases} \tag{8b}$$

The impact parameter $b(\theta)$ is a multiple-valued function of θ. The differential cross section is therefore

$$\frac{d\sigma}{d\Omega} = \left|\frac{b\,db}{d(\cos\theta)}\right| = \sum_\chi \left(\frac{\kappa}{2\mu v_0}\right)^2 \frac{1}{\sin^4\frac{\chi}{2}}\left|\frac{\sin\chi\,d\chi}{\sin\theta\,d\theta}\right|. \tag{9}$$

Representative results are given in Ref. [3].

The cross section becomes infinite in two circumstances; first, when

$$\sin\theta = 0 \quad (\sin\chi \neq 0), \quad \theta = \pi, \tag{10}$$

we have what is called a *glory*. For monopole-electron scattering this occurs for

$$\frac{\chi_g}{2} = 1.047,\ 1.318,\ 1.403,\ \ldots. \tag{11}$$

The other case in which the cross section diverges is when

$$\frac{d\theta}{d\chi} = 0. \tag{12}$$

This is called a *rainbow*. For monopole-electron scattering this occurs at

$$\theta_r = 140.1°,\ 156.7°,\ 163.5°,\ \ldots. \tag{13}$$

For small scattering angles we have the generalization of the Rutherford formula

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2\mu v_0)^2}\left\{\left(\frac{e_1g_2 - e_2g_1}{c}\right)^2 + \left(\frac{e_1e_2 + g_1g_2}{v_0}\right)^2\right\}\frac{1}{(\theta/2)^4}, \quad \theta \ll 1. \tag{14}$$

3. Quantum Theory

Dirac[4] showed in 1931 that quantum mechanics was consistent with the existence of magnetic monopoles provided the quantization condition (4) holds, which explains the quantization of electric charge. This was generalized by Schwinger to dyons:

$$e_1g_2 - e_2g_1 = \frac{n}{2}\hbar c. \tag{15}$$

(Schwinger sometimes argued that n was an even integer, or even 4 times an integer.)

One can see where this comes from by considering quantum mechanical scattering. To define the Hamiltonian, one must introduce a vector potential, which must be singular because $\mathbf{B} \neq \boldsymbol{\nabla} \times \mathbf{A}$. For example, a potential singular along the entire line $\hat{\mathbf{n}}$ is

$$\mathbf{A}(\mathbf{r}) = -\frac{g}{r}\frac{1}{2}\left(\frac{\hat{\mathbf{n}} \times \mathbf{r}}{r - \hat{\mathbf{n}} \cdot \mathbf{r}} - \frac{\hat{\mathbf{n}} \times \mathbf{r}}{r + \hat{\mathbf{n}} \cdot \mathbf{r}}\right) = -\frac{g}{r}\cot\theta\,\hat{\boldsymbol{\phi}} \quad \text{if} \quad \hat{\mathbf{n}} = \hat{\mathbf{z}}, \qquad (16)$$

which corresponds to the desired magnetic field from a magnetic monopole, $\mathbf{B}(\mathbf{r}) = g\frac{\mathbf{r}}{r^3}$. Invariance of the theory (wavefunctions must be single-valued) under string rotations implies the charge quantization condition. This is a nonperturbative statement.

Yang offered another approach, which is fundamentally equivalent.[5] He insisted that there be no singularities, but rather different potentials in different regions:

$$A_\phi^a = \frac{g}{r\sin\theta}(1 - \cos\theta) = \frac{g}{r}\tan\frac{\theta}{2}, \quad \theta < \pi, \qquad (17a)$$

$$A_\phi^b = -\frac{g}{r\sin\theta}(1 + \cos\theta) = -\frac{g}{r}\cot\frac{\theta}{2}, \quad \theta > 0. \qquad (17b)$$

These correspond to the same magnetic field, so must differ by a gradient:

$$A_\mu^a - A_\mu^b = \frac{2g}{r\sin\theta}\hat{\boldsymbol{\phi}} = \partial_\mu\lambda, \quad \lambda = 2g\phi. \qquad (18)$$

Requiring now that $e^{ie\lambda}$ be single valued leads to the quantization condition (4).

There is also a intrinsic spin formulation, pioneered by Goldhaber.[6] The energy (6) differs by a gauge transformation from

$$\mathcal{H} = \frac{1}{2\mu}\left(p_r^2 + \frac{J^2 - (\mathbf{J} \cdot \hat{\mathbf{r}})^2}{r^2}\right) + \frac{e_1 e_2 + g_1 g_2}{r}, \qquad (19)$$

where

$$\mathbf{J} = \mathbf{r} \times \mathbf{p} + \mathbf{S}, \quad \mu\mathbf{v} = \mathbf{p} + \frac{\mathbf{S} \times \mathbf{r}}{r^2}. \qquad (20)$$

The quantization condition appears as $\mathbf{S} \cdot \hat{\mathbf{r}} = m'$. This was elaborated long ago.[7]

The nonrelativistic Hamiltonian for a system of two interacting dyons is

$$\mathcal{H} = -\frac{\hbar^2}{2\mu}\left(\nabla^2 + \frac{2m'}{r^2}\frac{\cos\theta}{\sin^2\theta}\frac{1}{i}\frac{\partial}{\partial\phi} - \frac{m'^2}{r^2}\cot^2\theta\right) + \frac{q}{r}, \qquad (21)$$

where $m' = -(e_1 g_2 - e_2 g_1)/\hbar c$. The wavefunction separates: $\psi(\mathbf{r}) = R(r)\Theta(\theta)e^{im\phi}$, where

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + k^2 - \frac{2\mu}{\hbar^2}\frac{q}{r} - \frac{j(j+1) - m'^2}{r^2}\right)R = 0, \qquad (22a)$$

$$-\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right) - \frac{m^2 - 2mm'\cos\theta + m'^2}{\sin^2\theta}\right]\Theta = j(j+1)\Theta. \qquad (22b)$$

The solution to the θ equation is the rotation matrix element: $(x = \cos\theta)$

$$U_{m'm}^{(j)}(\theta) = \langle jm'|e^{iJ_2\theta/\hbar}|jm\rangle \propto (1-x)^{\frac{m'-m}{2}}(1+x)^{\frac{m'+m}{2}}P_{j-m}^{(m'-m,m'+m)}(x), \quad (23)$$

where $P_j^{(m,n)}$ are the Jacobi polynomials, or "monopole harmonics." This forces m' to be an integer. The radial solutions are confluent hypergeometric functions,

$$R_{kj}(r) = e^{-ikr}(kr)^L F(L+1-i\eta, 2L+2, 2ikr), \qquad (24a)$$

$$\eta = \frac{\mu q}{\hbar^2 k}, \quad k = \frac{\sqrt{2\mu E}}{\hbar}, \quad L+\frac{1}{2} = \sqrt{\left(j+\frac{1}{2}\right)^2 - m'^2}. \qquad (24b)$$

We solve the Schrödinger equation such that a distorted plane wave is incident,

$$\psi_{\text{in}} = \exp i\left[\mathbf{k}\cdot\mathbf{r} + \eta\ln(kr - \mathbf{k}\cdot\mathbf{r})\right]. \qquad (25)$$

Then the outgoing wave has the form

$$\psi_{\text{out}} \sim \frac{1}{r}e^{i(kr-\eta\ln 2kr)}e^{im'\bar{\phi}'}f(\theta), \qquad (26)$$

so up to an unobservable phase, the scattering amplitude is (here θ is the scattering angle)

$$2ikf(\theta) = \sum_{j=|m'|}^{\infty}(2j+1)U_{m'm'}^{(j)}(\pi-\theta)e^{-i(\pi L - 2\delta_L)}, \qquad (27)$$

in terms of the Coulomb phase shift, $\delta_L = \arg\Gamma(L+1+i\eta)$. Note that the integer quantization of m' results from the use of an infinite ("symmetric") string; an unsymmetric string allows $m' = \text{integer} + \frac{1}{2}$.

One can show that reorienting the string direction gives rise to an unobservable phase.[7] Note that this result is completely general: the incident wave makes an arbitrary angle with respect to the string direction. *Rotation of the string direction is a gauge transformation.*

Notice that small angle scattering is still given by the Rutherford formula:

$$\frac{d\sigma}{d\Omega} \approx \left(\frac{m'}{2k}\right)^2\frac{1}{\sin^4\theta/2}, \quad \theta \ll 1, \qquad (28)$$

for electron-monopole scattering. The classical result is good roughly up to the first classical rainbow. In general, one must proceed numerically. Various remarkable results are shown in Ref. [3].

We can also include the effect of a magnetic dipole moment interaction, by adding a term to the Hamiltonian,

$$-\frac{e\hbar}{2\mu c}\gamma\boldsymbol{\sigma}\cdot\mathbf{H}, \quad \mathbf{H} = g\frac{\mathbf{r}}{r^3}. \qquad (29)$$

Relativistic Cross Sections
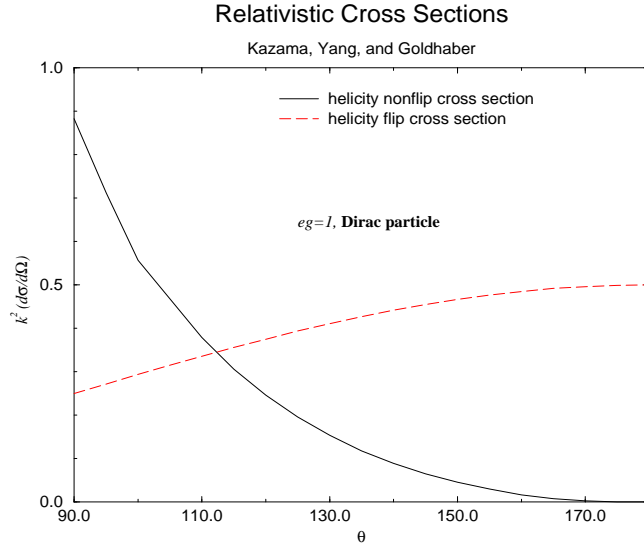
Kazama, Yang, and Goldhaber

Fig. 2. Relativistic scattering of a spin-1/2 electron by a heavy monopole.

For small scattering angles, the spin-flip and spin-nonflip cross sections are (for $\gamma = 1$, $\theta \ll 1$)

$$\left.\frac{d\sigma}{d\Omega}\right|_F \approx \left(\frac{m'}{2k}\right)^2 \frac{\sin^2\theta/2}{\sin^4\theta/2}, \quad \left.\frac{d\sigma}{d\Omega}\right|_{NF} \approx \left(\frac{m'}{2k}\right)^2 \frac{\cos^2\theta/2}{\sin^4\theta/2}. \tag{30}$$

Note that the spin-flip amplitude always vanishes in the backward direction; the spin-nonflip amplitude also vanishes there for conditions almost pertaining to an electron: for $m' > 0$, and $\gamma = 1$, $\frac{d\sigma}{d\Omega}(\pi) = 0$. Various results are shown in Ref. [3].

All of this work was done many years ago in Ref. [3], which just goes to show that "good work ages more slowly than its creators." There was of course much earlier work [8,9] A relativistic calculation of the scattering of a spin-1/2 Dirac particle by a heavy monopole was given by Kazama, Yang, and Goldhaber.[10] They also gave helicity-flip and helicity-nonflip cross sections which are shown in Fig. 2. (Note for $\theta = \pi$, helicity nonflip corresponds to spin flip, and vice versa.)

4. Quantum Field Theory

The quantum field theory of magnetic charge has been developed by many people, notably Schwinger[11] and Zwanziger.[12] A recent formulation suitable for eikonal calculations is given in Ref. [13]. Formal Lorentz invariance is demonstrated provided

the quantization condition holds (rationalized units):

$$\frac{e_a g_b - e_b g_a}{4\pi} = \left\{ \begin{array}{l} \frac{n}{2}\,,\text{unsymmetric} \\ n\,,\text{symmetric} \end{array} \right\}, \quad n \in Z. \tag{31}$$

("Symmetric" and "unsymmetric" refer to the presence or absence of dual symmetry in the solutions of Maxwell's equations.)

The electric and magnetic currents are the sources of the field strength and its dual:

$$\partial^\nu F_{\mu\nu} = j_\mu \quad \text{and} \quad \partial^\nu {}^*F_{\mu\nu} = {}^*j_\mu \,, \tag{32}$$

where

$$^*F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\sigma\tau}F^{\sigma\tau}, \tag{33}$$

which imply the dual conservation of electric and magnetic currents, j_μ and $^*j_\mu$, respectively,

$$\partial_\mu j^\mu = 0 \quad \text{and} \quad \partial_\mu {}^*j^\mu = 0\,. \tag{34}$$

The first-order form of the action describing the interaction of a spin-1/2 electron ψ and a spin-1/2 monopole χ is

$$W = \int (dx)\bigg\{ -\frac{1}{2}F^{\mu\nu}(x)\left(\partial_\mu A_\nu\,(x) - \partial_\nu A_\mu\,(x)\right) + \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$$

$$+ \, \bar\psi(x)\left(i\gamma\partial + e\gamma A(x) - m_\psi\right)\psi(x) + \bar\chi(x)\left(i\gamma\partial + g\gamma B(x) - m_\chi\right)\chi(x)\bigg\}, \tag{35}$$

where A_μ and $F_{\mu\nu}$ are independent field variables and

$$B_\mu(x) = -\int (dy)f^\nu\,(x - y)\,{}^*F_{\mu\nu}(y)\,, \tag{36}$$

Here $f_\mu(x)$ is the Dirac string function which satisfies the differential equation

$$\partial_\mu f^\mu(x) = \delta(x)\,, \tag{37}$$

a formal symmetric solution of which is given by

$$f^\mu(x) = n^\mu\,(n \cdot \partial)^{-1}\,\delta(x)\,, \quad f^\mu(x) = -f^\mu(-x), \tag{38}$$

where n^μ is an arbitrary vector. A corresponding dual form in terms of independent variables B_μ and $^*F_{\mu\nu}$ can be immediately written down. Although from these actions a complete path integral version of dual QED can be given,[13] all that is needed for our purposes here is the relativistic interaction between spinor electric and magnetic currents $j_\mu = e\bar\psi\gamma_\mu\psi$ and $^*j_\mu = g\bar\chi\gamma_\mu\chi$:

$$W(j, {}^*j) = \int (dx)(dx')(dx'')\,{}^*j^\mu(x)\epsilon_{\mu\nu\sigma\tau}\partial^\nu f^\sigma\,(x - x')\,D_+\,(x' - x'')\,j^\tau\,(x'')\,, \tag{39}$$

where the photon propagator is denoted by $D_+(x - x')$.

The modern path integral reformulation of the dual quantum electrodynamics of electric and magnetic charges were used in Ref. [13] to rederive and generalize the eikonal results of Urrutia[14] for high-energy, low momentum-transfer scattering between an electron and a monopole. A simplified version of the argument appears in Ref. [15]. In terms of the quantization condition (4), the scattering amplitude, for an arbitrary direction of the incident particle, turns out to be

$$I(\mathbf{q}) = -\frac{4\pi n}{q^2} e^{-2in\phi},\tag{40}$$

where $\mathbf{q}$ is the momentum transfer. Squaring this and putting in the kinematical factors we obtain Urrutia's result[14]

$$\frac{d\sigma}{dt} = \frac{(eg)^2}{4\pi} \frac{1}{t^2}, \quad t = q^2,\tag{41}$$

which is exactly the same as the nonrelativistic, small angle result found in Eq. (28). This calculation, however, points the way toward a proper relativistic treatment, and will be extended elsewhere to the crossed process, the production of monopole-antimonopole pairs through quark-antiquark annihilation.

5. Previous Searches for Magnetic Monopoles

In the context of "more unified" non-Abelian theories, classical composite monopole solutions were discovered. The mass of these monopoles would be of the order of the relevant gauge-symmetry breaking scale, which for grand unified theories is of order 10^{16} GeV or higher. But there are models where the electroweak symmetry breaking can give rise to monopoles of mass ~ 10 TeV. Even the latter are not yet accessible to accelerator experiments, so limits on heavy monopoles depend either on cosmological considerations,[16] or detection of cosmologically produced (relic) monopoles impinging upon the earth or moon.[17] However, *a priori*, there is no reason that Dirac/Schwinger monopoles or dyons of arbitrary mass might not exist: In this respect, it is important to set limits below the 1 TeV scale.

At the University of Oklahoma we are carrying out a direct search for monopoles produced at the Tevatron, which we will describe in the next section. But *indirect* searches have been proposed and carried out as well. De Rújula[18] proposed looking at the three-photon decay of the Z boson, where the process proceeds through a virtual monopole loop. If we use his formula for the branching ratio for the $Z \to 3\gamma$ process, compared to the current experimental upper limit[19] for the branching ratio of 10^{-5}, we can rule out monopole masses lower than about 400 GeV, rather than the 600 GeV quoted by De Rújula. Similarly, Ginzburg and Panfil[20] and more recently Ginzburg and Schiller[21] considered the production of two photons with high transverse momenta by the collision of two photons produced either from e^+e^- or quark-(anti-)quark collisions. Again the final photons are produced through a virtual monopole loop. Based on this theoretical scheme, an experimental limit

has appeared by the D0 collaboration,[22] which sets the following bounds on the monopole mass M:

$$\frac{M}{n} > \begin{cases} 610 \text{ GeV} & \text{for } S = 0 \\ 870 \text{ GeV} & \text{for } S = 1/2 \\ 1580 \text{ GeV} & \text{for } S = 1 \end{cases} , \tag{42}$$

where S is the spin of the monopole. It is worth noting that a lower mass limit of 120 GeV for a Dirac monopole has been set by Graf, Schäfer, and Greiner,[23] based on the monopole contribution to the vacuum polarization correction to the muon anomalous magnetic moment. (Actually, we believe that the correct limit, obtained from the well-known textbook formula for the g-factor correction due to a massive Dirac particle is 60 GeV.)

In Ref. [15] we have criticized these limits on theoretical grounds. They are based on a naive application of duality, in which the quantization plays no role. Thus gauge invariance is not demonstrated. The Euler-Heisenberg Lagrangian is used outside its range of validity for hard photon processes. That the Euler-Heisenberg Lagrangian is not an effective Lagrangian in the sense of capturing radiative corrections is demonstrated by the disparate work of Refs. [24,25]. Moreover, the *substitution* $e \to g$, or

$$\alpha \to \alpha_g = \frac{137}{4} n^2, \quad n = 1, 2, 3, \ldots, \tag{43}$$

is made, which implies the manifest inconsistency of perturbation theory (which is already precluded by the nonperturbative quantization condition). The expansion parameter is α_g, which is huge. Instead of radiative corrections being of the order of α for the electron-loop process, these corrections will be of order α_g, which implies an uncontrollable sequence of corrections. For example, the internal radiative corrections to the four-photon box diagram have been computed by Ritus[26] and by Reuter, Schmidt, and Schubert[27] in QED. In the $O(\alpha^2)$ term in the expansion of the EH Lagrangian, the coefficients of the $(F^2)^2$ and the $(F\,{}^*F)^2$ terms are multiplied by $\left(1 + \frac{40}{9}\frac{\alpha}{\pi} + O(\alpha^2)\right)$ and $\left(1 + \frac{1315}{252}\frac{\alpha}{\pi} + O(\alpha^2)\right)$ respectively. These corrections become meaningless when we *replace* $\alpha \to \alpha_g$. Moreover, it is easy to see[15] that unitarity is violated by the formulas used in the D0 analysis unless the monopole masses are above 1 TeV, so the limits quoted are meaningless.

6. Oklahoma Experiment: Fermilab E882

The best prior experimental limit on the direct accelerator production of magnetic monopoles is that of Bertani et al.[28]: $\sigma \leq 2 \times 10^{-34} \text{cm}^2$ for $M \leq 850 \text{ GeV}$. The fundamental mechanism is supposed to be a Drell-Yan process, $p + \bar{p} \to m + \bar{m} + X$, where the cross section is given by

$$\frac{d\sigma}{dM} = (68.5n)^2 \beta^3 \frac{8\pi\alpha^2}{9s} \int \frac{dx_1}{x_1} \sum_i Q_i^2 q_i(x_1) \bar{q}_i \left(\frac{\mathcal{M}^2}{sx_1}\right). \tag{44}$$

Here $\mathcal{M}$ is the invariant mass of the monopole-antimonopole pair, and we have included a factor of β^3 to reflect (1) phase space and (2) the velocity suppression of

the magnetic coupling. Note that we are unable to calculate the elementary process $q\bar{q} \to \gamma^* \to m\bar{m}$ perturbatively, so we must use nonperturbative estimates.

Any monopole produced at Fermilab loses energy as it passes through the detector by ionization (computed using the energy loss formula of Ahlen[29]) and is trapped in the detector elements with 100% probability due to interaction with the magnetic moments of the nuclei. The experiment consists of running samples obtained from the old D0 and CDF detectors through a superconducting induction detector. A schematic of our induction detector is shown on the web at `http://www.nhn.ou.edu/%7Egrk/apparatus.pdf`. We are able to set much better limits than Bertani et al. because the integrated luminosity delivered to D0 is 10^4 larger than that of the previous 1990 experiment: $\int \mathcal{L} = 172 \pm 8\,\mathrm{pb}^{-1}$.

If $q = e_1 e_2 + g_1 g_2 < 0$, $\mathcal{H}_{\mathrm{NR}}$ in Eq. (21) gives binding of dyons

$$E_{nj} = -\frac{\mu}{2} q^2 \left[n + \frac{1}{2} + \left((j+1/2)^2 - m'^2 \right)^{1/2} \right]^{-2}. \tag{45}$$

Monopoles will not bind this way—a magnetic moment coupling as in Eq. (29) is required, in terms the gyromagnetic ratio $\gamma = 1 + \kappa = \frac{g}{2}$. ($\gamma = 1$ or $g = 2$ is the "normal" value.) The theory[15] is somewhat complicated and most inconclusive, as Table 1 shows.

Table 1. Weakly bound states of nuclei to a magnetic monopole. The angular momentum quantum number J of the lowest bound state is indicated. In Notes, NR means nonrelativistic and R relativistic calculations; hc indicates an additional hard core interaction is assumed, while FF signifies use of a form factor. IM means induced magnetization, an additional interaction employed for the relativistic spin-1 calculation. We use $n = 1$ except for the deuteron, where $n = 2$ is required for binding.

Nucleus	Spin	γ	J	E_b	Notes	Ref
n	$\frac{1}{2}$	-1.91	$\frac{1}{2}$	350 keV	NR,hc	30
$^{1}_{1}\mathrm{H}$	$\frac{1}{2}$	2.79	$l - \frac{1}{2} = 0$	15.1 keV	NR,hc	31
				320 keV	NR,hc	30
				50–1000 keV	NR,FF	32
				263 keV	R	33
$^{2}_{1}\mathrm{H}$	1	0.857	$l - 1 = 0\ (n=2)$	$\frac{130}{\lambda}$ keV	R,IM	34
$^{3}_{2}\mathrm{He}$	$\frac{1}{2}$	-2.13	$l + \frac{1}{2} = \frac{3}{2}$	13.4 keV	NR,hc	31
$^{27}_{13}\mathrm{Al}$	$\frac{5}{2}$	3.63	$l - \frac{5}{2} = 4$	2.6 MeV	NR,FF	32
$^{27}_{13}\mathrm{Al}$	$\frac{5}{2}$	3.63	$l - \frac{5}{2} = 4$	560 keV	NR,hc	35
$^{113}_{48}\mathrm{Cd}$	$\frac{1}{2}$	-0.62	$l + \frac{1}{2} = \frac{49}{2}$	6.3 keV	NR,hc	31

Unfortunately, the simple theory says that Be (of which the beam pipe is made) will not bind to monopoles ($S = 3/2$, $\gamma < 0$), but the strength of the magnetic field in the vicinity of a monopole will mostly likely disrupt the nucleus, ensuring

binding. Al and Pb[a] are okay.

Estimates of binding energies are in the keV range and up. Simple estimations show that an energy of an eV is sufficient to bind a monopole to a nucleus with a 10 year lifetime, and then the monopole-atom complex will remain permanently bound to the material lattice. (Evidently, it would not do to melt the material down.)

A characteristic signal would be produced in a superconducting loop contained within a superconducting can by a magnetic monopole of strength g pulled through it. Note that if the shield were not present, the supercurrent induced in the loop of inductance L and radius R would be given by

$$I(t) = \frac{2\pi g}{Lc} \left(1 - \frac{z(t)}{\sqrt{R^2 + z(t)^2}} \right), \tag{46}$$

where $z(t)$ is the vertical position of the monopole relative to the position of the center of the loop. The theory including the shield correction can be verified with a pseudopole.

Background effects are enormous. All nonmagnetic but conducting samples possess:

- Permanent magnetic dipole moments (in the absence of boundaries):

$$I(t) = -\frac{2\pi\mu_z}{Lc} \frac{R^2}{[R^2 + z(t)^2]^{3/2}} \tag{47}$$

- Induced magnetization: (a is the radius of the superconducting cylinder)

$$I(t) = \frac{v}{c^3} \frac{1}{L} \int (d\mathbf{r}) r^2 \sigma(r) \frac{\partial B_z}{\partial z'}(z') \frac{1}{R} H\left(\frac{z'}{R}, \frac{a}{R}\right), \tag{48}$$

  where v is the velocity with which the sample is pulled through the detector, σ is the conductivity of the sample, and

$$H\left(\frac{z'}{R}, \frac{a}{R}\right) = \int_0^\infty dy\, y \cos y \frac{z'}{R} \left[K_1(y) - I_1(y) \frac{K_1(ya/R)}{I_1(ya/R)} \right] \tag{49a}$$

$$\to \frac{\pi}{2} \frac{R^3}{(R^2 + z'^2)^{3/2}}, \quad a/R \to \infty. \tag{49b}$$

Calibration and real data are shown in the Figures. The pseudopole data (Fig. 3) clearly shows that we could detect a Dirac pole. As one sees from Fig. 4 real samples have "large" dipole signals; what we are looking for is an asymptotic "step" indicating the presence of a magnetic charge. Steps seen are typically much smaller than that expected of a magnetic pole of Dirac strength. The histogram of steps is shown in Fig. 5. For $n = 1$ the 90% confidence upper limit is 4.2 signal events

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[a]22% of naturally occurring Pb is $^{207}_{82}$Pb, which has spin 1/2 and $\gamma = 0.582$, which is sufficient for binding in the lowest angular momentum state.[15] The other three stable isotopes have spin 0.
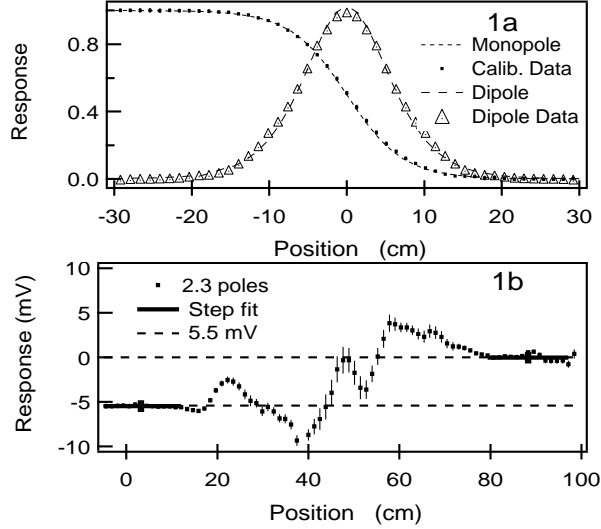
Fig. 3. "Pseudopole" curves. a) Comparison of theoretical monopole response to an experimental calibration and of a simple point dipole of one sample with that calculated from the theoretical response curve. b) The observed "step" for a pseudopole current, corresponding to 2.3 minimum Dirac poles, embedded in an Al sample.

Table 2. Acceptances, upper cross section limits, and lower mass limits, as determined in this work (at 90% CL).

Magnetic Charge	$|n| = 1$	$|n| = 2$	$|n| = 3$	$|n| = 6$
Sample	Al	Al	Be	Be
$\Delta\Omega/4\pi$ acceptance	0.12	0.12	0.95	0.95
Mass Acceptance	0.22	0.060	0.0018	0.11
Number of Poles	< 4.2	< 2.4	< 2.4	< 2.4
Cross section limit	0.84 pb	1.9 pb	8.4 pb	0.17 pb
Monopole Mass Limit	> 263 GeV	> 282 GeV	> 284 GeV	> 413 GeV

for 8 events observed when 10 were expected.[36] These 8 samples were remeasured and all fell within ± 1.47 mV of $n = 0$ (more than 1.28σ from $n = \pm 1$). For $n = 2$ the 90% confidence upper limit is 2.4 signal events for zero events observed and zero expected. Then, by putting in angular and mass acceptances we can get cross section limits as shown in Table 2. These numbers reflect a new analysis, and so differ somewhat from our published results.[37] To obtain the mass limits, we use the model cross sections referred to above.
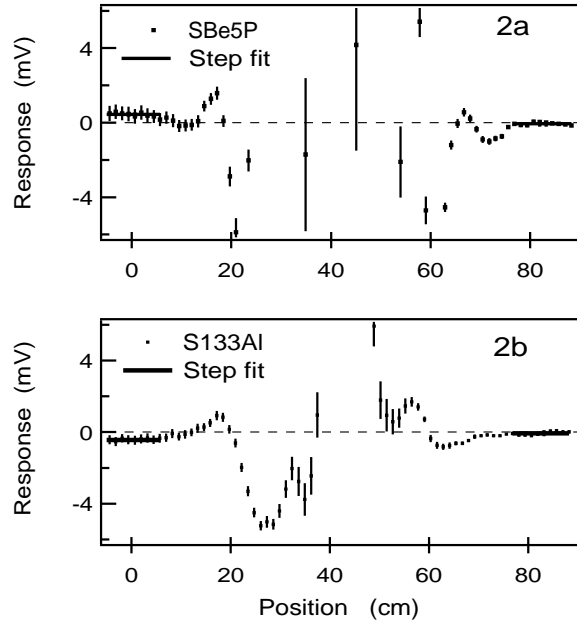
Acknowledgements

Fig. 4. Sample spectra. a) Beryllium sample "SBe5P," and b) aluminum sample "S133Al." The observed steps are −0.8 mV in a) and +0.4 mV in b). The dipole signals are off scale in the middle regions of the plot in this vertically expanded view.

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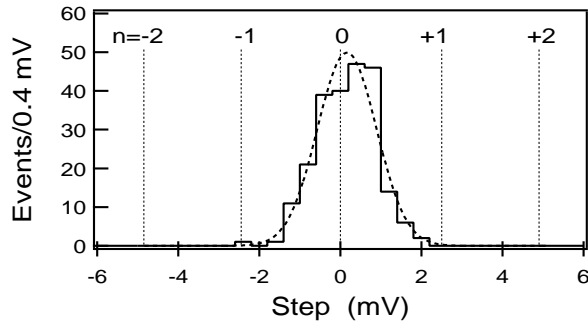
Fig. 5. Histogram of steps. Vertical lines (dashed) define the expected positions of signals for various n. The Gaussian curve (dashed) corresponds to 228 measurements having an average value of 0.16 mV and an rms sigma of 0.73 mV.

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