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Quantum (In)Stability of a Brane-World AdS_5 Universe at Nonzero Temperature

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Abstract

We consider the quantum effects of bulk matter (scalars, spinors) in the Randall-Sundrum AdS_5 brane-world at nonzero temperature. The thermodynamic energy (modulus potential) is evaluated at low and high temperatures. This potential has an extremum which could be a minimum in some cases (for example, for a single fermion). That suggests a new dynamical mechanism to stabilize the thermal AdS_5 brane-world. It is shown that the brane separation

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required to solve the hierarchy scale problem may occur at a quite low temperature. A natural generalization in terms of the AdS/CFT correspondence (through the supergravity thermal contribution) is also possible.

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I. INTRODUCTION

It is expected that the Randall-Sundrum scenario [1] (or warped compactification) should be realized completely in terms of the anti-deSitter/conformal field theory (AdS/CFT) correspondence [2]. There are various proposals for such a realization. For example, taking the quantum effects of brane CFT (via the corresponding conformal anomaly) may help in the construction of new brane-worlds [3–6] which occur in the AdS/CFT correspondence (as a kind of holographic renormalization group (RG) flow). It is remarkable that a calculation of corrections to Newton's law from quantum brane CFT coincides quite well with that computed from supergravity [7].

The role played by quantum effects suggests that they may be relevant in other aspects of warped (brane-world) compactifications. Indeed, for usual two-dimensional or four-dimensional AdS backgrounds the matter quantum effects may help to stabilize (or destabilize) the AdS background under consideration [3,8]. In the same way, one can expect that quantum bulk gravity may be important in the realization of warped compactification within the AdS/CFT scheme or in improving the Randall-Sundrum scenario. As a toy model for quantum bulk gravity one can consider a quantum bulk scalar [9–12].

As has been suggested in Refs. [9,11] bulk quantum effects may generate a modulus effective potential which will help in stabilizing the brane-world five-dimensional AdS space with a finite separation of (flat) branes, which also solves the hierarchy problem. Unfortunately, the explicit analysis done in Ref. [11] exhibits a power suppression of quantum bulk effects, which consequently have a negligible role in stabilizing the brane-world scenario with the necessary hierarchy. (In the original RS scenario this purpose is achieved by fine-tuning of parameters.)

Nevertheless, it may happen that, if we take into account other effects or a change of topology [13], bulk quantum fields may provide the dynamical mechanism to construct the stable AdS brane-world without fine-tuning. As one step in this direction, we consider the quantum effects of the bulk matter in a five-dimensional AdS universe at nonzero temperature. The corresponding high- and low-temperature limits of the thermodynamic energy are estimated and applied to the study of quantum stability of an AdS_5 (flat) brane-world. It is demonstrated that the thermal modulus potential may have an extremum which can be a minimum in some cases. Hence, a new thermal dynamical mechanism to stabilize a five-dimensional AdS brane-world is suggested. It is interesting that the brane separation required to solve the hierarchy problem may occur as a minimum of the effective potential at a quite low temperature.

II. EFFECTIVE POTENTIAL IN 5-DIMENSIONAL AdS SPACE AT NONZERO TEMPERATURE

Our purpose in this section will be the calculation of the energy (effective potential) for a bulk quantum field on a five-dimensional AdS background at nonzero temperature. We start with a textbook-like review of black body radiation. Since the energy of a single particle with momentum $\mathbf{p}$ and mass m is given by

$$E_p = \sqrt{\mathbf{p}^2 + m^2} \ , \tag{1}$$

we obtain the following expression for its contribution to the partition function Z

$$Z_p^b = \sum_{n=0}^{\infty} \mathrm{e}^{-\beta E_p \left(n+\frac{1}{2}\right)} = \frac{1}{2\sinh\left(\frac{\beta E_p}{2}\right)} \ , \tag{2a}$$

for a boson and

$$Z_p^f = \sum_{n=0}^{1} \mathrm{e}^{-\beta E_p \left(n-\frac{1}{2}\right)} = 2\cosh\left(\frac{\beta E_p}{2}\right) \ , \tag{2b}$$

for a fermion. Here β is the inverse of the temperature T, $\beta = 1/T$, and we sum over all possible states with n particles (quanta). In Eqs. (2a) and (2b) we express the zero-point energy by $\pm\frac{1}{2}E_p$. The total partition function of the particle with mass m in a $d-1$-dimensional volume V_{d-1} is given by summing over Z_k with respect to the momentum $\mathbf{p}$:

$$\beta F^b = -\ln Z^b = V_{d-1} \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \ln\left(2\sinh\left(\frac{\beta E_p}{2}\right)\right) \ , \tag{3a}$$

$$\beta F^f = -\ln Z^f = -V_{d-1} \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \ln\left(2\cosh\left(\frac{\beta E_p}{2}\right)\right) \ . \tag{3b}$$

Here $F^{b,f}$ is the free energy. Eqs. (3a) and (3b) diverge, and require regularization in general. For the supersymmetric case, we obtain a finite result:

$$\beta F^s = \beta\left(F^b + F^f\right) = V_{d-1} \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \ln\left(\tanh\left(\frac{\beta E_p}{2}\right)\right) \ . \tag{3c}$$

The average energy E is given by the derivative of the free energy,

$$E = \frac{\partial}{\partial\beta}(\beta F) \ . \tag{4}$$

Then for above cases (3a), (3b) and (3c), we find

$$E_d^b(\beta) = V_{d-1} \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{E_p}{2} \coth\left(\frac{\beta E_p}{2}\right) \ , \tag{5a}$$

$$E_d^f(\beta) = -V_{d-1} \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{E_p}{2} \tanh\left(\frac{\beta E_p}{2}\right) \ , \tag{5b}$$

$$E_d^s(\beta) = V_{d-1} \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{E_p}{\sinh \beta E_p} \ . \tag{5c}$$

This completes our elementary review of quantum statistics at nonzero temperature.

In the Randall-Sundrum model [1], the Standard Model fields are confined on one of the two 3-branes, which are the boundaries of the bulk AdS_5 space, whose metric is given by

$$ds^2 = -r_c^2 d\phi^2 + \mathrm{e}^{-2kr_c|\phi|} \eta_{\mu\nu} dx^\mu dx^\nu \ . \tag{6}$$

Here the coordinate ϕ takes a value in $[-\pi, \pi]$, k is a parameter of order the Planck scale, and r_c is the length of the orbifold. For simplicity, let us consider the bulk scalar field Φ, whose action is given by

$$S_\Phi = \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-G}\left\{G^{AB}\partial_A\Phi\partial_B\Phi - \left(m^2 + \frac{2\alpha k}{r_c}\left(\delta(\phi) - \delta(\phi-\pi)\right)\right)\Phi^2\right\} , \qquad (7)$$

where α parametrizes the mass on the boundaries. All fields in the bulk 5-dimensional spacetime can be regarded as Kaluza-Klein modes, which in their own right can be considered as 4-dimensional fields on the brane with an infinite tower of masses. The mass spectrum m_n of the Kaluza-Klein modes in Φ is given [14,15] by roots of

$$j_\nu(x_n)y_\nu(ax_n) - j_\nu(ax_n)y_\nu(x_n) = 0 , \qquad (8)$$

where $x_n = m_n/ak$, $a = \mathrm{e}^{-kr_c\pi}$, $\nu = \sqrt{4 + \frac{m^2}{k^2}}$, and the altered Bessel functions are

$$j_\nu(z) = (2-\alpha)J_\nu(z) + zJ'_\nu(z), \quad y_\nu(z) = (2-\alpha)Y_\nu(z) + zY'_\nu(z). \qquad (9)$$

The radius r_c can be regarded as the vacuum expectation value of the radion field $T(x)$. In the following, we will be interested in the effective potential (energy) for a.

If we assume that a function $f(x)$ is not singular for real positive x, while another function $g(x)$ has first-order zeroes when $x = x_n > 0$, $(n = 1, 2, \cdots)$, then we obtain by an elementary argument the following formula

$$\sum_n f(x_n) = -\frac{1}{2\pi i}\int_C dz f'(z) \ln g(z) . \qquad (10)$$

Here the contour C can be taken to encircle the real axis,

$$\int_C dz \cdots = \left(\int_{+\infty-i\epsilon}^{\epsilon'-i\epsilon} + \int_{\epsilon'-i\epsilon}^{\epsilon'+i\epsilon} + \int_{\epsilon'+i\epsilon}^{+\infty+i\epsilon}\right) dz \cdots , \qquad (11)$$

where ϵ and ϵ' are infinitesimally small positive constants.

Using Eq. (10), the total free energy can be obtained by summing up the KK modes given by Eq. (8):

$$\beta F^{bKK} = \mathcal{F}\left[\ln\left(2\sinh\left(\frac{\beta\sqrt{\mathbf{p}^2 + a^2k^2x^2}}{2}\right)\right)\right], \qquad (12a)$$

$$\beta F^{fKK} = -\mathcal{F}\left[\ln\left(2\cosh\left(\frac{\beta\sqrt{\mathbf{p}^2 + a^2k^2x^2}}{2}\right)\right)\right], \qquad (12b)$$

$$\beta F^{sKK} = \mathcal{F}\left[\ln\tanh\left(\frac{\beta\sqrt{\mathbf{p}^2 + a^2k^2x^2}}{2}\right)\right] , \qquad (12c)$$

where the functional $\mathcal{F}$ is defined by

$$\mathcal{F}[f(p,x)] = V_{d-1}\int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{i}{2\pi}\int_C dx \frac{d}{dx} f(p,x) \ln\left(j_\nu(x)y_\nu(ax) - j_\nu(ax)y_\nu(x)\right) . \qquad (13)$$

The corresponding energies are given by

$$E^{bKK} = \mathcal{F}\left[\frac{\sqrt{\mathbf{p}^2+a^2k^2x^2}}{2}\coth\left(\frac{\beta\sqrt{\mathbf{p}^2+a^2k^2x^2}}{2}\right)\right] , \tag{14a}$$

$$E^{fKK} = -\mathcal{F}\left[\frac{\sqrt{\mathbf{p}^2+a^2k^2x^2}}{2}\tanh\left(\frac{\beta\sqrt{\mathbf{p}^2+a^2k^2x^2}}{2}\right)\right] , \tag{14b}$$

$$E^{sKK} = \mathcal{F}\left[\frac{\sqrt{\mathbf{p}^2+a^2k^2x^2}}{\sinh\left(\beta\sqrt{\mathbf{p}^2+a^2k^2x^2}\right)}\right] . \tag{14c}$$

It is interesting that one can consider the thermodynamic correction coming from the quantum radion itself by starting from the following Lagrangian [11]:

$$\mathcal{L} = \frac{f^2}{2}\left(\partial a\right)^2 - \delta V_v a^4 \ . \tag{15}$$

Here we regard a as the dynamical field $a = \mathrm{e}^{-k\pi T(x)}$ expressed in terms of the radion field $T(x)$, while δV_v is a small classical shift in the TeV brane tension relative to the value which generates the background metric. Note that f is given in terms of the five-dimensional Planck scale M_5 as $f = \sqrt{24M_5^3/k}$. If one divides a into a sum of background and fluctuation parts by

$$a \longrightarrow a + \frac{1}{f}\delta a \ , \tag{16}$$

the Lagrangian (15) assumes the following form

$$\mathcal{L} = \frac{1}{2}\left(\partial\delta a\right)^2 - \delta V_v\left(a^4 + \frac{4}{f}a^3\delta a + \frac{6}{f^2}a^2\delta a^2 + \frac{4}{f^3}a\delta a^3 + \frac{1}{f^4}\delta a^4\right) . \tag{17}$$

Then the effective mass $\hat{m}$ is given by

$$\hat{m}^2 = \frac{12\delta V_v a^2}{f^2} \tag{18}$$

and from Eqs. (1), (3a), and (5a), the free energy and energy have the following forms:

$$\beta F^r = V_{d-1}\int\frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}}\ln\left(2\sinh\left(\frac{\beta\sqrt{\mathbf{p}^2+\frac{12\delta V_v a^2}{f^2}}}{2}\right)\right) , \tag{19}$$

$$E^r = V_{d-1}\int\frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}}\frac{\sqrt{\mathbf{p}^2+\frac{12\delta V_v a^2}{f^2}}}{2}\coth\left(\frac{\beta\sqrt{\mathbf{p}^2+\frac{12\delta V_v a^2}{f^2}}}{2}\right) . \tag{20}$$

Similarly, the contribution of other (higher spin) bulk fields may be taken into account.

III. HIGH TEMPERATURE LIMIT

Let us consider the high temperature limit where β is small. Since the zero temperature contributions have been evaluated in Ref. [11], we subtract these contributions from $E_d^b(\beta)$

in Eq. (5a) and $E_d^f(\beta)$ in Eq. (5b). ($E_d^s(\beta)$ in Eq. (5c) does not contain a zero temperature contribution):

$$\tilde{E}_d^b(\beta) = E_d^b(\beta) - E_d^b(\infty) \ , \quad \tilde{E}_d^f(\beta) = E_d^f(\beta) - E_d^f(\infty) \ . \tag{21}$$

We should note that $\tilde{E}_d^b(\beta)$ and $\tilde{E}_d^f(\beta)$ are finite. If we change the momentum variable $\mathbf{p}$ to

$$|\mathbf{p}| = \frac{q}{\beta} \ , \tag{22}$$

Eq. (21) may be rewritten as follows

$$\tilde{E}_d^{b,f}(\beta) = \frac{V_{d-1}}{2^{d-2}\pi^{\frac{d-1}{2}}\Gamma\left(\frac{d-1}{2}\right)\beta^d}\int_0^\infty dq\, q^{d-2}\frac{\sqrt{q^2+\beta^2m^2}}{\mathrm{e}^{\sqrt{q^2+\beta^2m^2}} \mp 1} \ . \tag{23}$$

When $d=4$, the expressions in Eq. (23) can be expanded with respect to β as follows:

$$\tilde{E}_4^{b,f}(\beta) = \frac{V_3}{2\pi^2\beta^4}\int_0^\infty dq \left\{ \frac{q^3}{\mathrm{e}^q \mp 1} + \beta^2 m^2 \left(\frac{q}{2\left(\mathrm{e}^q \mp 1\right)} - \frac{q^2 \mathrm{e}^q}{2\left(\mathrm{e}^q \mp 1\right)^2} \right) + \mathcal{O}\left(\beta^4\right) \right\} \ . \tag{24}$$

Using the well-known formulas

$$\int_0^\infty \frac{x^{s-1}}{\mathrm{e}^x - 1}dx = \Gamma(s)\zeta(s) \ , \quad \int_0^\infty \frac{x^{s-1}}{\mathrm{e}^x + 1}dx = (1-2^{1-s})\Gamma(s)\zeta(s) \ , \tag{25}$$

we obtain

$$\tilde{E}_4^b(\beta) = \frac{V_3}{2\pi^2\beta^4}\left(\frac{\pi^4}{15} - \frac{\pi^2\beta^2m^2}{12} + \mathcal{O}\left(\beta^4\right)\right) \ , \tag{26a}$$

$$\tilde{E}_4^f(\beta) = \frac{V_3}{2\pi^2\beta^4}\left(\frac{7\pi^4}{120} - \frac{\pi^2\beta^2m^2}{24} + \mathcal{O}\left(\beta^4\right)\right) \ , \tag{26b}$$

$$E_4^s(\beta) = \tilde{E}_d^b(\beta) + \tilde{E}_d^f(\beta) = \frac{V_3}{2\pi^2\beta^4}\left(\frac{\pi^4}{8} - \frac{\pi^2\beta^2m^2}{8} - \mathcal{O}\left(\beta^4\right)\right) \ . \tag{26c}$$

The first term corresponds to Wien's law for black body radiation of massless fields.

In order to sum up the contribution from the Kaluza-Klein modes, we use the following formula from Ref. [11] (which is obtained from Eq. (10) by unfolding C so that it lies along the imaginary axis): (the large t behavior for $a \ll 1$ has been removed)

$$\begin{aligned} A_s(a) &\equiv \sum_n x_n^{-s} \\ &= \frac{s}{\pi}\sin\left(\frac{\pi s}{2}\right)\int_0^\infty dt\, t^{-s-1}\ln\left[\frac{2}{t\sqrt{a}}\mathrm{e}^{-t(1-a)}\left\{k_\nu(t)i_\nu(at) - k_\nu(at)i_\nu(t)\right\}\right] \end{aligned} \tag{27}$$

for $s = 0, -2$. (This expression may be obtained on the basis of the zeta-regularization technique, see Ref. [16]). Here the Bessel functions of imaginary argument are

$$\begin{aligned} i_\nu(z) &= (2-\alpha)I_\nu(z) + zI'_\nu(z) \\ k_\nu(z) &= (2-\alpha)K_\nu(z) + zK'_\nu(z) \ . \end{aligned} \tag{28}$$

($I_\nu(z)$ and $K_\nu(z)$ are modified Bessel functions). The expression in Eq. (27) is valid when $-1 < s < 0$ and we now consider its analytic continuation to $s = 0$ or $s = -2$.

When $s = 0$, the integration in Eq. (27) contains a divergence coming from the integration when $t \sim 0$ because

$$\begin{aligned} &\ln\left[\frac{2}{t\sqrt{a}}\mathrm{e}^{-t(1-a)}\left\{k_\nu(t)i_\nu(at) - k_\nu(at)i_\nu(t)\right\}\right] \\ &\quad \sim \ln\left[\frac{(2-\alpha-\nu)(2-\alpha+\nu)}{\nu t\sqrt{a}}\left(a^{-\nu} - a^{\nu}\right)\right] . \end{aligned} \tag{29}$$

Thus we divide the region of the integration into two parts ($\epsilon \ll 1$)

$$\int_0^\infty \cdots \to \left(\int_0^\epsilon + \int_\epsilon^\infty\right) \cdots , \tag{30}$$

and rewrite Eq. (27) in the following form:

$$A_s(a) = \frac{s}{\pi}\sin\left(\frac{\pi s}{2}\right)\left(A_s^{(1)}(a) + A_s^{(2)}(a)\right) . \tag{31}$$

When $s \to 0-$, $A_s^{(1)}(a)$ is finite but $A_s^{(2)}(a)$ behaves as

$$A_s^{(2)}(a) \sim -\int_0^\epsilon dt\, t^{-s-1}\ln t \sim \frac{\epsilon^{-s}}{s^2} . \tag{32}$$

Then in the limit of $s \to 0-$, we obtain

$$A_0(a) = \frac{1}{2} . \tag{33}$$

On the other hand, when $s \to -2$, the divergence of the integration in Eq. (27) comes from $t \to \infty$ since

$$\begin{aligned} &\ln\left[\frac{2}{t\sqrt{a}}\mathrm{e}^{-t(1-a)}\left\{k_\nu(t)i_\nu(at) - k_\nu(at)i_\nu(t)\right\}\right] \sim \left(\frac{13}{8} - \alpha - \frac{\nu^2}{2}\right)\left(1 - \frac{1}{a}\right)t^{-1} \\ &+ \left(\frac{\nu^2}{4} - \frac{\alpha^2}{2} + \frac{3\alpha}{2} - \frac{19}{16}\right)\left(1 + \frac{1}{a^2}\right)t^{-2} \equiv h(t) . \end{aligned} \tag{34}$$

Then we divide the region of the integration into two regions ($\Lambda \gg 1$)

$$\int_0^\infty \cdots \to \left(\int_0^\Lambda + \int_\Lambda^\infty\right) \cdots , \tag{35}$$

and rewrite Eq. (27) in the following form:

$$\begin{aligned} A_s(a) &= \frac{s}{\pi}\sin\left(\frac{\pi s}{2}\right)\left(\hat{A}_s^{(1)}(a) + \hat{A}_s^{(2)}(a)\right) , \\ \hat{A}_s^{(2)}(a) &\equiv \int_\Lambda^\infty dt\, t^{-s-1}h(t). \end{aligned} \tag{36}$$

One should note that $\hat{A}_s^{(1)}(a)$ is finite when $-3 < s < 0$. When $-1 < s < 0$, the integrals appearing in $\hat{A}_s^{(2)}(a)$ have the following form:

$$\int_\Lambda^\infty dt\, t^{-s-2} = \frac{\Lambda^{-s-1}}{s+1}, \quad \int_\Lambda^\infty dt\, t^{-s-3} = \frac{\Lambda^{-s-2}}{s+2}. \tag{37}$$

Then by analytically continuing to $s \to -2$, one obtains

$$A_{-2}(a) = \left[\frac{\nu^2}{4} - \frac{\alpha^2}{2} + \frac{3\alpha}{2} - \frac{19}{16}\right]\left(1 + \frac{1}{a^2}\right) . \tag{38}$$

An important remark is in order. The same technique may be applied to the calculation of the contribution to the energy from higher spin fields (vectors, tensors) [9] at nonzero temperature. Only the corresponding values for α, ν and the mass m will be changed.

The energy $E(a)$ can be regarded as an effective potential with respect to the expectation value of the radion field or a, which determines the distance between the two branes. Note that there appears a nontrivial potential even in the supersymmetric case due to the finite temperature effect, which breaks the supersymmetry explicitly. In the high-temperature limit, from Eqs. (26), the effective potentials have the following form

$$\tilde{E}_4^{bKK}(\beta) = \frac{V_3}{2\beta^4}\frac{\pi^2}{15}A_0(a) - \frac{V_3}{2\beta^2}\frac{a^2k^2}{12}A_{-2}(a) + \mathcal{O}\left(1\right) , \tag{39a}$$

$$\tilde{E}_4^{fKK}(\beta) = \frac{V_3}{2\beta^4}\frac{7\pi^2}{120}A_0(a) - \frac{V_3}{2\beta^2}\frac{a^2k^2}{24}A_{-2}(a) + \mathcal{O}\left(1\right) , \tag{39b}$$

$$E_4^{sKK}(\beta) = \frac{V_3}{2\beta^4}\frac{\pi^2}{8}A_0(a) - \frac{V_3}{2\beta^2}\frac{a^2k^2}{8}A_{-2}(a) + \mathcal{O}\left(1\right) . \tag{39c}$$

Using the explicit forms of $A_0(a)$ given in Eq. (33) and $A_{-2}(a)$ given in Eq. (38), the above energies have the following form:

$$\tilde{E}_4^{iKK}(\beta; a) = \frac{B_0^i}{\beta^4} + \frac{B_1^i k^2}{\beta^2}\left(1 + a^2\right) + \mathcal{O}\left(\beta^0\right) . \tag{40}$$

Here $i = b,\, f,\, s$ and $\tilde{E}_4^{sKK} = E_4^{sKK}$.

We now add the contributions from the zero-point energies, which were subtracted in Eq. (21). The contributions were evaluated in [11] and have the following form:

$$\begin{aligned} E_4^{bKK}(\beta=\infty) = -E_4^{fKK}(\beta=\infty) &= \frac{k^4a^4V_3}{16\pi^2}\int_0^\infty dt\, t^3 \ln\left[1 - \frac{k_\nu(t)i_\nu(at)}{k_\nu(at)i_\nu(t)}\right] , \\ E_4^{sKK}(\beta=\infty) &= 0 . \end{aligned} \tag{41}$$

Here we neglect terms that can be absorbed into the redefinition of the brane tensions. For small a, we have

$$\int_0^\infty dt\, t^3 \ln\left[1 - \frac{k_\nu(t)i_\nu(at)}{k_\nu(at)i_\nu(t)}\right] = \frac{2}{\nu\Gamma(\nu)^2}\left(\frac{\nu-\alpha+2}{\alpha+\nu-2}\right)\left(\frac{a}{2}\right)^{2\nu}\int_0^\infty dt\, t^{2\nu+3}\frac{k_\nu(t)}{i_\nu(t)} + \cdots \tag{42}$$

when $\alpha + \nu \neq 2$ and

$$\int_0^\infty dt\, t^3 \ln\left[1 - \frac{k_\nu(t) i_\nu(at)}{k_\nu(at) i_\nu(t)}\right] = \frac{2(\nu-1)}{\Gamma(\nu)^2}\left(\frac{a}{2}\right)^{2\nu-2} \int_0^\infty dt\, t^{2\nu+1} \frac{k_\nu(t)}{i_\nu(t)} + \cdots \tag{43}$$

when $\alpha + \nu = 2$. The a dependent part of the total effective action has the following form:

$$V^i(a) = \frac{B_1^i k^2}{\beta^2} a^2 + B_2^i k^4 a^{2\mu} \ . \tag{44}$$

Here $\mu = \nu + 2$ when $\alpha + \nu \neq 2$ or $\mu = \nu + 1$ when $\alpha + \nu = 2$. If $\mu \neq 1$ and $-\frac{B_1^i}{\mu B_2^i} > 0$, V^i has non-trivial extremum at

$$a_m^2 = \left(-\frac{B_1^i}{\mu k^2 \beta^2 B_2^i}\right)^{\frac{1}{\mu-1}} \ , \tag{45}$$

which determines the distance between the branes. As an example we consider the case in which $\alpha = 2$ and $\nu > 0$. Then from Eq. (28), we find

$$i_\nu(z) = z I_\nu'(z) > 0 \ , \quad k_\nu(z) = z K_\nu'(z) < 0 \ , \tag{46}$$

and therefore from Eq. (41)

$$B_2^b = -B_2^f = \frac{V_3}{16\pi^2} \frac{2^{1-2\nu}}{\nu \Gamma(\nu)^2} \int_0^\infty dt\, t^{2\nu+3} \frac{K_\nu'(t)}{I_\nu'(t)} < 0 \ . \tag{47}$$

On the other hand, from Eq. (38) with $\alpha = 2$, Eqs. (39a)–(39c) and Eq. (40), we find B_1^i is negative if $\nu^2 > 3/4$. Then, in case of fermions, V^f can have an extremum, which is a minimum. In the special case of $\nu = \frac{5}{2}$, as an example, using numerical integration one gets

$$\int_0^\infty dt\, t^8 \frac{k_{\frac{5}{2}}(t)}{i_{\frac{5}{2}}(t)} = -1492.97\ldots \ . \tag{48}$$

Then

$$\begin{aligned} \frac{B_1^f}{V_3} &= -\frac{11}{384} = -0.0286458 \\ \frac{B_2^f}{V_3} &= 0.133752\ldots \ , \end{aligned} \tag{49}$$

and

$$a_m = \left(\frac{0.0475938}{k^2 \beta^2}\right)^{\frac{1}{7}} \ . \tag{50}$$

Thus, we explicitly demonstrate that quantum spontaneous compactification of the 5-dimensional AdS brane-world at nonzero temperature is possible.

Now one can try to estimate the value of temperature which is necessary to solve the hierarchy problem. Since $k^2 = (10^{19}\text{GeV})^2$, in order that a_m be the ratio of the weak scale to the Planck scale, 10^{-17}, we find $\frac{1}{\beta} \sim 10^{-40}\text{GeV} \sim 10^{-27}\,\text{K}$. This is an extremely low temperature. Of course, this also means the high temperature expansion is not valid since the dimensionless expansion parameter is $k\beta \sim 10^{59}$.

IV. LOW-TEMPERATURE LIMIT

Thus, we are led to consider the low temperature limit. We start from Eq. (23). By changing the variable from q to s

$$q = \sqrt{s^2 + 2\beta m s}\ , \tag{51}$$

the energies $\tilde{E}^b_d(\beta)$ and $\tilde{E}^f_d(\beta)$ for $d = 4$ in Eq. (23) have the following forms:

$$\tilde{E}^{b,f}_4(\beta) = \frac{V_3}{2\pi^2\beta^4}\int_0^\infty ds \frac{(s+\beta m)^2\sqrt{s^2+2\beta m s}}{\mathrm{e}^{s+\beta m}\mp 1}\ . \tag{52}$$

Then in the low temperature limit, where $\beta \to \infty$, one gets

$$\tilde{E}^{b,f}_4(\beta) \to \frac{V_3 m^{\frac{5}{2}}\mathrm{e}^{-\beta m}}{\sqrt{2}\pi^2\beta^{\frac{3}{2}}}\int_0^\infty ds s^{\frac{1}{2}}\mathrm{e}^{-s} = \frac{V_3 m^{\frac{5}{2}}\mathrm{e}^{-\beta m}}{(2\pi\beta)^{3/2}} \tag{53}$$

We now consider the sum of the Kaluza-Klein modes, but due to the factor of $\mathrm{e}^{-\beta m}$, when β is large we need to include only the lowest root of Eq. (8), $x = x_1$. In general x_1 depends on a but Eq. (8) reduces to

$$-(2-\alpha-\nu)\Gamma(\nu)\left(\frac{a x_1}{2}\right)^{-\nu} j_\nu(x_1) = 0 \tag{54}$$

when a is small. Then x_1 satisfies $j_\nu(x_1) = 0$ ($\alpha+\nu \neq 2$) in the limit and one can regard that x_1 is of order unity. Adding the contribution from the zero-point energies as in Eq. (44), we find the following effective potential

$$\begin{aligned} V^i(a) &= B^i_2 k^4 a^{2\mu} + B^i_3 \beta^{-\frac{3}{2}}(ka)^{\frac{5}{2}}\mathrm{e}^{-\beta k a x_1} \\ &= k^4 B^i_2\left(a^{2\mu} + \frac{B^i_3}{B^i_2}(\beta k)^{-3/2}a^{5/2}\mathrm{e}^{-\beta k a x_1}\right)\ . \end{aligned} \tag{55}$$

Here

$$B^b_3 = B^f_3 = \frac{B^s_3}{2} = \frac{V_3 x_1^{\frac{5}{2}}}{(2\pi)^{3/2}} > 0\ . \tag{56}$$

Since B^i_3 is positive, the second term in the effective potential (55) has a maximum when $\beta k a \sim 1$ and exponentially approaches zero after the maximum. Then if βk is large, the effective potential (55) has a nontrivial minimum if $\mu > 5/4$ and B^i_2 is positive. Thus when βk is large, the order of magnitude of the minimum a_m in the effective potential (55) is given roughly by

$$a_m \sim \frac{1}{\beta k}\ln \beta k\ . \tag{57}$$

Then since $k \sim 10^{19}$GeV, if $\frac{1}{\beta} \sim 10\,$GeV, we have $a_m \sim 10^{-17}$ and the weak scale can be generated : $ak \sim 10^2\,$GeV.

As an example of how this minimum can be achieved, we consider the case that $\nu > 0$ and $\alpha = 2$. Then because $\mu = \nu + 2 > 2$ and $B_2^f > 0$ from Eq. (47), we have the minimum of the potential for the fermionic case. If $\nu = 5/2$ or $\mu = 9/2$, $x_1 = 3.6328$, $B_3^f / B_2^f = 11.9408$, and we find the minimum occurs at $a_m = 5.15 \times 10^{-17}$. In other words, a bulk quantum fermion may generate a thermal (flat) 5-dimensional AdS brane-world with the necessary hierarchy scale! This example proves that quantum bulk effects in a brane-world AdS_5 at nonzero temperature may not only stabilize the brane-world (quantum spontaneous compactification occurs) but also provide the dynamical mechanism for the resolution of the hierarchy problem (with no fine-tuning). Note, however, that the temperature in above example is much less that the characteristic temperature in hot inflationary universe. It is very important to recognize that despite the fact that our toy example involved only bulk matter fields it is expected that such a dynamical mechanism occurs in terms of the AdS/CFT correspondence, due to the contribution of thermal 5-dimensional gravitons and matter supermultiplets which appear after sphere compactification of IIB supergravity on the AdS_5 space.

V. DISCUSSION

In summary, we have formulated quantum bulk scalar (or spinor) dynamics in a brane-world AdS_5 at nonzero temperature. Such a universe is a natural generalization of the Randall-Sundrum scenario. The calculation of the thermodynamic energy (effective potential) is done in the high- and low-temperature limits. Thermal quantum corrections generate a modulus potential which in some explicit examples has been shown to have a minimum. Hence, a new dynamical mechanism to stabilize the thermal brane-world universe has been suggested. Moreover, the finite separation of flat branes may be fixed in terms of the Planck constant and the temperature so that the hierarchy problem is naturally solved.

The unsatisfactory feature of our mechanism is that although quantum spontaneous compactification occurs, the natural hierarchy is generated at quite small values of temperature, much smaller than the temperature which is characteristic for a hot inflationary universe. The good feature of our proposal is that it is directly applicable to the RS warped compactification in terms of the AdS/CFT correspondence. The reason is that the required thermal modulus potential may be easily generated by bulk quantum supergravity. Only the numerical coefficients in the potential (55) (or Eq. (44)) will be changed due to the contribution of the thermal graviton (and superpartners). It remains to be checked that these explicit values correspond to the extremum of graviton potential being a minimum, a maximum, or an inflection point.

In any case the possibility of obtaining a thermal modulus potential with a possible extremum opens a new perspective on the dynamical formulation of brane-worlds. So far the only known (classical) dynamical mechanism to solve the hierarchy problem has been proposed in Ref. [17]. Unlike our scheme, that mechanism does not permit the extension to bulk gravity (AdS/CFT correspondence).

As a final remark let us note that it would be very interesting to generalize our scenario to the case where the branes are curved (but with no changes in the bulk space). It could perhaps result in the construction of a new inflationary brane-world at nonzero temperature where thermal quantum effects generate not only inflation (as in case of the nonthermal scenario of Refs. [3,4]) but also a hierarchy of scales corresponding to a physically reasonable hot universe.

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