Physics 6433, Quantum Field Theory
Assignment #8
Due Friday, November 6, 2009

October 30, 2009

1. Do the integral (4.26) given in class.

2. Using the proper-time representation for the propagation function, derive the following dimensional regularization formulæ:

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2 + M^2 + 2l \cdot p)^\alpha} = \frac{\Gamma(\alpha - d/2)}{(4\pi)^{d/2}\Gamma(\alpha)} \frac{1}{(M^2 - p^2)^{\alpha - d/2}},$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_\mu}{(l^2 + M^2 + 2l \cdot p)^\alpha} = -\frac{\Gamma(\alpha - d/2)}{(4\pi)^{d/2}\Gamma(\alpha)} \frac{p_\mu}{(M^2 - p^2)^{\alpha - d/2}},$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_\mu l_\nu}{(l^2 + M^2 + 2l \cdot p)^\alpha} = \frac{1}{(4\pi)^{d/2}\Gamma(\alpha)} \Big[p_\mu p_\nu \frac{\Gamma(\alpha - d/2)}{(M^2 - p^2)^{\alpha - d/2}} + \frac{1}{2}\delta_{\mu\nu} \frac{\Gamma(\alpha - 1 - d/2)}{(M^2 - p^2)^{\alpha - 1 - d/2}} \Big],$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_\mu l_\nu l_\rho}{(l^2 + M^2 + 2l \cdot p)^\alpha} = -\frac{1}{(4\pi)^{d/2}\Gamma(\alpha)} \Big[p_\mu p_\nu p_\rho \frac{\Gamma(\alpha - d/2)}{(M^2 - p^2)^{\alpha - d/2}} + \frac{1}{2}(\delta_{\mu\nu} p_\rho + \delta_{\mu\rho} p_\nu + \delta_{\nu\rho} p_\mu) \frac{\Gamma(\alpha - 1 - d/2)}{(M^2 - p^2)^{\alpha - 1 - d/2}} \Big],$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_\mu l_\nu l_\rho l_\sigma}{(l^2 + M^2 + 2l \cdot p)^\alpha} = \frac{1}{(4\pi)^{d/2}\Gamma(\alpha)} \Big[p_\mu p_\nu p_\rho p_\sigma \frac{\Gamma(\alpha - d/2)}{(M^2 - p^2)^{\alpha - d/2}} + \frac{1}{2}(\delta_{\mu\nu} p_\rho p_\sigma + \delta_{\nu\sigma} p_\mu p_\rho + \delta_{\rho\sigma} p_\mu p_\nu + \delta_{\mu\rho} p_\nu p_\sigma + \delta_{\nu\rho} p_\mu p_\sigma + \delta_{\mu\sigma} p_\nu p_\rho) \times \frac{\Gamma(\alpha - 1 - d/2)}{(M^2 - p^2)^{\alpha - 1 - d/2}} + \frac{1}{4}(\delta_{\mu\nu}\delta_{\sigma\rho} + \delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho}) \frac{\Gamma(\alpha - 2 - d/2)}{(M^2 - p^2)^{\alpha - 2 - d/2}}.$$

3. The "Spence function" or dilogarithm occurs frequently in the evaluation of two-loop Feynman diagrams. It is defined by for real x

$$f(x) = -\int_0^x \frac{dt}{t} \ln|1-t|.$$

Establish the identities

$$f(x) + f(1-x) + \ln|x| \ln|1-x| = \frac{\pi^2}{6},$$
$$f(x) + f(1/x) + \frac{1}{2}\ln^2|x| = \begin{cases} \frac{1}{3}\pi^2, & x > 0, \\ -\frac{1}{6}\pi^2, & x < 0, \end{cases}$$

and determine the specific values

$$f(1) = -2f(-1) = \frac{2}{3}f(2) = 2f(1/2) + \ln^2 2 = \frac{1}{6}\pi^2.$$