Physics 6433, Quantum Field Theory
Assignment #2
Due Monday, February 4, 2002

January 28, 2002

1. Verify that the coordinate transformation law for the field strength tensor,
$$\delta F_{\mu\nu} = -\delta x^\lambda \partial_\lambda F_{\mu\nu} - (\partial_\mu \delta x^\lambda) F_{\lambda\nu} - (\partial_\nu \delta x^\lambda) F_{\mu\lambda},$$
is consistent with that for the vector potential
$$\delta A_\mu = -\delta x^\lambda \partial_\lambda A_\mu - (\partial_\mu \delta x^\lambda) A_\lambda.$$

2. Using the free Maxwell equations, verify directly the energy-momentum conservation equation,
$$\partial_\mu t^{\mu\nu} = 0.$$
How is this equation modified if a charge density j_μ is present?

3. A scalar particle of mass m is described by a field $\phi(x)$ and is governed by the Lagrange density
$$\mathcal{L} = -\frac{1}{2}\left(\partial^\mu \phi \partial_\mu \phi + m^2 \phi^2\right).$$
Under an infinitesimal coordinate transformation, $\phi(x)$ transforms according to
$$\delta\phi(x) = -\delta x^\nu(x) \partial_\nu \phi(x).$$
As in the lecture, compute the corresponding energy-momentum tensor $t^{\mu\nu}$.

4. Scale invariance is violated by the energy-momentum tensor computed in the above problem, because, as $m \to 0$, $t = t^{\mu}{}_{\mu} \not\to 0$. To what limit does t converge? We can, however, introduce a new energy-momentum tensor, $\theta^{\mu\nu}$, by adding to $t^{\mu\nu}$ an identically conserved term,

$$\theta^{\mu\nu} = t^{\mu\nu} + a\left(\partial^{\mu}\partial^{\nu} - g^{\mu\nu}\partial^{2}\right)\phi^{2}.$$

Determine the number a so that $\theta = \theta^{\mu}{}_{\mu} \to 0$ as $m \to 0$. This is the so-called conformal stress tensor. Argue that $\theta^{\mu\nu}$ and $t^{\mu\nu}$ are equally as good at describing the energy and momentum of the system. [There is an inherent arbitrariness in defining the energy-momentum tensor.]