Physics 6433, Quantum Field Theory
Assignment #11
Due Wednesday, May 1, 2002

April 24, 2002

1. Verify the form found in class for the two-point function using the renormalization group ($g = 4!\lambda/(4\pi)^2$):

$$\Gamma^{(2)}(p) = -p^2 - m^2 + \frac{1}{2} g m^2 \left[\psi(2) - \ln \hat{m}^2\right] + g^2 m^2 f\left(\frac{m^2}{\mu^2}, \frac{p^2}{m^2}\right),$$

with

$$f\left(\frac{m^2}{\mu^2}, \frac{p^2}{m^2}\right) = -\frac{1}{2}\ln^2 \hat{m}^2 + \left[\psi(2) + \frac{1}{4} + \frac{p^2}{12m^2}\right]\ln \hat{m}^2 + g\left(\frac{p^2}{m^2}\right),$$

by computing f from the diagrams for $\Sigma(p)$ through second order in g, using the minimal subtraction scheme of 't Hooft and Weinberg.

2. Verify that the engineering dimension of $\Gamma^{(n)}(p_1, \ldots, p_n)$ is $4-n+\frac{\epsilon}{2}(n-2)$.

3. Calculate the lowest-order renormalization-group coefficients β, γ, and δ in the renormalization scheme based on "physical renormalization" points, that is, with the counterterms chosen so that the pole of $G^{(2)}$ is at $p^2 = -m_R^2$, and this pole has residue 1, while the renormalized coupling constant is defined by $\lambda_R = -\frac{1}{4!}\Gamma_R^{(4)}|_{p_i=0}$, [Hint: expressions for F_1 and G_1 were given in class for this case.]

4. Show that the sign of $d\beta/d\lambda$ at a fixed point is renormalization-scheme independent, if masses are neglected.