Physics 6433, Quantum Field Theory Assignment #11 Due Friday, May 8, by 5pm without fail!!

April 28, 2015

1. To establish the unitarity of the free photon description, given by the vacuum persistence amplitude expression

$$\langle 0_+|0_-\rangle_0^J = \exp\left[\frac{i}{2}\int (dx)(dx') J^\mu(x)D_+(x-x')J_\mu(x')\right]$$

supplemented by the constraint

$$\partial^{\mu}J_{\mu} = 0,$$

proceed as follows. For a massless particle, p^{μ} is a null vector,

$$p^2 = 0.$$

Define a second null vector from the components of $p^{\mu} = (p^0, \mathbf{p})$:

$$\overline{p}^{\mu} = (p^0, -\mathbf{p}), \quad \overline{p}^2 = 0.$$

Show that $p + \overline{p}$ is a time-like vector, while $p - \overline{p}$ is space-like. Let the two remaining space-like directions be spanned by the orthonormal vectors e_{p1}^{μ} , e_{p2}^{μ} :

$$g^{\mu\nu} = \frac{(p+\overline{p})^{\mu}(p+\overline{p})^{\nu}}{(p+\overline{p})^{2}} + \frac{(p-\overline{p})^{\mu}(p-\overline{p})^{\nu}}{(p-\overline{p})^{2}} + \sum_{\lambda=1}^{2} e^{\mu}_{p\lambda} e^{\nu*}_{p\lambda}$$
$$= \frac{p^{\mu}\overline{p}^{\nu} + \overline{p}^{\mu}p^{\nu}}{(p\overline{p})} + \sum_{\lambda=1}^{2} e^{\mu}_{p\lambda} e^{\nu*}_{p\lambda}.$$



Figure 1: Diagram giving rise to the anomalous magnetic moment of the electron. Solid lines stand for electrons (or positrons), while dashed lines stand for photons.

Then show that

$$\int d\tilde{p} J^{\mu}(-p) J_{\mu}(p) = \sum_{\mathbf{p}\lambda} |J_{\mathbf{p}\lambda}|^2,$$

where

$$J_{\mathbf{p}\lambda} = \sqrt{d\tilde{p}} \, e_{p\lambda}^{\mu*} J_{\mu}(p),$$

and from this show that

$$|\langle 0_+|0_-\rangle_0^J|^2 \le 1.$$

- 2. Consider electron-electron scattering (like charges). Draw the relevant Feynman diagrams to order e^2 and write down the corresponding amplitudes.
- 3. Compute the anomalous magnetic moment of the electron from the diagram in Fig. 1. The external electrons are on the mass shell, so that on the left $m + \gamma p_1 \rightarrow 0$, while on the right $m \gamma p'_1 \rightarrow 0$. Pick out the magnetic moment term, using the identity

$$(p_1 - p_1')^{\mu} = 2m\gamma^{\mu} + i\sigma^{\mu\nu}k_{\nu}$$

valid when sandwiched between on-shell spinors. (Prove this lemma.) Use dimensional regularization to evaluate the integrals. Content youself with finding the magnetic moment at $k^2 = 0$.