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Difficulties with Photonic Searches for Magnetic Monopoles

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Abstract

Recently, there have been proposals that the classic Euler-Heisenberg Lagrangian together with duality could be employed to set limits on magnetic monopoles having masses less than 1 TeV. The D0 collaboration at Fermilab has used such a proposal to set mass limits based on the nonobservation of pairs of photons each with high transverse momentum. In this note, we critique the underlying theory, by showing that at the quoted limits the cross section violates unitarity and is unstable with respect to radiative corrections. It is proposed that the correct coupling of magnetic monopoles to photons leads to an effective softening of the interaction, leading to a much smaller cross section, from which no significant limit can be obtained from the current experiments. Previous limits based on virtual monopole loops are similarly criticized.

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I. INTRODUCTION

The notion of magnetic charge has intrigued physicists since Dirac [1] showed that it was consistent with quantum mechanics provided a suitable quantization condition was satisfied: For a monopole of magnetic charge g in the presence of an electric charge e, that quantization condition is (in this paper we use rationalized units)

$$\frac{eg}{4\pi} = \frac{n}{2}\hbar c, \tag{1}$$

where n is an integer. For a pair of dyons, that is, particles carrying both electric and magnetic charge, the quantization condition is replaced by [2]

$$\frac{e_1 g_2 - e_2 g_1}{4\pi} = \frac{n}{2}\hbar c, \tag{2}$$

where (e_1, g_1) and (e_2, g_2) are the charges of the two dyons.

With the advent of "more unified" non-Abelian theories, classical composite monopole solutions were discovered [3]. The mass of these monopoles would be of the order of the relevant gauge-symmetry breaking scale, which for grand unified theories is of order 10^{16} GeV or higher. But there are models where the electroweak symmetry breaking can give rise to monopoles of mass ~ 10 TeV [4]. Even the latter are not yet accessible to accelerator experiments, so limits on heavy monopoles depend either on cosmological considerations [5], or detection of cosmologically produced (relic) monopoles impinging upon the earth or moon [6]. However, *a priori*, there is no reason that Dirac/Schwinger monopoles or dyons of arbitrary mass might not exist: It is important to set limits below the 1 TeV scale.

Such an experiment is currently in progress at the University of Oklahoma [7], where we expect to be able to set limits on *direct* monopole production at Fermilab up to several hundred GeV. This will be a substantial improvement over previous limits [8]. But *indirect* searches have been proposed and carried out as well. De Rújula [9] proposed looking at the three-photon decay of the Z boson, where the process proceeds through a virtual monopole loop. If we use his formula [9] for the branching ratio for the $Z \to 3\gamma$ process, compared to the current experimental upper limit [10] for the branching ratio of 10^{-5}, we can rule out monopole masses lower than about 400 GeV, rather than the 600 GeV quoted in Ref. [9]. Similarly, Ginzburg and Panfil [11] and very recently Ginzburg and Schiller [12] considered the production of two photons with high transverse momenta by the collision of two photons produced either from e^+e^- or quark-(anti-)quark collisions. Again the final photons are produced through a virtual monopole loop. Based on this theoretical scheme, an experimental limit has appeared by the D0 collaboration [13], which sets the following bounds on the monopole mass M:

$$\frac{M}{n} > \begin{cases} 610 \text{ GeV} & \text{for } S = 0 \\ 870 \text{ GeV} & \text{for } S = 1/2 \\ 1580 \text{ GeV} & \text{for } S = 1 \end{cases}, \tag{3}$$

where S is the spin of the monopole. It is worth noting that a mass limit of 120 GeV for a Dirac monopole has been set by Graf, Schäfer, and Greiner [14], based on the monopole contribution to the vacuum polarization correction to the muon anomalous magnetic moment. (Actually, we believe that the correct limit, obtained from the well-known textbook formula [15] for the g-factor correction due to a massive Dirac particle is 60 GeV.)

The purpose of this paper is to critique the theory of Refs. [9], [11], [12], and [14]. We will show that it is based on a naive application of electromagnetic duality; the resulting cross section cannot be valid because unitarity is violated for monopole masses as low as the quoted limits, and the process is subject to enormous, uncontrollable radiative corrections. It is not correct, in any sense, as Refs. [12] and [13] state, that the effective expansion parameter is $g\omega/M$, where ω is some external photon energy; rather, the factors of ω/M emerge kinematically from the requirements of gauge invariance at the one-loop level. If, in fact, a correct calculation introduced such additional factors of ω/M, arising from the complicated coupling of magnetic charge to photons, we argue that no limit could be deduced for monopole masses from the current experiments. It may even be the case, based on preliminary field-theoretic calculations, that processes involving the production of real photons vanish.

II. DUALITY AND THE EULER-HEISENBERG LAGRANGIAN

Let us concentrate on the process contemplated in Refs. [12] and [13], that is

$$\begin{pmatrix} qq & \to & qq \\ \bar{q}q & \to & \bar{q}q \end{pmatrix} + \gamma\gamma, \quad \gamma\gamma \to \gamma\gamma, \tag{4}$$

where the photon scattering process is given by the one-loop light-by-light scattering graph shown in Fig. 1. If the particle in the loop is an ordinary electrically charged electron, this process is well-known [16,15,17]. If, further, the photons involved are of very low momentum compared the the mass of the electron, then the result may be simply derived from the well-known Euler-Heisenberg Lagrangian [18], which for a spin 1/2 charged-particle loop in the presence of homogeneous electric and magnetic fields is[1]

$$\mathcal{L} = -\mathcal{F} - \frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-m^2 s}\left[(es)^2\mathcal{G}\frac{\mathrm{Re}\cosh esX}{\mathrm{Im}\cosh esX} - 1 - \frac{2}{3}(es)^2\mathcal{F}\right]. \tag{5}$$

Here the invariant field strength combinations are

$$\mathcal{F} = \frac{1}{4}F^2 = \frac{1}{2}(\mathbf{H}^2 - \mathbf{E}^2), \quad \mathcal{G} = \frac{1}{4}F\tilde{F} = \mathbf{E}\cdot\mathbf{H}, \tag{6}$$

$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ being the dual field strength tensor, and the argument of the hyperbolic cosine in Eq. (5) is given in terms of

$$X = [2(\mathcal{F} + i\mathcal{G})]^{1/2} = [(\mathbf{H} + i\mathbf{E})^2]^{1/2}. \tag{7}$$

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[1]We emphasize that Eq. (5) is only valid when $\partial_\alpha F_{\mu\nu} = 0$.

If we pick out those terms quadratic, quartic and sextic in the field strengths, we obtain[2]

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{\alpha^2}{360}\frac{1}{m^4}[4(F^2)^2 + 7(F\tilde{F})^2]$$
$$- \frac{\pi\alpha^3}{630}\frac{1}{m^8}F^2[8(F^2)^2 + 13(F^*F)^2] + \dots. \tag{8}$$

The Lagrangian for a spin-0 and spin-1 charged particle in the loop is given by similar formulas which are derived in Ref. [16,15] and (implicitly) in Ref. [19], respectively.

Given this homogeneous-field effective Lagrangian, it is a simple matter to derive the cross section for the $\gamma\gamma \to \gamma\gamma$ process in the low energy limit. (These results can, of course, be directly calculated from the corresponding one-loop Feynman graph with on-mass-shell photons. See Refs. [15,17].) Explicit results for the differential cross section are given by Ref. [17]:

$$\frac{d\sigma}{d\Omega} = \frac{139}{32400\pi^2}\alpha^4\frac{\omega^6}{m^8}(3 + \cos^2\theta)^2, \tag{9}$$

and the total cross section for a spin-1/2 charged particle in the loop is[3]

$$\sigma = \frac{973}{10125\pi}\alpha^4\frac{\omega^6}{m^8}. \tag{10}$$

Here, ω is the energy of the photon in the center of mass frame, $s = 4\omega^2$. This result is valid provided $\omega/m \ll 1$. The dependence on m and ω is evident from the Lagrangian (8), the ω dependence coming from the field strength tensor. Further note that perturbative quantum corrections are small, because they are of relative order $3\alpha \sim 10^{-2}$ [20]. Processes in which four final-state photons are produced, which may be easily calculated from the last displayed term in Eq. (8), are even smaller, being of relative order $\sim \alpha^2(\omega/m)^8$. So light-by-light scattering, which has been indirectly observed through its contribution to the anomalous magnetic moment of the electron [21], is completely under control for electron loops.

How is this applicable to photon scattering through a monopole loop? At first blush this calculation seems formidable. The interaction of a magnetically charged particle with a photon involves a "string," that is, an arbitrary vector function $f_\mu(x-x')$ that satisfies

$$\partial_\mu f^\mu(x-x') = \delta(x-x'), \tag{11}$$

which can be realized by a line integral, for example, the semi-infinite one

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[2]Incidentally, note that the coefficient of the last term is 36 times larger than that given in Ref. [9].

[3]The numerical coefficient in the total cross section for a spin-0 and spin-1 charged particle in the loop is $119/20250\pi$ and $2751/250\pi$, respectively. Numerically the coefficients are 0.00187, 0.0306, and 3.50 for spin 0, spin 1/2, and spin 1, respectively.

$$f_\mu(x) = \int_0^\infty d\xi_\mu \, \delta(x-\xi), \tag{12}$$

where the ξ integration follows some path from the origin to infinity. For the case of a straight line with direction n_μ, this can be written in the form

$$f_\mu(x) = \frac{n_\mu}{i} \int \frac{(dq)}{(2\pi)^4} \frac{e^{iqx}}{n \cdot q - i\epsilon}. \tag{13}$$

The interaction between a magnetic current J_m^μ and the electromagnetic field is given by

$$W_{\rm int} = \int (dx)(dx') \tilde{F}_{\mu\nu}(x') f^\nu(x'-x) J_m^\mu(x), \tag{14}$$

where the magnetic current must be conserved, $\partial_\mu J_m^\mu = 0$. Here, the string-dependent field strength tensor [1,2] is

$$F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu + \epsilon_{\mu\nu\sigma\tau} \int (dy) f^\sigma(x-y) J_m^\tau(y). \tag{15}$$

The interaction (14) corresponds to a coupling between electric and magnetic currents of

$$W^{(eg)} = -\epsilon_{\mu\nu\sigma\tau} \int (dx)(dx')(dx'') J_e^\mu(x) \partial^\sigma D_+(x-x') f^\tau(x'-x'') J_m^\nu(x''). \tag{16}$$

From Eqs. (13)–(16) one obtains the relevant string-dependent monopole-photon coupling vertex in momentum space,

$$\Gamma_\mu(q) = ig \frac{\epsilon_{\mu\nu\sigma\tau} n^\nu q^\sigma \gamma^\tau}{n \cdot q - i\epsilon}. \tag{17}$$

The choice of the string is arbitrary; reorienting the string is a kind of gauge transformation. In fact, it is this requirement that leads to the quantization conditions (1) and (2). The consistency of magnetic charge has been demonstrated in quantum mechanics (for example, see Refs. [22] and [23]), but never completely in quantum field theory.[4] The use of the string-dependent vertex (17) directly is not meaningful. In this regard, the "remedy" proposed by Deans [25] and cited as a solution to the gauge-string dependence of Drell-Yan processes is implausible.

The authors of Refs. [9], [11], and [12] do not attempt a calculation of the "box" diagram with the interaction (14). Rather, they (explicitly or implicitly) appeal to duality, that is, the symmetry that the introduction of magnetic charge brings to Maxwell's equations:

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[4]Arguments have been given to demonstrate the relativistic invariance of the theory, and the string independence of the action for classical particle currents [2]. See also Ref. [24]. This consistency is a consequence of the quantization condition (1) or (2). We should also bear in mind Schwinger's warning, in the first reference in Ref. [2]: "Relativistic invariance will appear to be violated in any treatment based on a perturbation expansion. Field theory is more than a set of 'Feynman's rules.' "

$$\mathbf{E} \to \mathbf{H}, \quad \mathbf{H} \to -\mathbf{E}, \tag{18}$$

and similarly for charges and currents. Thus the argument is that for low energy photon processes it suffices to compute the fermion loop graph in the presence of zero-energy photons, that is, in the presence of static, constant fields. The box diagram shown in Fig. 1 with a spin-1/2 monopole running around the loop in the presence of a homogeneous $\mathbf{E}, \mathbf{H}$ field is then obtained from that analogous process with an electron in the loop in the presence of a homogeneous $\mathbf{H}, -\mathbf{E}$ field, with the substitution $e \to g$. Since the Euler-Heisenberg Lagrangian (8) is invariant under the substitution (18) on the fields alone, this means we obtain the low energy cross section $\sigma_{\gamma\gamma\to\gamma\gamma}$ through the monopole loop from Eq. (10) by the substitution $e \to g$, or

$$\alpha \to \alpha_g = \frac{137}{4} n^2, \quad n = 1, 2, 3, \ldots . \tag{19}$$

III. INCONSISTENCY OF THE DUALITY APPROXIMATION

It is critical to emphasize that the Euler-Heisenberg Lagrangian is an effective Lagrangian for calculations at the *one fermion loop level* for low energy, i.e., $\omega/M \ll 1$. It is commonly asserted that the Euler-Heisenberg Lagrangian is an *effective Lagrangian* in the sense used in chiral perturbation theory [26,27]. This is not true. The QED expansion generates derivative terms which do not arise in the effective Lagrangian expansion of the Euler-Heisenberg Lagrangian [20]. One can only say that the Euler-Heisenberg Lagrangian is a good approximation for light-by-light scattering (without monopoles) at low energy because radiative corrections are down by factors of α. However, it becomes unreliable if radiative corrections are large.

In this regard, both the Ginzburg [11,12] and the De Rújula [9] articles, particularly Ref. [12], are rather misleading as to the validity of the approximation sketched in the previous section. They state that the expansion parameter is not g but $g\omega/M$, M being the monopole mass, so that the perturbation expansion may be valid for large g if ω is small enough. But this is an invalid argument. It is only when external photon lines are attached that extra factors of ω/M occur, due to the appearance of the field strength tensor in the Euler-Heisenberg Lagrangian. Moreover, the powers of g and ω/M are the same only for the F^4 process. The expansion parameter is α_g, which is huge. Instead of radiative corrections being of the order of α for the electron-loop process, these corrections will be of order α_g, which implies an uncontrollable sequence of corrections. For example, the internal radiative correction to the box diagram in Fig. 1 have been computed by Ritus [28] and by Reuter, Schmidt, and Schubert [29] in QED. In the $O(\alpha^2)$ term in Eq. (8) the coefficients of the $(F^2)^2$ and the $(F\tilde{F})^2$ terms are multiplied by $\left(1 + \frac{40}{9}\frac{\alpha}{\pi} + O(\alpha^2)\right)$ and $\left(1 + \frac{1315}{252}\frac{\alpha}{\pi} + O(\alpha^2)\right)$, respectively. The corrections become meaningless when we *replace* $\alpha \to \alpha_g$.

This would seem to be a devastating objection to the results quoted in Ref. [12] and used in Ref. [13]. But even if one closes one's eyes to higher order effects, it seems clear that the mass limits quoted are inconsistent.

If we take the cross section given by Eq. (10) and make the substitution (19), we obtain for the low energy light-by-light scattering cross section in the presence of a monopole loop

$$\sigma_{\gamma\gamma\to\gamma\gamma} \approx \frac{973}{2592000\pi}\frac{n^8}{\alpha^4}\frac{\omega^6}{M^8} = 4.2\times 10^4\, n^8 \frac{1}{M^2}\left(\frac{\omega}{M}\right)^6. \tag{20}$$

If the cross section were dominated by a single partial wave of angular momentum J, the cross section would be bounded by

$$\sigma \le \frac{\pi(2J+1)}{s} \sim \frac{3\pi}{s}, \tag{21}$$

if we take $J = 1$ as a typical partial wave. Comparing this with the cross section given in Eq. (20), we obtain the following inequality for the cross section to be consistent with unitarity,

$$\frac{M}{\omega} \gtrsim 3n. \tag{22}$$

But the limits quoted for the monopole mass are less than this:

$$\frac{M}{n} > 870 \text{ Gev}, \quad \text{spin } 1/2, \tag{23}$$

because, at best, a minimum $\langle\omega\rangle \sim 300$ GeV; the theory cannot sensibly be applied below a monopole mass of about 1 TeV. (Note that changing the value of J in the unitarity limits has very little effect on the bound (22) since an 8th root is taken: replacing J by 50 reduces the limit (22) only by 50%.)

Similar remarks can be directed toward the De Rújula limits [9]. That author, however, notes the "perilous use of a perturbative expansion in g." However, he fails to use the correct vertex, Eq. (17), instead appealing to duality, and even so he admittedly omits enormous radiative corrections of $O(\alpha_g)$ without any justification other than what we believe is a specious reference to the use of effective Lagrangian techniques for these processes.

IV. PROPOSED REMEDIES

Apparently, then, the formal small ω result obtained from the Euler-Heisenberg Lagrangian cannot be valid beyond a photon energy $\omega/M \gtrsim 0.1$. The reader might ask why one cannot use duality to convert the monopole coupling with an arbitrary photon to the ordinary vector coupling. The answer is that little is thereby gained, because the coupling of the photon to ordinary charged particles is then converted into a complicated form analogous to Eq. (14). This point is stated and then ignored in Ref. [9] in the calculation of $Z \to 3\gamma$. There is, in general, no way of avoiding the complication of including the string.

We are currently undertaking realistic calculations of virtual (monopole loop) and real (monopole production) magnetic monopole processes [30]. These calculations are, as the reader may infer, somewhat difficult and involve subtle issues of principle involving the string, and it will be some time before we have results to present. Therefore, here we wish to offer plausible qualitative considerations, which we believe suggest bounds that call into question the results of Ginzburg et al. [11,12].

Our point is very simple. The interaction (14) couples the magnetic current to the dual field strength. This corresponds to the velocity suppression in the interaction of magnetic fields with electrically charged particles, or to the velocity suppression in the interaction of electric fields with magnetically charged particles, as most simply seen in the magnetic analog of the Lorentz force,

$$\mathbf{F} = g(\mathbf{B} - \frac{\mathbf{v}}{c} \times \mathbf{E}). \tag{24}$$

That is, the force between an electric charge e and magnetic charge g, moving with relative velocity $\mathbf{v}$ and with relative separation $\mathbf{r}$ is

$$\mathbf{F} = -\frac{eg}{c}\frac{\mathbf{v} \times \mathbf{r}}{4\pi r^3}. \tag{25}$$

This velocity suppression is reflected in nonrelativistic calculations. For example, the energy loss in matter of a magnetically charge particle is approximately obtained from that of a particle with charge Ze by the substitution [31]

$$\frac{Ze}{v} \to \frac{g}{c}. \tag{26}$$

And the classical nonrelativistic dyon-dyon scattering cross section near the forward direction is [22]

$$\frac{d\sigma}{d\Omega} \approx \frac{1}{(2\mu v)^2}\left[\left(\frac{e_1 g_2 - e_2 g_1}{4\pi c}\right)^2 + \left(\frac{e_1 e_2 + g_1 g_2}{4\pi v}\right)^2\right]\frac{1}{(\theta/2)^4}, \quad \theta \ll 1, \tag{27}$$

the expected generalization of the Rutherford scattering cross section at small angles.

Of course, the true structure of the magnetic interaction and the resulting scattering cross section is much more complicated. For example, classical electron-monopole or dyon-dyon scattering exhibits rainbows and glories, and the quantum scattering exhibits a complicated oscillatory behavior in the backward direction [22]. These reflect the complexities of the magnetic interaction between electrically and magnetically charged particles, which can be represented as a kind of angular momentum [23,32]. Nevertheless, for the purpose of extracting qualitative information, the naive substitution,

$$e \to \frac{v}{c}g, \tag{28}$$

seems a reasonable first step.[5] Indeed, such a substitution was used in the proposal [7] to estimate production rates of monopoles at Fermilab.

The situation is somewhat less clear for the virtual processes considered here. Nevertheless, the interaction (14) suggests there should, in general, be a softening of the vertex. In the current absence of a valid calculational scheme, we will merely suggest two plausible alternatives to the mere replacement procedure adopted in Refs. [9,11,12,14].

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[5] This, and the extension of this idea to virtual processes, leaves aside the troublesome issue of radiative corrections. The hope is that an effective Lagrangian can be found by approximately integrating over the fermions which incorporates these effects.

We first suggest, as seemingly Ref. [12] does, that the approximate effective vertex incorporates an additional factor of ω/M. Thus we propose the following estimate for the $\gamma\gamma$ cross section in place of Eq. (20),

$$\sigma_{\gamma\gamma\to\gamma\gamma} \sim 10^4 n^8 \frac{1}{M^2}\left(\frac{\omega}{M}\right)^{14}, \tag{29}$$

since there are four suppression factors in the amplitude. Now a considerably larger value of ω is consistent with unitarity,

$$\frac{M}{\omega} \gtrsim \sqrt{3n}, \tag{30}$$

if we take $J = 1$ again. We now must re-examine the $\sigma_{pp\to\gamma\gamma X}$ cross section.

In the model given in Ref. [12], where the photon energy distribution is given in terms of the functions $f(y)$, $y = \omega/E$, the physical cross section is given by

$$\sigma_{pp\to\gamma\gamma X} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{dy_1}{y_1}\frac{dy_2}{y_2} f(y_1)f(y_2)\sigma_{\gamma\gamma\to\gamma\gamma} = \int dy_1 dy_2 \frac{d\sigma}{dy_1\, dy_2}, \tag{31}$$

where now (*cf.* Eq. (25) of Ref. [12])

$$\frac{d\sigma}{dy_1\, dy_2} = \left(\frac{\alpha}{\pi}\right)^2 RE^6 \left(\frac{E}{M}\right)^8 y_1^6 f(y_1) y_2^6 f(y_2), \tag{32}$$

where, for spin 1/2, (up to factors of order unity)

$$R \sim \frac{10^{-4}}{\alpha^4}\left(\frac{n}{M}\right)^8. \tag{33}$$

The result in (32) differs from that in Ref. [12] by a factor of $(E/M)^8 y_1^4 y_2^4$. The photon distribution function $y^2 f(y)$ used is rather strongly peaked at $y \sim 0.3$. (This peaking is necessary to have any chance of satisfying the low-frequency criterion.) When we multiply by y^4, the amplitude is greatly reduced and the peak is shifted above $y = 1/2$, violating even the naive criterion for the validity of perturbation theory. Nevertheless, the integral of the distribution function is reduced by two orders of magnitude, that is,

$$\frac{\int_0^1 dy\, y^6 f(y)}{\int_0^1 dy\, y^2 f(y)} \sim 10^{-2}. \tag{34}$$

This reduces the mass limit quoted in [13] by a factor of $1/\sqrt{3}$, to about 500 GeV, where $\langle\omega\rangle/M \approx 0.9$. This dubious result makes us conclude that it is impossible to derive any limit for the monopole mass from the present data.

As for the De Rújula limit[6] from the $Z \to 3\gamma$ process, if we insert a suppression factor of ω/M at each vertex and integrate over the final state photon distributions, given by Eq. (18)

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[6]We note that De Rújula also considers the monopole vacuum polarization correction to g_V/g_A, g_A, and m_W/m_Z, proportional to $(m_Z/M)^2$ in each case, once again ignoring both the string and the radiative correction problem. He assumes that the monopole is a heavy vector-like fermion, and obtains a limit of $M/n > 8m_Z$. Our ansatz changes $(m_Z/M)^2$ to $(m_Z/M)^4$, so that $M/\sqrt{n} > \sqrt{8}m_Z \approx 250$ GeV, a substantial reduction.

of Ref. [9], the mass limit is reduced to $M/\sqrt{n} \gtrsim 1.4 m_Z \sim 120$ GeV, again grossly violating the low energy criterion. And the limit deduced from the vacuum polarization correction to the anomalous magnetic moment of the muon due to virtual monopole pairs [14] is reduced to 2 GeV.

The reader might object that this ω/M softening of the vertex has little field-theoretic basis. Therefore, we propose a second possibility that does have such a basis. The vertex (17) suggests, and detailed calculation supports (based on the tensor structure of the photon amplitudes[7]) the introduction of the string-dependent factor $\sqrt{q^2/(n\cdot q)^2}$ at each vertex, where q is the photon momentum. Such a factor is devastating to the indirect monopole searches—for any process involving a real photon, such as that of the D0 experiment [13] or for $Z \to 3\gamma$ discussed in [9], the amplitude vanishes. Because such factors can and do appear in full monopole calculations, it is clearly premature to claim any limits based on virtual processes involving real final-state photons.

V. CONCLUSIONS

We do not take our reduced limits on monopole masses very seriously. Rather, we believe they demonstrate our point that given the dual difficulties of theoretically treating monopoles, that is, incorporating the string and dealing with enormously strong coupling,[8] it is premature to attempt to set any limits on monopole masses based on virtual effects. A direct search stills seems much less problematic.

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[7]For example, the naive monopole loop contribution to vacuum polarization differs from that of an electron loop (apart from charge and mass replacements) entirely by the replacement in the latter of $(g_{\mu\nu} - q_\mu q_\nu/q^2) \to (\mathbf{q}^2/q_0^2)(\delta_{ij} - q_i q_j/\mathbf{q}^2)$, when n^μ points in the time direction. Apart from this different tensor structure, the vacuum polarization is given by exactly the usual formula, found, for example in Ref. [15]. Details of this and related calculations will be given in Ref. [30].

[8]None of the papers dealing with virtual monopole effects, [9,11,12,14], in fact, incorporate these nontrivial effects.

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FIGURES

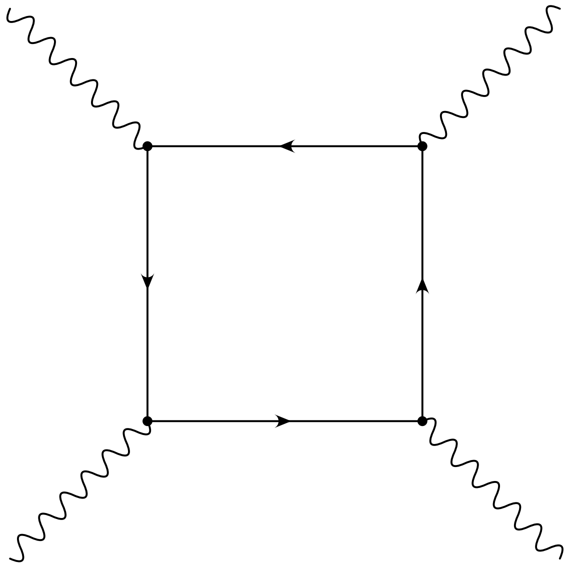

FIG. 1. The light-by-light scattering graph for either an electron or a monopole loop.