Physics 5970, Advanced Quantum Field Theory
Homework 2

Due Wednesday, February 16, 2005

February 7, 2005

1. Problem 5 from Assignment 1.

2. Using the construction of $u_{p\sigma}$ in terms of v_σ prove that

$$\frac{m - \gamma p}{2m} = \sum_\sigma u_{p\sigma} u^\dagger_{p\sigma} \gamma^0$$

and

$$u^\dagger_{p\sigma} \gamma^0 \gamma^\mu u_{p\sigma'} = \delta_{\sigma\sigma'} \frac{p^\mu}{m}.$$

3. Using the properties of the γ^μ matrices prove that $\gamma_5 = \gamma^0\gamma^1\gamma^2\gamma^3$ has the properties

$$(i\gamma_5)^2 = 1, \quad \{\gamma_5, \gamma_\mu\} = 0, \quad \gamma_5^\dagger = -\gamma_5.$$

4. The rest-frame spinors satisfy

$$\Sigma_3 v_\sigma = \sigma v_\sigma, \quad \sigma = \pm 1,$$

$$\gamma^0 v_\sigma = v_\sigma,$$

$$\frac{1}{2}(\Sigma_1 + i\Sigma_2) v_- = v_+, \quad \frac{1}{2}(\Sigma_1 - i\Sigma_2) v_+ = v_-,$$

while

$$\Sigma_3 v^*_\sigma = -\sigma v^*_\sigma,$$

$$\gamma^0 v_\sigma^* = -v_\sigma^*,$$

$$\frac{1}{2}(\Sigma_1 \mp i\Sigma_2)v_\mp^* = -v_\pm^*,$$

for imaginary γ^0 and $\mathbf{\Sigma}$. Prove that an acceptable relation is

$$v_{-\sigma}^* = i\gamma_5 \sigma v_\sigma.$$

5. Prove that $\overline{\psi}\psi = \psi^\dagger\gamma^0\psi$ is a Lorentz scalar, while $\overline{\psi}\sigma^{\mu\nu}\psi = \psi^\dagger\gamma^0\sigma^{\mu\nu}\psi$ is a Lorentz tensor of the second rank.