Physics 6433, Quantum Field Theory
Assignment #5
Due Monday, March 2, 2015

February 24, 2015

1. As in the calculation for the Hamiltonian given in class, obtain the expression for the linear momentum operator $\mathbf{P}$ in terms of creation and annihilation operators $a_k^\dagger$ and a_k, for a free scalar field. What is the momentum of the vacuum state, the lowest energy state of the system?

2. The time-ordered product of two fields is defined by ($x^0 = t$, $x'^0 = t'$)
$$T(\phi(x)\phi(x')) = \eta(t-t')\phi(x)\phi(x') + \eta(t'-t)\phi(x')\phi(x).$$
Show by direct differentiation that
$$(-\partial^2 + m^2)G(x-x') = \delta(x-x'),$$
where
$$G(x-x') = i\langle 0|T(\phi(x)\phi(x'))|0\rangle.$$
By using the decomposition of the free field ϕ into creation and annihilation operators, show that $G(x-x') = \Delta_+(x-x')$.

3. Derive the following form for the Minkowski-space propagation function in the vicinity of the light-cone
$$m^2|(x-x')^2| \ll 1: \quad \Delta_+(x-x') \approx \frac{i}{4\pi^2}\frac{1}{(x-x')^2 + i\epsilon}$$
$$= \frac{i}{4\pi^2}P\frac{1}{(x-x')^2} + \frac{1}{4\pi}\delta((x-x')^2),$$
where P stands for the Cauchy principal value.

4. Derive the following asymptotic forms for the Minkowski-space propagation function for large spacelike, and large timelike intervals, respectively,

$$R = [(x-x')^2]^{1/2}, mR \gg 1 : \quad \Delta_+(x-x') \approx i\frac{(2m)^{1/2}}{(4\pi R)^{3/2}}e^{-mR},$$

$$T = [-(x-x')^2]^{1/2}, mT \gg 1 : \quad \Delta_+(x-x') \approx e^{-i\pi/4}\frac{(2m)^{1/2}}{(4\pi T)^{3/2}}e^{-imT}.$$

Discuss the correspondence between the Euclidean and Minkowskian propagation functions in these limits.