

Physics 6433, Quantum Field Theory  
Assignment #8  
Due Monday, April 13, 2015

April 1, 2015

1. Compute the  $\mathcal{O}(1/\epsilon)$  terms in  $K(p)$  and  $K_\mu(p)$  given in the notes, and thereby establish that

$$\Sigma^{(2a)}(0) = -\frac{3\lambda^2}{8\pi^4} \left[ \frac{6m^2}{\epsilon^2} + \frac{6m^2}{\epsilon} \left( \frac{3}{2} + \psi(1) + \ln \frac{4\pi\mu^2}{m^2} \right) + \mathcal{O}(1) \right].$$

2. For  $m = 0$  evaluate  $K(p)$  and  $K_\mu(p)$  through  $\mathcal{O}(1)$ , and thereby find the finite part of  $\Sigma^{(2a)}(p)$  when  $m = 0$ .
3. Compute  $\Sigma^{(2a)}(p)$  on the mass shell,  $p^2 = -m^2$ . The answer is

$$\begin{aligned} \Sigma^{(2a)}(p) \Big|_{p^2=-m^2} = & -96\hat{\lambda}^2 m^2 \left\{ \frac{6}{\epsilon^2} + \frac{1}{\epsilon} \left( \frac{17}{2} + 6\psi(1) - 6\ln^2 \hat{m}^2 \right) \right. \\ & + \frac{71}{8} + \frac{\pi^2}{4} + \frac{17}{2} [\psi(1) - \ln \hat{m}^2] + 3\psi^2(1) \\ & \left. + 3\ln^2 \hat{m}^2 - 6\psi(1) \ln \hat{m}^2 \right\}, \end{aligned}$$

where

$$\hat{\lambda} = \frac{\lambda}{(4\pi)^2}, \quad \hat{m}^2 = \frac{m^2}{4\pi\mu^2}.$$