Physics 5970. Homework 7
Advanced Quantum Field Theory

Due Monday, April 18, 2005

1. Consider electron-electron scattering (like charges). Draw the relevant Feynman diagrams to order e^2 and write down the corresponding amplitudes.

2. Using the result of Problem 1, compute the amplitude in the center-of-mass frame for equal initial helicities $\sigma_2 = \sigma_{2'}$ in the high energy limit. That is, if M is the center of mass energy, let $M \gg m$, m being the electron mass.

3. To find the corresponding cross section, square this amplitude, divide by the incident flux, and integrate over the momentum-conserving δ function. The relative flux for two particles is given by

$$
\begin{aligned}
F &= d\tilde{p}_a d\tilde{p}_b 4[(p_a p_b)^2 - m_a^2 m_b^2]^{1/2} \\
&= 2 d\tilde{p}_a d\tilde{p}_b [M^2 - (m_a + m_b)^2]^{1/2} [M^2 - (m_a - m_b)^2]^{1/2}.
\end{aligned}
$$

   Show that the integration over the momentum-conserving δ function involves the same expression:

$$
\begin{aligned}
&\int d\tilde{p}_a d\tilde{p}_b (2\pi)^4 \delta(p_a + p_b - P) \\
&= \frac{d\Omega}{32\pi^2} \frac{1}{M^2} [M^2 - (m_a + m_b)^2]^{1/2} [M^2 - (m_a - m_b)^2]^{1/2},
\end{aligned}
$$

   where $d\Omega$ is the element of solid angle into which one of the particles is scattered. Show that the result for the differential cross section is

$$
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{M^2} \left(\frac{1}{\sin^2 \frac{1}{2}\theta} + \frac{1}{\cos^2 \frac{1}{2}\theta} \right)^2, \quad \sigma_2 = \sigma_2', \quad M \gg m.
$$

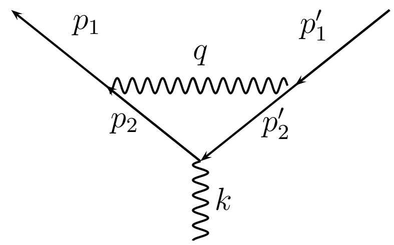

Figure 1: Feynman diagram giving rise to the electric and magnetic form factors of the electron in order α.

4. Similarly, compute the cross section in the high energy limit for opposite initial helicities, $\sigma_2 = -\sigma_2'$. Average these two results, and obtain the differential cross section for an *unpolarized* initial beam in the high energy limit.

5. Compute the anomalous magnetic moment of the electron from the diagram in the Figure. The external electrons are on the mass shell, so that on the left $m + \gamma p_1 \to 0$, while on the right $m - \gamma p_1' \to 0$. Pick out the magnetic moment term, using the identity

$$(p_1 - p_1')^\mu = 2m\gamma^\mu + i\sigma^{\mu\nu}k_\nu$$

valid when sandwiched between on-shell spinors. (Prove this lemma.) Content youself with finding the magnetic moment at $k^2 = 0$.

Hints:

- The Feynman rules for Fermions include the following
  - For an incoming electron, use the factor $\sqrt{2m\,d\tilde{p}}\,\overline{u}_{p\sigma}$, and for an outgoing electron the factor $\sqrt{2m\,d\tilde{p}}\,u_{p\sigma}$. For a positron, $u_{p\sigma} \to u^*_{p\sigma}$, which satisfies $(m - \gamma p)u^*_{p\sigma} = 0$.
  - Because $\eta(p_1)\eta(p_2) = -\eta(p_2)\eta(p_1)$, when external Fermion lines are crossed, there is a change of sign. (For the same reason, there is a -1 factor associated with each closed Fermion loop.

- A useful identity, which you should prove, is the following for arbitrary 2×2 matrices X and Y:

$$\mathrm{Tr}(\sigma_\mu X)\mathrm{Tr}(\sigma^\mu Y) = 2\left[\mathrm{Tr}\,XY - (\mathrm{Tr}\,X)(\mathrm{Tr}\,Y)\right],$$

where $\sigma^\mu = (1, \boldsymbol{\sigma})$.

- Use

$$\begin{aligned} u_{p\sigma} &= \frac{1}{\sqrt{2m}} \left[\sqrt{E+m} + \sqrt{E-m}\, i\gamma_5 \sigma \right] v_\sigma \\ &\approx \sqrt{\frac{E}{2m}} (1 + i\gamma_5 \sigma) v_\sigma, \end{aligned}$$

  where the latter holds if $E = M/2 \gg m$.