Physics 6433, Quantum Field Theory
Assignment #9
Due Monday, November 16, 2009

November 6, 2009

1. Compute the $\mathcal{O}(1/\epsilon)$ terms in $K(p)$ and $K_\mu(p)$ given in the notes, and thereby establish that

$$\Sigma^{(2a)}(0) = -\frac{3\lambda^2}{8\pi^4}\left[\frac{6m^2}{\epsilon^2} + \frac{6m^2}{\epsilon}\left(\frac{3}{2} + \psi(1) + \ln\frac{4\pi\mu^2}{m^2}\right) + \mathcal{O}(1)\right].$$

2. For $m = 0$ evaluate $K(p)$ and $K_\mu(p)$ through $\mathcal{O}(1)$, and thereby find the finite part of $\Sigma^{(2a)}(p)$ when $m = 0$.

3. Compute $\Sigma^{(2a)}(p)$ on the mass shell, $p^2 = -m^2$. The answer is

$$\begin{aligned}\Sigma^{(2a)}(p)\Big|_{p^2=-m^2} &= -96\hat{\lambda}^2 m^2\left\{\frac{6}{\epsilon^2} + \frac{1}{\epsilon}\left(\frac{17}{2} + 6\psi(1) - 6\ln^2\hat{m}^2\right)\right.\\ &\quad + \frac{71}{8} + \frac{\pi^2}{4} + \frac{17}{2}\left[\psi(1) - \ln\hat{m}^2\right] + 3\psi^2(1)\\ &\quad \left. + 3\ln^2\hat{m}^2 - 6\psi(1)\ln\hat{m}^2\right\},\end{aligned}$$

where

$$\hat{\lambda} = \frac{\lambda}{(4\pi)^2}, \quad \hat{m}^2 = \frac{m^2}{4\pi\mu^2}.$$