Physics 5970. Homework 9

Due Friday, May 13, at 5pm

1. Show that the SU(2) field strength transforms under a finite gauge transformation according to

$$F^a_{\mu\nu} \to (F^U_{\mu\nu})^a = \frac{1}{2}\mathrm{Tr}\,[\tau_a U \tau_b U^\dagger] F^b_{\mu\nu},$$

   and that hence $-\frac{1}{4}\mathbf{F}_{\mu\nu}\cdot\mathbf{F}^{\mu\nu}$ is gauge invariant because

$$\sum_{a=1}^{3}(\tau_a)_{ij}(\tau_a)_{kl} = 2(\delta_{il}\delta_{kj} - \frac{1}{2}\delta_{ij}\delta_{kl}).$$

   Prove this last statement.

2. Prove that

$$(\mathcal{D}_\mu)_{ab} = \delta_{ab}\partial_\mu + g\epsilon_{abc}A^c_\mu$$

   is the gauge covariant derivative for the spin-1 (adjoint) representation of SU(2), by showing that

$$\mathcal{D}_\mu \to U\mathcal{D}_\mu U^\dagger$$

   under a gauge transformation. It may help to first prove that if ω is in the spin-1 representation, $\omega = \omega^a\tau^a/2$, that

$$\mathcal{D}_\mu\omega = \partial_\mu\omega + ig[A_\mu, \omega].$$

3. Calculate the structure constants f_{abc} for SU(3). Also calculate d_{abc} defined by

$$\{\lambda_a, \lambda_b\} = \frac{4}{3}\delta_{ab} + 2d_{abc}\lambda_c.$$