Physics 6433, Quantum Field Theory Assignment #3 Due Monday, February 16, 2015

February 10, 2015

- 1. How is the calculation of the transformation function for the harmonic oscillator, $\langle a^{\dagger\prime}, t_1 | a^{\prime\prime}, t_2 \rangle$, given in class, modified when the zero-point energy, $H_0 = \frac{1}{2}\omega$, is included in the Hamiltonian? What is the effect on the energy eigenvalues and the energy eigenfunctions?
- 2. From the relation between annihilation eigenvectors and energy eigenvectors,

$$|a'\rangle = \sum_{n=0}^{\infty} \frac{(a')^n}{\sqrt{n!}} |n\rangle,$$
$$\langle a^{\dagger\prime}| = \sum_{n=0}^{\infty} \langle n| \frac{(a^{\dagger\prime})^n}{\sqrt{n!}},$$

deduce a closed form for $\langle a^{\dagger\prime} | a^{\prime\prime} \rangle$ analogous to

$$\langle p'|q'\rangle = \frac{1}{\sqrt{2\pi}}e^{-ip'q'}$$

3. Using

$$G_q - G_p = \delta\left(\sum_a q_a \cdot p_a\right),$$

write $\delta \langle q' | p' \rangle$, and evaluate $\langle q' | p' \rangle$.