

Physics 6433, Quantum Field Theory
Assignment #7
Due Wednesday, April 1, 2015

March 25, 2015

1. Do the integral (4.26) given in class.
2. Using the proper-time representation for the propagation function, derive the following dimensional regularization formulæ:

$$\begin{aligned}
 \int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2 + M^2 + 2l \cdot p)^\alpha} &= \frac{\Gamma(\alpha - d/2)}{(4\pi)^{d/2} \Gamma(\alpha)} \frac{1}{(M^2 - p^2)^{\alpha - d/2}}, \\
 \int \frac{d^d l}{(2\pi)^d} \frac{l_\mu}{(l^2 + M^2 + 2l \cdot p)^\alpha} &= -\frac{\Gamma(\alpha - d/2)}{(4\pi)^{d/2} \Gamma(\alpha)} \frac{p_\mu}{(M^2 - p^2)^{\alpha - d/2}}, \\
 \int \frac{d^d l}{(2\pi)^d} \frac{l_\mu l_\nu}{(l^2 + M^2 + 2l \cdot p)^\alpha} &= \frac{1}{(4\pi)^{d/2} \Gamma(\alpha)} \left[p_\mu p_\nu \frac{\Gamma(\alpha - d/2)}{(M^2 - p^2)^{\alpha - d/2}} \right. \\
 &\quad \left. + \frac{1}{2} \delta_{\mu\nu} \frac{\Gamma(\alpha - 1 - d/2)}{(M^2 - p^2)^{\alpha - 1 - d/2}} \right], \\
 \int \frac{d^d l}{(2\pi)^d} \frac{l_\mu l_\nu l_\rho}{(l^2 + M^2 + 2l \cdot p)^\alpha} &= -\frac{1}{(4\pi)^{d/2} \Gamma(\alpha)} \left[p_\mu p_\nu p_\rho \frac{\Gamma(\alpha - d/2)}{(M^2 - p^2)^{\alpha - d/2}} \right. \\
 &\quad \left. + \frac{1}{2} (\delta_{\mu\nu} p_\rho + \delta_{\mu\rho} p_\nu + \delta_{\nu\rho} p_\mu) \frac{\Gamma(\alpha - 1 - d/2)}{(M^2 - p^2)^{\alpha - 1 - d/2}} \right], \\
 \int \frac{d^d l}{(2\pi)^d} \frac{l_\mu l_\nu l_\rho l_\sigma}{(l^2 + M^2 + 2l \cdot p)^\alpha} &= \frac{1}{(4\pi)^{d/2} \Gamma(\alpha)} \left[p_\mu p_\nu p_\rho p_\sigma \frac{\Gamma(\alpha - d/2)}{(M^2 - p^2)^{\alpha - d/2}} \right. \\
 &\quad + \frac{1}{2} (\delta_{\mu\nu} p_\rho p_\sigma + \delta_{\nu\sigma} p_\mu p_\rho + \delta_{\rho\sigma} p_\mu p_\nu + \delta_{\mu\rho} p_\nu p_\sigma + \delta_{\nu\rho} p_\mu p_\sigma + \delta_{\mu\sigma} p_\nu p_\rho) \\
 &\quad \times \frac{\Gamma(\alpha - 1 - d/2)}{(M^2 - p^2)^{\alpha - 1 - d/2}} + \frac{1}{4} (\delta_{\mu\nu} \delta_{\rho\sigma} + \delta_{\mu\rho} \delta_{\nu\sigma} + \delta_{\mu\sigma} \delta_{\nu\rho}) \frac{\Gamma(\alpha - 2 - d/2)}{(M^2 - p^2)^{\alpha - 2 - d/2}}.
 \end{aligned}$$

3. The “Spence function” or dilogarithm occurs frequently in the evaluation of two-loop Feynman diagrams. It is defined by for real x

$$f(x) = - \int_0^x \frac{dt}{t} \ln |1 - t|.$$

Establish the identities

$$\begin{aligned} f(x) + f(1-x) + \ln |x| \ln |1-x| &= \frac{\pi^2}{6}, \\ f(x) + f(1/x) + \frac{1}{2} \ln^2 |x| &= \begin{cases} \frac{1}{3} \pi^2, & x > 0, \\ -\frac{1}{6} \pi^2, & x < 0, \end{cases} \end{aligned}$$

and determine the specific values

$$f(1) = -2f(-1) = \frac{2}{3}f(2) = 2f(1/2) + \ln^2 2 = \frac{1}{6}\pi^2.$$