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Direct and Indirect Searches for Low-Mass Magnetic Monopoles*

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Abstract

Recently, there has been renewed interest in the search for low-mass magnetic monopoles. At the University of Oklahoma we are performing an experiment (Fermilab E882) using material from the old D0 and CDF detectors to set limits on the existence of Dirac monopoles of masses of the order of 500 GeV. To set such limits, estimates must be made of the production rate of such monopoles at the Tevatron collider, and of the binding strength of any such produced monopoles to matter. Here we sketch the still primitive theory of such interactions, and indicate why we believe a credible limit may still be obtained. On the other hand, there have been proposals that the classic Euler-Heisenberg Lagrangian together with duality could be employed to set limits on magnetic monopoles having masses less than 1 TeV, based on virtual, rather than real processes. The D0 collaboration at Fermilab has used such a proposal to set mass limits based on the nonobservation of pairs of photons each with high transverse momentum. We critique the underlying theory, by showing that the cross section violates unitarity at the quoted limits and is unstable with respect to radiative corrections. We therefore believe that no significant limit can be obtained from the current experiments, based on virtual monopole processes.

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I. INTRODUCTION

The notion of magnetic charge has intrigued physicists since Dirac [1] showed that it was consistent with quantum mechanics provided a suitable quantization condition was satisfied: For a monopole of magnetic charge g in the presence of an electric charge e, that quantization condition is (in this paper we use rationalized units)

$$\frac{eg}{4\pi} = \frac{N}{2}\hbar c, \tag{1}$$

where N is an integer. For a pair of dyons, that is, particles carrying both electric and magnetic charge, the quantization condition is replaced by [2,3]

$$\frac{e_1 g_2 - e_2 g_1}{4\pi} = \frac{N}{2}\hbar c, \tag{2}$$

where (e_1, g_1) and (e_2, g_2) are the charges of the two dyons.[1]

With the advent of "more unified" non-Abelian theories, classical composite monopole solutions were discovered [4]. The mass of these monopoles would be of the order of the relevant gauge-symmetry breaking scale, which for grand unified theories is of order 10^{16} GeV or higher. But there are models where the electroweak symmetry breaking can give rise to monopoles of mass ~ 10 TeV [5]. Even the latter are not yet accessible to accelerator experiments, so limits on heavy monopoles depend either on cosmological considerations [6], or detection of cosmologically produced (relic) monopoles impinging upon the earth or moon [7]. However, *a priori*, there is no reason that Dirac/Schwinger monopoles or dyons of arbitrary mass might not exist: In this respect, it is important to set limits below the 1 TeV scale.

Such an experiment is currently in progress at the University of Oklahoma [8], where we expect to be able to set limits on *direct* monopole production at Fermilab up to several hundred GeV. This will be a substantial improvement over previous limits [9]. But *indirect* searches have been proposed and carried out as well. De Rújula [10] proposed looking at the three-photon decay of the Z boson, where the process proceeds through a virtual monopole loop. If we use his formula [10] for the branching ratio for the $Z \to 3\gamma$ process, compared to the current experimental upper limit [11] for the branching ratio of 10^{-5}, we can rule out monopole masses lower than about 400 GeV, rather than the 600 GeV quoted in Ref. [10]. Similarly, Ginzburg and Panfil [12] and more recently Ginzburg and Schiller [13] considered the production of two photons with high transverse momenta by the collision of two photons produced either from e^+e^- or quark-(anti-)quark collisions. Again the final photons are produced through a virtual monopole loop. Based on this theoretical scheme, an experimental limit has appeared by the D0 collaboration [14], which sets the following bounds on the monopole mass M:

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[1] An additional factor of 2 appears on the right hand side of these conditions if a symmetrical solution is adopted—see Eq. (16) below.

$$\frac{M}{N} > \begin{cases} 610 \text{ GeV} & \text{for } S = 0 \\ 870 \text{ GeV} & \text{for } S = 1/2 \\ 1580 \text{ GeV} & \text{for } S = 1 \end{cases} , \tag{3}$$

where S is the spin of the monopole. It is worth noting that a mass limit of 120 GeV for a Dirac monopole has been set by Graf, Schäfer, and Greiner [15], based on the monopole contribution to the vacuum polarization correction to the muon anomalous magnetic moment. (Actually, we believe that the correct limit, obtained from the well-known textbook formula [16] for the g-factor correction due to a massive Dirac particle is 60 GeV.)

The purpose of the present paper is to, first, describe the key theoretical elements necessary for the establishment of a direct limit for production of monopoles at Fermilab: An estimate of the production cross section for monopole-antimonopole pairs at the collider must be made, and then an estimate of the binding probability of such produced monopoles with matter must be given, so that we can predict how many monopoles would be bound to the detector elements that are run through our induction detector. Such estimates were given in our proposal to Fermilab; the complete analysis will be given in the experimental papers to follow. Here the emphasis will be on the elementary processes involved.

A secondary purpose of this paper is to critique the theory of Refs. [10], [12], [13], and [15], and thereby demonstrate that experimental limits, such as that of Ref. [14], based on virtual processes are unreliable. We will show that the theory is based on a naive application of electromagnetic duality; the resulting cross section cannot be valid because unitarity is violated for monopole masses as low as the quoted limits, and the process is subject to enormous, uncontrollable radiative corrections. It is not correct, in any sense, as Refs. [13] and [14] state, that the effective expansion parameter is $g\omega/M$, where ω is some external photon energy; rather, the factors of ω/M emerge kinematically from the requirements of gauge invariance at the one-loop level. If, in fact, a correct calculation introduced such additional factors of ω/M, arising from the complicated coupling of magnetic charge to photons, we argue that no limit could be deduced for monopole masses from the current experiments. It may even be the case, based on preliminary field-theoretic calculations, that processes involving the production of real photons vanish.

II. EIKONAL APPROXIMATION FOR ELECTRON- (OR QUARK-) MONOPOLE SCATTERING

It is envisaged that if monopoles are sufficiently light, they would be produced by a Drell-Yan type of process occurring in $p\overline{p}$ collisions at the Tevatron. The difficulty is to make a believable estimate of the elementary process $q\overline{q} \to \gamma^* \to M\overline{M}$, where q stands for quark and M for magnetic monopole. It is not known how to calculate such a process using perturbation theory; indeed, perturbation theory is inapplicable to monopole processes because of the quantization condition (1). It is only because of that consistency condition that the Dirac string, for example, disappears from the result.

Only formally has it been shown that the quantum field theory of electric and magnetic charges is independent of the string orientation, or, more generally, is gauge and rotationally invariant [2,17]. It has not yet proved possible to develop generally consistent schemes for calculating processes involving real or virtual magnetically charged particles. Partly this is because a sufficiently general field theoretic formulation has not yet been given; this defect will be addressed elsewhere [18]. However, the nonrelativistic scattering of magnetically charged particles is well understood[2] [19,20]. Thus it should not be surprising that an eikonal approximation gives a string-independent result for electron-monopole scattering provided the condition (1) is satisfied. More than two decades ago Schwinger proposed [3] and Urrutia carried out such a calculation [21]. Indeed, this subject has had arrested development. Since this is the only successful field-theoretic calculation yet presented, it may be useful to review it here. (A more detailed discussion will be presented in Ref. [18].)

The interaction between electric (J^μ) and magnetic ($^*J^\mu$) currents is given by

$$W^{(eg)} = -\epsilon_{\mu\nu\sigma\tau} \int (dx)(dx')(dx'') J^\mu(x) \partial^\sigma D_+(x-x') f^\tau(x'-x'')^* J^\nu(x''). \tag{4}$$

Here D_+ is the usual photon propagator, and the arbitrary "string" function $f_\mu(x-x')$ satisfies

$$\partial_\mu f^\mu(x-x') = \delta(x-x'). \tag{5}$$

It turns out to be convenient for this calculation to choose a symmetrical string, which satisfies

$$f^\mu(x) = -f^\mu(-x). \tag{6}$$

In the following we choose a string lying along the straight line n^μ, in which case the function may be written as a Fourier transform

$$f_\mu(x) = \frac{n_\mu}{2i} \int \frac{(dk)}{(2\pi)^4} e^{ikx} \left(\frac{1}{n\cdot k - i\epsilon} + \frac{1}{n\cdot k + i\epsilon} \right). \tag{7}$$

In the high-energy, low-momentum-transfer regime, the scattering amplitude between electron and monopole is obtained from Eq. (4) by inserting the classical currents,

$$J^\mu(x) = e\int_{-\infty}^{\infty} d\lambda \, \frac{p_2^\mu}{m}\, \delta\left(x - \frac{p_2}{m}\lambda\right), \tag{8a}$$

$$^*J^\mu(x) = e\int_{-\infty}^{\infty} d\lambda' \, \frac{p_2'^\mu}{M}\, \delta\left(x + b - \frac{p_2'}{M}\lambda'\right), \tag{8b}$$

where m and M are the masses of the electron and monopole, respectively. Let us choose a coordinate system such that the incident momenta of the two particles have spatial components along the z-axis:

$$p_2 = (p,0,0,p), \quad p_2' = (p,0,0,-p), \tag{9}$$

[2]Contrary to the statement in the second reference in Ref. [13], the nonrelativistic calculation is exact, employs the quantization condition, and uses no "unjustified extra prescription."

and the impact parameter lies in the xy plane:

$$b = (0, \mathbf{b}, 0). \tag{10}$$

Apart from kinematical factors, the scattering amplitude is simply the transverse Fourier transform of the eikonal phase,

$$I(\mathbf{q}) = \int d^2b\, e^{-i\mathbf{b}\cdot\mathbf{q}} \left(e^{i\chi} - 1\right), \tag{11}$$

where χ is simply $W^{(eg)}$ with the classical currents substituted, and $\mathbf{q}$ is the momentum transfer.

First we calculate χ; it is immediately seen to be, if n^μ has no time component,

$$\chi = \frac{eg}{2} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{\hat{\mathbf{z}}\cdot(\hat{\mathbf{n}}\times\mathbf{k}_\perp)}{k_\perp^2 - i\epsilon} e^{i\mathbf{k}_\perp\cdot\mathbf{b}} \left(\frac{1}{\hat{\mathbf{n}}\cdot\mathbf{k}_\perp - i\epsilon} + \frac{1}{\hat{\mathbf{n}}\cdot\mathbf{k}_\perp + i\epsilon}\right), \tag{12}$$

where $\mathbf{k}_\perp$ is the component of the photon momentum perpendicular to the z axis. From this expression we see that the result is independent of the angle $\mathbf{n}$ makes with the z axis. We next use proper-time representations for the denominators in Eq. (12),

$$\frac{1}{k_\perp^2} = \int_0^\infty ds\, e^{-sk_\perp^2}, \tag{13a}$$

$$\frac{1}{\hat{\mathbf{n}}\cdot\mathbf{k}_\perp - i\epsilon} + \frac{1}{\hat{\mathbf{n}}\cdot\mathbf{k}_\perp + i\epsilon} = \frac{1}{i}\left[\int_0^\infty d\lambda - \int_{-\infty}^0 d\lambda\right] e^{i\lambda\hat{\mathbf{n}}\cdot\mathbf{k}_\perp} e^{-|\lambda|\epsilon}. \tag{13b}$$

We then complete the square in the exponential and perform the Gaussian integration to obtain

$$\chi = \frac{eg}{4\pi}\hat{\mathbf{z}}\cdot(\hat{\mathbf{n}}\times\mathbf{b}) \int d\lambda \frac{1}{(\lambda + \mathbf{b}\cdot\hat{\mathbf{n}})^2 + b^2 - (\mathbf{b}\cdot\hat{\mathbf{n}})^2}, \tag{14}$$

or

$$\chi = \frac{eg}{2\pi} \tan^{-1}\left(\frac{\hat{\mathbf{n}}\cdot\mathbf{b}}{\hat{\mathbf{z}}\cdot(\mathbf{b}\times\hat{\mathbf{n}})}\right). \tag{15}$$

Because $e^{i\chi}$ must be continuous when $\hat{\mathbf{n}}$ and $\mathbf{b}$ lie in the same direction, we must have the Schwinger quantization condition for an infinite string,

$$eg = 4\pi N, \tag{16}$$

where N is an integer.

To carry out the integration in Eq. (11), choose $\mathbf{b}$ to make an angle ψ with $\mathbf{q}$, and the projection of $\hat{\mathbf{n}}$ in the xy plane to make an angle ϕ with $\mathbf{q}$; then

$$\chi = \frac{eg}{2\pi}(\psi - \phi - \pi/2). \tag{17}$$

To avoid the appearance of a Bessel function, we first integrate over $b = |\mathbf{b}|$, and then over ψ:

$$
\begin{aligned}
I(\mathbf{q}) &= \int_0^{2\pi} d\psi \int_0^\infty b\,db\, e^{-ibq(\cos\psi - i\epsilon)} e^{2iN(\psi-\phi-\pi/2)} \\
&= \frac{4}{i}\frac{e^{-2iN(\phi+\pi/2)}}{q^2}\oint_C \frac{dz\, z^{2N-1}}{(z+1/z-i\epsilon)^2} \\
&= -\frac{4\pi N}{q^2} e^{-2iN\phi},
\end{aligned}
\tag{18}
$$

where C is a unit circle about the origin, and where again the quantization condition (16) has been used. Squaring this and putting in the kinematical factors we obtain Urrutia's result [21]

$$
\frac{d\sigma}{dt} = \frac{(eg)^2}{4\pi}\frac{1}{t^2}, \quad t = q^2, \tag{19}
$$

which is exactly the same as the nonrelativistic, small angle result found, for example, in Ref. [19]. This calculation, however, points the way toward a proper relativistic treatment, and will be extended to the crossed process, quark-antiquark production of monopole-antimonopole pairs elsewhere.

III. BINDING OF MONOPOLES TO MATTER

Once the monopoles are produced in a collision at the Tevatron, they travel through the detector, losing energy in a well-known manner (see, e.g., Ref. [22]), presumably ranging out, and eventually binding to matter in the detector (Be, Al, Pb, for example). The purpose of this section is to review the theory of the binding of magnetic charges to matter.

We consider the binding of a monopole of magnetic charge g to a nucleus of charge Ze, mass $\mathcal{M} = Am_p$, and magnetic moment

$$
\boldsymbol{\mu} = \frac{e}{m_p}\gamma\mathbf{S}, \tag{20}
$$

$\mathbf{S}$ being the spin of the nucleus. (We will assume here that the monopole mass $\gg \mathcal{M}$, which restriction could be easily removed.) Other notations for the magnetic moment are

$$
\gamma = 1 + \kappa = \frac{g_S}{2}. \tag{21}
$$

The charge quantization condition is given by Eq. (1). Because the nuclear charge is Ze, the relevant angular momentum quantum number is [recall N is the magnetic charge quantization number in Eq. (1)]

$$
l = \frac{NZ}{2}. \tag{22}
$$

We do not address the issue of dyons [23], which for the correct sign of the electric charge will always bind electrically to nuclei.

A. Nonrelativistic binding for $S = 1/2$

In this subsection we follow the early work of Malkus [24] and the more recent paper of Bracci and Fiorentini [25]. (There are also the results given in Ref. [26], but this reference seems to contain errors.)

The neutron ($Z = 0$) is a special case. Binding will occur in the lowest angular momentum state, $J = 1/2$, if

$$|\gamma| > \frac{3}{2N} \tag{23}$$

Since $\gamma_n = -1.91$, this condition is satisfied for all N.

In general, it is convenient to define a reduced gyromagnetic ratio,

$$\hat{\gamma} = \frac{A}{Z}\gamma, \quad \hat{\kappa} = \hat{\gamma} - 1. \tag{24}$$

This expresses the magnetic moment in terms of the mass and charge of the nucleus. Binding will occur in the special lowest angular momentum state $J = l - \frac{1}{2}$ if

$$\hat{\gamma} > 1 + \frac{1}{4l}. \tag{25}$$

Thus binding can occur here only if the anomalous magnetic moment $\hat{\kappa} > 1/4l$. The proton, with $\kappa = 1.79$, will bind.

Binding can occur in higher angular momentum states J if and only if

$$|\hat{\kappa}| > \kappa_c = \frac{1}{l}\left|J^2 + J - l^2\right|. \tag{26}$$

For example, for $J = l + \frac{1}{2}$, $\kappa_c = 2 + 3/4l$, and for $J = l + \frac{3}{2}$, $\kappa_c = 4 + 15/4l$. Thus ${}^3_2$He, which is spin 1/2, will bind in the first excited angular momentum state because $\hat{\kappa} = -4.2$.

Unfortunately, to calculate the binding energy, one must regulate the potential at $r = 0$. The results shown in Table 1 assume a hard core.

B. Nonrelativistic binding for general S

The reference here is [27]. The assumption made here is that $l \geq S$. (There are only 3 exceptions, apparently: ^{2}H, ^{8}Li, and ^{10}B.)

Binding in the lowest angular momentum state $J = l - S$ is given by the same criterion (25) as in spin 1/2. Binding in the next state, with $J = l - S + 1$ occurs if $\lambda_\pm > \frac{1}{4}$ where

$$\lambda_\pm = \left(S - \frac{1}{2}\right)\frac{\hat{\gamma}}{S}l - 2l - 1 \pm \sqrt{(1+l)^2 + (2S - 1 - l)\frac{\hat{\gamma}}{S}l + \frac{1}{4}l^2\left(\frac{\hat{\gamma}}{S}\right)^2}. \tag{27}$$

The previous result for $S = 1/2$ is recovered, of course. $S = 1$ is a special case: Then λ_- is always negative, while $\lambda_+ > \frac{1}{4}$ if $\hat{\gamma} > \gamma_c$, where

$$\gamma_c = \frac{3}{4l}\frac{(3 + 16l + 16l^2)}{9 + 4l}. \tag{28}$$

For higher spins, both $\lambda_\pm$ can exceed 1/4:

$$\lambda_+ > \frac{1}{4} \text{ for } \hat{\gamma} > \gamma_{c-} \tag{29}$$

$$\lambda_- > \frac{1}{4} \text{ for } \hat{\gamma} > \gamma_{c+} \tag{30}$$

where for $S = \frac{3}{2}$

$$(\gamma_c)_\mp = \frac{3}{4l}(6 + 4l \mp \sqrt{33 + 32l}). \tag{31}$$

For ${}^9_4$Be, for which $\hat{\gamma} = -2.66$, we cannot have binding because $3 > \gamma_{c-} > 1.557$, $3 < \gamma_{c+} < 8.943$, where the ranges come from considering different values of N from 1 to ∞. For $S = \frac{5}{2}$,

$$(\gamma_c)_\mp = \frac{36 + 28l \mp \sqrt{1161 + 1296l + 64l^2}}{12l}. \tag{32}$$

So ${}^{27}_{13}$Al will bind in either of these states, or the lowest angular momentum state, because $\hat{\gamma} = 7.56$, and $1.67 > \gamma_{c-} > 1.374$, $1.67 < \gamma_{c+} < 4.216$.

C. Relativistic spin-1/2

Kazama and Yang treated the Dirac equation [28]. See also [29] and [23].

In addition to the bound states found nonrelativistically, deeply bound states, with $E_{\text{binding}} = M$ are found. These states always exist for $J \geq l + 1/2$. For $J = l - 1/2$, these (relativistic) $E = 0$ bound states exist only if $\kappa > 0$. Thus (modulo the question of form factors) Kazama and Yang [28] expect that electrons can bind to monopoles. (We suspect that one must take the existence of these deeply bound states with a fair degree of skepticism. See also [30].)

As expected, for $J = l - 1/2$ we have weakly bound states only for $\kappa > 1/4l$, which is the same as the nonrelativistic condition (25), and for $J \geq l + 1/2$, only if $|\hat{\kappa}| > \kappa_c$, where κ_c is given in Eq. (26).

D. Relativistic spin-1

Olsen, Osland, and Wu considered this situation [31].

In this case, no bound states exist, unless an additional interaction is introduced (this is similar to what happens nonrelativistically, because of the bad behavior of the Hamiltonian at the origin). Bound states are found if an "induced magnetization" interaction (quadratic in the magnetic field) is introduced. Binding is then found for the lowest angular momentum state $J = l - 1$ again if $\hat{\kappa} > 1/4l$. For the higher angular momentum states, the situation is more complicated:

- for $J = l$: bound states require $l \geq 16$, and
- for $J \geq l + 1$: bound states require $J(J+1) - l^2 \geq 25$.

But these results are probably highly dependent on the form of the additional interaction. The binding energies found are inversely proportional to the strength λ of this extra interaction.

Nucleus	Spin	γ	$\hat{\gamma}$	J	E_b	Notes	Ref
n	$\frac{1}{2}$	-1.91		$\frac{1}{2}$	350 keV	NR,hc	[26]
${}^1_1$H	$\frac{1}{2}$	2.79	2.79	$l - \frac{1}{2} = 0$	15.1 keV	NR,hc	[25]
					320 keV	NR,hc	[26]
					50–1000 keV	NR,FF	[27]
					263 keV	R	[29]
${}^2_1$H	1	0.857	1.71	$l - 1 = 0$ $(N = 2)$	$\frac{130}{\lambda}$ keV	R,IM	[31]
${}^3_2$He	$\frac{1}{2}$	-2.13	-3.20	$l + \frac{1}{2} = \frac{3}{2}$	13.4 keV	NR,hc	[25]
${}^{27}_{13}$Al	$\frac{5}{2}$	3.63	7.56	$l - \frac{5}{2} = 4$	2.6 MeV	NR,FF	[27]
${}^{27}_{13}$Al	$\frac{5}{2}$	3.63	7.56	$l - \frac{5}{2} = 4$	560 keV	NR,hc	[33]
${}^{113}_{48}$Cd	$\frac{1}{2}$	-0.62	-1.46	$l + \frac{1}{2} = \frac{49}{2}$	6.3 keV	NR,hc	[25]

TABLE I. Weakly bound states of nuclei to a magnetic monopole. The angular momentum quantum number J of the lowest bound state is indicated. In Notes, NR means nonrelativistic and R relativistic calculations; hc indicates an additional hard core interaction is assumed, while FF signifies use of a form factor. IM=induced magnetization, the additional interaction employed for the relativistic spin-1 calculation. We use $N = 1$ except for the deuteron, where $N = 2$ is required for binding.

E. Remarks on binding

Clearly, this summary indicates that the theory of monopole binding to nuclear magnetic dipole moments is rather primitive. The angular momentum criteria for binding is straightforward; but in general (except for relativistic spin 1/2) additional interactions have to be inserted by hand to regulate the potential at $r = 0$. The results for binding energies clearly are very sensitive to the nature of that additional interaction. It cannot even be certain that binding occurs in the allowed states. In fact, however, it seems nearly certain that monopoles will bind to all nuclei, even, for example, Be, because the magnetic field in the vicinity of the monopole is so strong that the monopole will disrupt the nucleus and will bind to the nuclear, or even the subnuclear, constituents.

F. Binding of monopole-nucleus complex to material lattice

Now the question arises: Is the bound complex of nucleus and monopole rigidly attached to the crystalline lattice of the material? This is a simple tunneling situation. The decay rate is estimated by the WKB formula

$$\Gamma \sim \frac{1}{a} e^{-2\int_a^b \sqrt{2\mathcal{M}(V-E)}}, \tag{33}$$

where the potential is crudely

$$V = -\frac{\mu g}{4\pi r^2} - gBr, \tag{34}$$

$\mathcal{M}$ is the nuclear mass $\ll$ monopole mass, and the inner and outer turning points, a and b are the zeroes of $E - V$. Provided the following equality holds,

$$(-E)^3 \gg \frac{g^3 \mu B^2}{4\pi}, \tag{35}$$

which should be very well satisfied, since the right hand side equals $10^{-20} N^3$ MeV^3, we can write the decay rate as

$$\Gamma \sim N^{-1/2} 10^{23} \mathrm{s}^{-1} \exp\left[-\frac{8\sqrt{2}}{3 \cdot 137}\left(\frac{-E}{m_e}\right)^{3/2} \frac{B_0}{NB} A^{1/2} \left(\frac{m_p}{m_e}\right)^{1/2}\right], \tag{36}$$

where the characteristic field, defined by $eB_0 = m_e^2$, is 4×10^9 T. If we put in $B = 1.5$T, and $A = 27$, $-E = 2.6$MeV, appropriate for ${}^{27}_{13}$Al, we have for the exponent, for $N = 1$, -2×10^{11}, corresponding to a rather long time! To get a 10 yr lifetime, the binding energy would have to be only of the order of 1eV. Monopoles bound with kilovolt or more energies will stay around forever.

Then the issue is whether the entire Al atom can be extracted with the 1.5 T magnetic field present in CDF. The answer seems to be unequivocally NO. The point is that the atoms are rigidly bound in a lattice, with no nearby site into which they can jump. A major disruption of the lattice would be required to dislodge the atoms, which would probably require kilovolts of energy [32]. Some such disruption was made by the monopole when it came to rest and was bound in the material, but that disruption would be very unlikely to be in the direction of the accelerating magnetic field. Again, a simple Boltzmann argument shows that any effective binding slightly bigger than 1 eV will result in monopole trapping "forever." This argument applies equally well to binding of monopoles in ferromagnets. If monopoles bind strongly to nuclei there, they will not be extracted by 5 T fields, contrary to the arguments of Goto et al. [34] The corresponding limits on monopoles from ferromagnetic samples of Carrigan et al. [35] are suspect.

IV. DUALITY AND THE EULER-HEISENBERG LAGRANGIAN

Finally, let us consider the process contemplated in Refs. [13] and [14], that is

$$\begin{pmatrix} qq & \to & qq \\ \bar{q}q & \to & \bar{q}q \end{pmatrix} + \gamma\gamma, \quad \gamma\gamma \to \gamma\gamma, \tag{37}$$

where the photon scattering process is given by the one-loop light-by-light scattering graph shown in Fig. 1. If the particle in the loop is an ordinary electrically charged electron, this

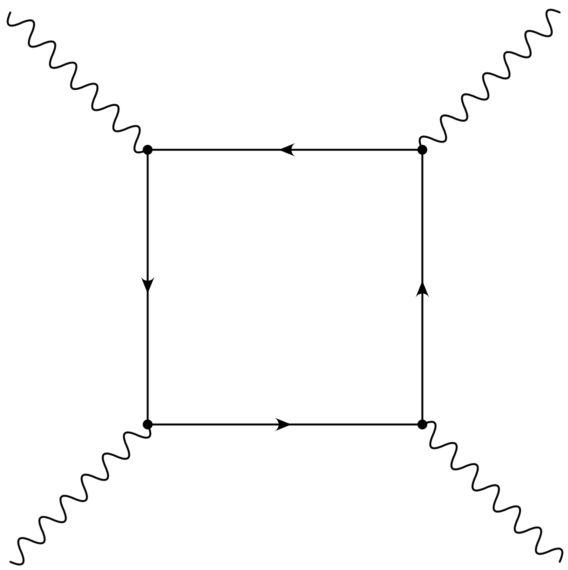

FIG. 1. The light-by-light scattering graph for either an electron or a monopole loop.

process is well-known [36,16,37]. If, further, the photons involved are of very low momentum compared the the mass of the electron, then the result may be simply derived from the well-known Euler-Heisenberg Lagrangian [38], which for a spin 1/2 charged-particle loop in the presence of homogeneous electric and magnetic fields is[3]

$$\mathcal{L} = -\mathcal{F} - \frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-m^2 s}\left[(es)^2\mathcal{G}\frac{\mathrm{Re}\cosh esX}{\mathrm{Im}\cosh esX} - 1 - \frac{2}{3}(es)^2\mathcal{F}\right]. \tag{38}$$

Here the invariant field strength combinations are

$$\mathcal{F} = \frac{1}{4}F^2 = \frac{1}{2}(\mathbf{H}^2 - \mathbf{E}^2), \quad \mathcal{G} = \frac{1}{4}F^*F = \mathbf{E}\cdot\mathbf{H}, \tag{39}$$

${}^*F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ being the dual field strength tensor, and the argument of the hyperbolic cosine in Eq. (38) is given in terms of

$$X = [2(\mathcal{F} + i\mathcal{G})]^{1/2} = [(\mathbf{H} + i\mathbf{E})^2]^{1/2}. \tag{40}$$

If we pick out those terms quadratic, quartic and sextic in the field strengths, we obtain[4]

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^2 + \frac{\alpha^2}{360}\frac{1}{m^4}[4(F^2)^2 + 7(F^*F)^2] \\ &- \frac{\pi\alpha^3}{630}\frac{1}{m^8}F^2[8(F^2)^2 + 13(F^*F)^2] + \dots .\end{aligned} \tag{41}$$

[3] We emphasize that Eq. (38) is only valid when $\partial_\alpha F_{\mu\nu} = 0$.

[4] Incidentally, note that the coefficient of the last term is 36 times larger than that given in Ref. [10].

The Lagrangian for a spin-0 and spin-1 charged particle in the loop is given by similar formulas which are derived in Ref. [36,16] and (implicitly) in Ref. [39], respectively.

Given this homogeneous-field effective Lagrangian, it is a simple matter to derive the cross section for the $\gamma\gamma \to \gamma\gamma$ process in the low energy limit. (These results can, of course, be directly calculated from the corresponding one-loop Feynman graph with on-mass-shell photons. See Refs. [16,37].) Explicit results for the differential cross section are given by Ref. [37]:

$$\frac{d\sigma}{d\Omega} = \frac{139}{32400\pi^2}\alpha^4\frac{\omega^6}{m^8}(3+\cos^2\theta)^2, \tag{42}$$

and the total cross section for a spin-1/2 charged particle in the loop is[5]

$$\sigma = \frac{973}{10125\pi}\alpha^4\frac{\omega^6}{m^8}. \tag{43}$$

Here, ω is the energy of the photon in the center of mass frame, $s = 4\omega^2$. This result is valid provided $\omega/m \ll 1$. The dependence on m and ω is evident from the Lagrangian (41), the ω dependence coming from the field strength tensor. Further note that perturbative quantum corrections are small, because they are of relative order $3\alpha \sim 10^{-2}$ [40]. Processes in which four final-state photons are produced, which may be easily calculated from the last displayed term in Eq. (41), are even smaller, being of relative order $\sim \alpha^2(\omega/m)^8$. So light-by-light scattering, which has been indirectly observed through its contribution to the anomalous magnetic moment of the electron [41], is completely under control for electron loops.

How is this applicable to photon scattering through a monopole loop? At first blush this calculation seems formidable. The interaction of a magnetically charged particle with a photon involves a "string," as described by the function f_μ given in Eq. (7). The interaction between electric and magnetic charges is given by the complicated expression (4). This coupling is equivalent to the interaction between the magnetic current ${}^*J^\mu$ and the electromagnetic field,

$$W_{\rm int} = \int (dx)(dx')^* F_{\mu\nu}(x')f^\nu(x'-x)^*J^\mu(x). \tag{44}$$

From Eqs. (4) and (7) one obtains the relevant string-dependent monopole-photon coupling vertex in momentum space,

$$\Gamma_\mu(q) = ig\frac{\epsilon_{\mu\nu\sigma\tau}n^\nu q^\sigma\gamma^\tau}{n\cdot q - i\epsilon}, \tag{45}$$

where we have, for variety's sake, chosen a semi-infinite string. As we have noted, the choice of the string is arbitrary; reorienting the string is a kind of gauge transformation. In fact, it is this requirement that leads to the quantization conditions (1) and (2).

[5] The numerical coefficient in the total cross section for a spin-0 and spin-1 charged particle in the loop is $119/20250\pi$ and $2751/250\pi$, respectively. Numerically the coefficients are 0.00187, 0.0306, and 3.50 for spin 0, spin 1/2, and spin 1, respectively.

The authors of Refs. [10], [12], and [13] do not attempt a calculation of the "box" diagram with the interaction (44). Rather, they (explicitly or implicitly) appeal to duality, that is, the symmetry that the introduction of magnetic charge brings to Maxwell's equations:

$$\mathbf{E} \to \mathbf{H}, \quad \mathbf{H} \to -\mathbf{E}, \tag{46}$$

and similarly for charges and currents. Thus the argument is that for low energy photon processes it suffices to compute the fermion loop graph in the presence of zero-energy photons, that is, in the presence of static, constant fields. The box diagram shown in Fig. 1 with a spin-1/2 monopole running around the loop in the presence of a homogeneous $\mathbf{E}, \mathbf{H}$ field is then obtained from that analogous process with an electron in the loop in the presence of a homogeneous $\mathbf{H}, -\mathbf{E}$ field, with the substitution $e \to g$. Since the Euler-Heisenberg Lagrangian (41) is invariant under the substitution (46) on the fields alone, this means we obtain the low energy cross section $\sigma_{\gamma\gamma\to\gamma\gamma}$ through the monopole loop from Eq. (43) by the substitution $e \to g$, or

$$\alpha \to \alpha_g = \frac{137}{4} N^2, \quad N = 1, 2, 3, \ldots. \tag{47}$$

A. Inconsistency of the Duality Approximation

It is critical to emphasize that the Euler-Heisenberg Lagrangian is an effective Lagrangian for calculations at the *one fermion loop level* for low energy, i.e., $\omega/M \ll 1$. It is commonly asserted that the Euler-Heisenberg Lagrangian is an *effective Lagrangian* in the sense used in chiral perturbation theory [42,43]. This is not true. The QED expansion generates derivative terms which do not arise in the effective Lagrangian expansion of the Euler-Heisenberg Lagrangian [40]. One can only say that the Euler-Heisenberg Lagrangian is a good approximation for light-by-light scattering (without monopoles) at low energy because radiative corrections are down by factors of α. However, it becomes unreliable if radiative corrections are large.[6]

In this regard, both the Ginzburg [12,13] and the De Rújula [10] articles, particularly Ref. [13], are rather misleading as to the validity of the approximation sketched in the previous section. They state that the expansion parameter is not g but $g\omega/M$, M being the monopole mass, so that the perturbation expansion may be valid for large g if ω is small enough. But this is an invalid argument. It is only when external photon lines are attached that extra factors of ω/M occur, due to the appearance of the field strength tensor in the Euler-Heisenberg Lagrangian. Moreover, the powers of g and ω/M are the same only for the F^4 process. The expansion parameter is α_g, which is huge. Instead of radiative corrections being of the order of α for the electron-loop process, these corrections will be of order α_g, which implies an uncontrollable sequence of corrections. For example, the internal radiative correction to the box diagram in Fig. 1 have been computed by Ritus [45] and by Reuter,

[6] The same has been noted in another context by Bordag et al. [44].

Schmidt, and Schubert [46] in QED. In the $O(\alpha^2)$ term in Eq. (41) the coefficients of the $(F^2)^2$ and the $(F\tilde{F})^2$ terms are multiplied by $\left(1+\frac{40}{9}\frac{\alpha}{\pi}+O(\alpha^2)\right)$ and $\left(1+\frac{1315}{252}\frac{\alpha}{\pi}+O(\alpha^2)\right)$, respectively. The corrections become meaningless when we *replace* $\alpha \to \alpha_g$.

This would seem to be a devastating objection to the results quoted in Ref. [13] and used in Ref. [14]. But even if one closes one's eyes to higher order effects, it seems clear that the mass limits quoted are inconsistent.

If we take the cross section given by Eq. (43) and make the substitution (47), we obtain for the low energy light-by-light scattering cross section in the presence of a monopole loop

$$\sigma_{\gamma\gamma\to\gamma\gamma} \approx \frac{973}{2592000\pi}\frac{N^8}{\alpha^4}\frac{\omega^6}{M^8} = 4.2\times 10^4\, N^8 \frac{1}{M^2}\left(\frac{\omega}{M}\right)^6. \tag{48}$$

If the cross section were dominated by a single partial wave of angular momentum J, the cross section would be bounded by

$$\sigma \le \frac{\pi(2J+1)}{s} \sim \frac{3\pi}{s}, \tag{49}$$

if we take $J = 1$ as a typical partial wave. Comparing this with the cross section given in Eq. (48), we obtain the following inequality for the cross section to be consistent with unitarity,

$$\frac{M}{\omega} \gtrsim 3N. \tag{50}$$

But the limits quoted [14] for the monopole mass are less than this:

$$\frac{M}{N} > 870 \text{ GeV}, \quad \text{spin } 1/2, \tag{51}$$

because, at best, a minimum $\langle\omega\rangle \sim 300$ GeV; the theory cannot sensibly be applied below a monopole mass of about 1 TeV. (Note that changing the value of J in the unitarity limits has very little effect on the bound (50) since an 8th root is taken: replacing J by 50 reduces the limit (50) only by 50%.)

Similar remarks can be directed toward the De Rújula limits [10]. That author, however, notes the "perilous use of a perturbative expansion in g." However, although he writes down the correct vertex, Eq. (45), he does not, in fact, use it, instead appealing to duality, and even so he admittedly omits enormous radiative corrections of $O(\alpha_g)$ without any justification other than what we believe is a specious reference to the use of effective Lagrangian techniques for these processes.

B. Proposed Remedies

Apparently, then, the formal small ω result obtained from the Euler-Heisenberg Lagrangian cannot be valid beyond a photon energy $\omega/M \gtrsim 0.1$. The reader might ask why one cannot use duality to convert the monopole coupling with an arbitrary photon to the ordinary vector coupling. The answer is that little is thereby gained, because the coupling of the photon to ordinary charged particles is then converted into a complicated form analogous to Eq. (44). This point is stated and then ignored in Ref. [10] in the calculation of $Z \to 3\gamma$. There is, in general, no way of avoiding the complication of including the string.

We are currently undertaking realistic calculations of virtual (monopole loop) and real (monopole production) magnetic monopole processes. These calculations are, as the reader may infer, somewhat difficult and involve subtle issues of principle involving the string, and it will be some time before we have results to present. Therefore, here we wish to offer plausible qualitative considerations, which we believe suggest bounds that call into question the results of Ginzburg et al. [12,13].

Our point is very simple. The interaction (44) couples the magnetic current to the dual field strength. This corresponds to the velocity suppression in the interaction of magnetic fields with electrically charged particles, or to the velocity suppression in the interaction of electric fields with magnetically charged particles, as most simply seen in the magnetic analog of the Lorentz force,

$$\mathbf{F} = g(\mathbf{B} - \frac{\mathbf{v}}{c} \times \mathbf{E}). \tag{52}$$

That is, the force between an electric charge e and magnetic charge g, moving with relative velocity $\mathbf{v}$ and with relative separation $\mathbf{r}$ is

$$\mathbf{F} = -\frac{eg}{c}\frac{\mathbf{v} \times \mathbf{r}}{4\pi r^3}. \tag{53}$$

This velocity suppression is reflected in nonrelativistic calculations. For example, the energy loss in matter of a magnetically charge particle is approximately obtained from that of a particle with charge Ze by the substitution [22]

$$\frac{Ze}{v} \to \frac{g}{c}. \tag{54}$$

And the classical nonrelativistic dyon-dyon scattering cross section near the forward direction is [19]

$$\frac{d\sigma}{d\Omega} \approx \frac{1}{(2\mu v)^2}\left[\left(\frac{e_1 g_2 - e_2 g_1}{4\pi c}\right)^2 + \left(\frac{e_1 e_2 + g_1 g_2}{4\pi v}\right)^2\right]\frac{1}{(\theta/2)^4}, \quad \theta \ll 1, \tag{55}$$

the expected generalization of the Rutherford scattering cross section at small angles.

Of course, the true structure of the magnetic interaction and the resulting scattering cross section is much more complicated. For example, classical electron-monopole or dyon-dyon scattering exhibits rainbows and glories, and the quantum scattering exhibits a complicated oscillatory behavior in the backward direction [19]. These reflect the complexities of the magnetic interaction between electrically and magnetically charged particles, which can be represented as a kind of angular momentum [20,47]. Nevertheless, for the purpose of extracting qualitative information, the naive substitution,

$$e \to \frac{v}{c} g, \tag{56}$$

seems a reasonable first step.[7] Indeed, such a substitution was used in the proposal [8] to estimate production rates of monopoles at Fermilab.

The situation is somewhat less clear for the virtual processes considered here. Nevertheless, the interaction (44) suggests there should, in general, be a softening of the vertex. In the current absence of a valid calculational scheme, we will merely suggest two plausible alternatives to the mere replacement procedure adopted in Refs. [10,12,13,15].

We first suggest, as seemingly Ref. [13] does, that the approximate effective vertex incorporates an additional factor of ω/M. Thus we propose the following estimate for the $\gamma\gamma$ cross section in place of Eq. (48),

$$\sigma_{\gamma\gamma\to\gamma\gamma} \sim 10^4 N^8 \frac{1}{M^2}\left(\frac{\omega}{M}\right)^{14}, \tag{58}$$

since there are four suppression factors in the amplitude. Now a considerably larger value of ω is consistent with unitarity,

$$\frac{M}{\omega} \gtrsim \sqrt{3N}, \tag{59}$$

if we take $J = 1$ again. We now must re-examine the $\sigma_{pp\to\gamma\gamma X}$ cross section.

In the model given in Ref. [13], where the photon energy distribution is given in terms of the functions $f(y)$, $y = \omega/E$, the physical cross section is given by

$$\sigma_{pp\to\gamma\gamma X} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{dy_1}{y_1}\frac{dy_2}{y_2} f(y_1) f(y_2) \sigma_{\gamma\gamma\to\gamma\gamma} = \int dy_1 dy_2 \frac{d\sigma}{dy_1\, dy_2}, \tag{60}$$

where now (*cf.* Eq. (25) of in the first reference in Ref. [13])

$$\frac{d\sigma}{dy_1\, dy_2} = \left(\frac{\alpha}{\pi}\right)^2 R E^6 \left(\frac{E}{M}\right)^8 y_1^6 f(y_1) y_2^6 f(y_2), \tag{61}$$

where, for spin 1/2, (up to factors of order unity)

$$R \sim \frac{10^{-4}}{\alpha^4}\left(\frac{N}{M}\right)^8. \tag{62}$$

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[7]This, and the extension of this idea to virtual processes, leaves aside the troublesome issue of radiative corrections. The hope is that an effective Lagrangian can be found by approximately integrating over the fermions which incorporates these effects. A first estimate of the effect of incorporating radiative correction, however, may be made by applying Padé summation of the leading corrections found in Refs. [45,46]:

$$\sigma \sim \frac{\sigma^{\rm PT}}{(1-\alpha_g)^2} \sim \frac{\sigma^{\rm PT}}{1000 N^4}, \tag{57}$$

which reduces the quoted mass limits of Ref. [14] a bit.

The result in (61) differs from that in Ref. [13] by a factor of $(E/M)^8 y_1^4 y_2^4$. The photon distribution function $y^2 f(y)$ used is rather strongly peaked at $y \sim 0.3$. (This peaking is necessary to have any chance of satisfying the low-frequency criterion.) When we multiply by y^4, the amplitude is greatly reduced and the peak is shifted above $y = 1/2$, violating even the naive criterion for the validity of perturbation theory. Nevertheless, the integral of the distribution function is reduced by two orders of magnitude, that is,

$$\frac{\int_0^1 dy\, y^6 f(y)}{\int_0^1 dy\, y^2 f(y)} \sim 10^{-2}. \tag{63}$$

This reduces the mass limit quoted in [14] by a factor of $1/\sqrt{3}$, to about 500 GeV, where $\langle\omega\rangle/M \approx 0.9$. This dubious result makes us conclude that it is impossible to derive any limit for the monopole mass from the present data.

As for the De Rújula limit[8] from the $Z \to 3\gamma$ process, if we insert a suppression factor of ω/M at each vertex and integrate over the final state photon distributions, given by Eq. (18) of Ref. [10], the mass limit is reduced to $M/\sqrt{N} \gtrsim 1.4 m_Z \sim 120$ GeV, again grossly violating the low energy criterion. And the limit deduced from the vacuum polarization correction to the anomalous magnetic moment of the muon due to virtual monopole pairs [15] is reduced to about 2 GeV.

The reader might object that this ω/M softening of the vertex has little field-theoretic basis. Therefore, we propose a second possibility that does have such a basis. The vertex (45) suggests, and detailed calculation supports (based on the tensor structure of the photon amplitudes[9]) the introduction of the string-dependent factor $\sqrt{q^2/(n\cdot q)^2}$ at each vertex, where q is the photon momentum. Such a factor is devastating to the indirect monopole searches—for any process involving a real photon, such as that of the D0 experiment [14] or for $Z \to 3\gamma$ discussed in [10], the amplitude vanishes. Because such factors can and do appear in full monopole calculations, it is clearly premature to claim any limits based on virtual processes involving real final-state photons.

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[8] We note that De Rújula also considers the monopole vacuum polarization correction to g_V/g_A, g_A, and m_W/m_Z, proportional to $(m_Z/M)^2$ in each case, once again ignoring both the string and the radiative correction problem. He assumes that the monopole is a heavy vector-like fermion, and obtains a limit of $M/N > 8m_Z$. Our ansatz changes $(m_Z/M)^2$ to $(m_Z/M)^4$, so that $M/\sqrt{N} > \sqrt{8} m_Z \approx 250$ GeV, a substantial reduction.

[9] For example, the naive monopole loop contribution to vacuum polarization differs from that of an electron loop (apart from charge and mass replacements) entirely by the replacement in the latter of $(g_{\mu\nu} - q_\mu q_\nu/q^2) \to (\mathbf{q}^2/q_0^2)(\delta_{ij} - q_i q_j/\mathbf{q}^2)$, when n^μ points in the time direction. Apart from this different tensor structure, the vacuum polarization is given by exactly the usual formula, found, for example in Ref. [16]. Details of this and related calculations will be given elsewhere.

V. CONCLUSIONS

The field theory of magnetic charge is still in a rather primitive state. Indeed, it has been in an arrested state of development for the past two decades. With serious limits now being announced based on laboratory measurements, it is crucial that the theory be raised to a useful level.

At the present time, we believe that theoretical estimates for the production of real monopoles are more reliable than are those for virtual processes. This is because, in effect, the former are dominated by tree-level processes. We have indicated why the indirect limits cannot be taken seriously at present; and of course only the real production processes offer the potential of discovery. Perhaps the arguments here will stimulate readers to contribute to the further development of the theory, for it remains an embarrassment that there is no well-defined quantum field theory of magnetic charge.

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