Physics 5970
Advanced Quantum Field Theory
Homework 5

Due Friday, March 25, 2005

1. A way to regulate divergent Feynman integrals is so-called dimensional regularization due to 't Hooft and Veltman. (Nobel Prize, 1999). For simplicity we work in Euclidean space. The idea is illustrated by the following. (Fill in the steps.) To define

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2+m^2)^{n+1}}$$

let the dimension 4 be replaced by a continuous variable d, and show that the denominator can be written in so-called proper-time form:

$$\frac{1}{(l^2+m^2)^{n+1}} = \frac{1}{\Gamma(n+1)} \int_0^\infty ds\, s^n e^{-s(l^2+m^2)}.$$

Then do the l integration before the s integration and show that

$$\int \frac{d^d l}{(2\pi)^d} e^{-sl^2} = \frac{1}{2^d \pi^{d/2} s^{d/2}}.$$

Finally, perform the s integral by using the definition of the gamma function,

$$\int_0^\infty \frac{dt}{t} t^\alpha e^{-t} = \Gamma(\alpha).$$

The result is

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2+m^2)^{n+1}} = \frac{m^{d-2n-2}}{2^d \pi^{d/2}} \frac{\Gamma(n+1-d/2)}{\Gamma(n+1)}.$$

2. Use the result of Problem 1 to show that

    (a) a quadratically divergent diagram has, when dimensionally regularized, poles at $d = 2, 4, 6, \ldots$,

    (b) while a logarithmically divergent one has poles at $d = 4, 6, \ldots$.

3. Use dimensional regularization to evaluate the amputated diagram (that is, remove the external propagators), shown in the figure. When

    

    we continue in dimension space, we want the coupling constant λ to remain dimensionless, so let

    $$\lambda \to \lambda \mu^{4-d},$$

    where μ is an arbitrary mass scale. Express your answer in terms of $\epsilon = d - 4$, and assume ϵ is small. The answer will involve the digamma function defined by

    $$\psi(x) = \frac{d}{dx} \ln \Gamma(x).$$

    Determine, therefore, the mass counterterm δm^2 to order λ.

4. Work out directly, using the residue theorem, the finite integral

    $$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 + m^2 - i\epsilon)^3}$$

    in Minkowski space using the residue theorem, and compare with the result given by dimensional regularization in Problem 1. How are the results related?