## Homework 8 Physics 5970, Advanced Quantum Field Theory

## Due April 25, 2005

1. By using the modified photon propagation function derived in class, show how the Coulomb potential between static charges is modified. In particular, show that at short distance the modified potential is

$$2mr \ll 1$$
:  $\mathcal{D}(r) = \frac{1}{4\pi r} \left[ 1 + \frac{2\alpha}{3\pi} \left( \ln \frac{1}{mr} - C - \frac{5}{6} \right) \right],$ 

where C = 0.57721... is Euler's constant.

- 2. Apply the above modification to the hydrogen atom, and calculate the change in the s-state energy levels using first order perturbation theory.
- 3. it was stated in class that the principal origin of the anomalous magnetic moments of the electron and muon,

$$\frac{g_{\mu} - g_2}{2} = 6.26 \times 10^{-6},$$

is due to the vacuum polarization due to the electron in the muonic mass operator. Estimate the size of this effect, and then do the calculation. You should obtain the result

$$\frac{g_{\mu} - g_2}{2} = \frac{\alpha}{2\pi} \frac{2\alpha}{3\pi} \left( \ln \frac{m_{\mu}}{m_e} - \frac{25}{12} + \frac{3}{4} \pi^2 \frac{m_e}{m_{\mu}} \right).$$

Put in the numbers, and verify that this accounts for more than 90% of the discrepancy.