Physics 6433, Quantum Field Theory
Assignment #13
Due Friday, December 18, 2009

December 3, 2009

1. To establish the unitarity of the free photon description, given by the vacuum persistence amplitude expression

$$\langle 0_+|0_-\rangle_0^J = \exp\left[\frac{i}{2}\int(dx)(dx')\,J^\mu(x)D_+(x-x')J_\mu(x')\right]$$

supplemented by the constraint

$$\partial^\mu J_\mu = 0,$$

proceed as follows. For a massless particle, p^μ is a null vector,

$$p^2 = 0.$$

Define a second null vector from the components of $p^\mu = (p^0, \mathbf{p})$:

$$\overline{p}^\mu = (p^0, -\mathbf{p}), \quad \overline{p}^2 = 0.$$

Show that $p + \overline{p}$ is a time-like vector, while $p - \overline{p}$ is space-like. Let the two remaining space-like directions be spanned by the orthonormal vectors e^μ_{p1}, e^μ_{p2}:

$$\begin{aligned} g^{\mu\nu} &= \frac{(p+\overline{p})^\mu(p+\overline{p})^\nu}{(p+\overline{p})^2} + \frac{(p-\overline{p})^\mu(p-\overline{p})^\nu}{(p-\overline{p})^2} + \sum_{\lambda=1}^{2} e^\mu_{p\lambda}e^{\nu *}_{p\lambda} \\ &= \frac{p^\mu\overline{p}^\nu + \overline{p}^\mu p^\nu}{(p\overline{p})} + \sum_{\lambda=1}^{2} e^\mu_{p\lambda}e^{\nu *}_{p\lambda}. \end{aligned}$$

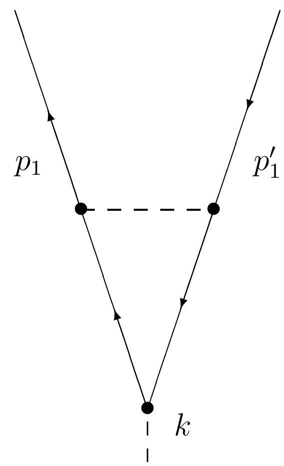

Figure 1: Diagram giving rise to the anomalous magnetic moment of the electron. Solid lines stand for electrons (or positrons), while dashed lines stand for photons.

Then show that

$$\int d\tilde{p}\, J^\mu(-p) J_\mu(p) = \sum_{\mathbf{p}\lambda} |J_{\mathbf{p}\lambda}|^2,$$

where

$$J_{\mathbf{p}\lambda} = \sqrt{d\tilde{p}}\, e^{\mu *}_{p\lambda} J_\mu(p),$$

and from this show that

$$|\langle 0_+ | 0_- \rangle_0^J|^2 \leq 1.$$

2. Consider electron-electron scattering (like charges). Draw the relevant Feynman diagrams to order e^2 and write down the corresponding amplitudes.

3. Compute the anomalous magnetic moment of the electron from the diagram in Fig. 1. The external electrons are on the mass shell, so that on the left $m + \gamma p_1 \to 0$, while on the right $m - \gamma p_1' \to 0$. Pick out the magnetic moment term, using the identity

$$(p_1 - p_1')^\mu = 2m\gamma^\mu + i\sigma^{\mu\nu} k_\nu$$

valid when sandwiched between on-shell spinors. (Prove this lemma.) Use dimensional regularization to evaluate the integrals. Content youself with finding the magnetic moment at $k^2 = 0$.