August 22, 2011


1. Prove that
\[
\lim_{n \to \infty} (\sqrt{n+1} - \sqrt{n})(\sqrt{n+\frac{1}{2}}) = \frac{1}{2}.
\]

2. Let \( a_n = \frac{10^n}{n!} \). (a) To what limit does \( a_n \) converge as \( n \to \infty \)? (b) Is the sequence monotonic? (c) Is it monotonic from a certain \( n \) onwards? (d) Give an estimate of the difference between \( a_n \) and the limit. (e) From what value of \( n \) onwards is this difference less than \( \frac{1}{100} \)?

3. Prove that the sequence \( \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots \), converges. Find its limit.

4. For what real values of \( \alpha \) does
\[
\sum_{n=1}^{\infty} \frac{n!}{(1+\alpha)(2+\alpha)\cdots(n+\alpha)}
\]
converge?

5. Test for convergence:

(a)
\[
\sum_{n=1}^{\infty} \frac{1}{n(n+1)}
\]
(b) \[ \sum_{n=1}^{\infty} \log(1 + \frac{1}{n}) \]

(c) \[ \sum_{n=2}^{\infty} \frac{1}{n \log n} \]

(d) \[ \sum_{n=1}^{\infty} \frac{1}{n n^{1/n}} \]

(e) \[ \sum_{n=1}^{\infty} \frac{1}{n 2^n} \]

6. For what real values of \( p \) and \( q \) will the following series converge?

\[ \sum_{n=2}^{\infty} \frac{1}{n^p (\ln n)^q} \]