Physics 6433, Quantum Field Theory
Assignment #5
Due Monday, October 12, 2009

October 5, 2009

1. Show that for a system with a single degree of freedom, if $t_2 < t', t'' < t_1$,

$$\int [dq][dp]\, q(t')q(t'') \exp\left[i\int_{t_2}^{t_1} dt\,(p\dot{q} - H)\right] = \langle q', t_1 | T(q(t')q(t''))|q'', t_2\rangle,$$

where T means the time-ordered product of the two position operators,

$$T(q(t')q(t'')) = \begin{cases} q(t')q(t'') & \text{if } t' > t'', \\ q(t'')q(t) & \text{if } t'' > t'. \end{cases}$$

Here $\langle q', t_1 | q(t_1) = q' \langle q', t_1|$, $q(t_2)|q'', t_2\rangle = q''|q'', t_2\rangle$.

2. By transforming to Euclidean time, $\tau = it$, show that we obtain the following form for the ground-state persistence amplitude for the harmonic oscillator:

$$Z_E[F] = \int [dq] \exp\left\{-\int d\tau \left[\frac{1}{2}\left(\frac{dq}{d\tau}\right)^2 + \frac{1}{2}\omega^2 q^2 - Fq\right]\right\}.$$

As in class, carry out the functional integral, and find

$$Z_E[F] = Z_E[0] \exp\left[-\frac{1}{2}\int d\tau\, d\sigma\, F(\tau) G_E(\tau - \sigma) F(\sigma)\right],$$

and give the form of the Euclidean Green's function G_E. Then use analytic continuation to obtain the G found in class.

3. As in the calculation for the Hamiltonian given in class, obtain the expression for the linear momentum operator $\mathbf{P}$ in terms of creation and annihilation operators $a_k^\dagger$ and a_k, for a free scalar field. What is the momentum of the vacuum state, the lowest energy state of the system?

4. The time-ordered product of two fields is defined by ($x^0 = t$, $x'^0 = t'$)

$$T(\phi(x)\phi(x')) = \eta(t-t')\phi(x)\phi(x') + \eta(t'-t)\phi(x')\phi(x).$$

   Show by direct differentiation that

$$(-\partial^2 + m^2)G(x-x') = \delta(x-x'),$$

   where

$$G(x-x') = i\langle 0|T(\phi(x)\phi(x'))|0\rangle.$$

   By using the decomposition of the free field ϕ into creation and annihilation operators, show that $G(x-x') = \Delta_+(x-x')$.