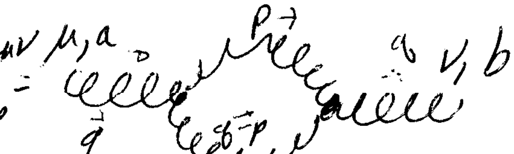
Homework # 10

We computed Z_3 due to the fermion loop in class (6.54).

1. Next consider

$i\Pi^{\mu\nu}_{ab} =$ 

(in the following we suppress initially the integration over loop momentum)

$$= \frac{g^2}{2} f_{acd}\left[g_{\mu\alpha}(q+p)_\beta + g_{\alpha\beta}(-p-p+q)_\mu + g_{\beta\mu}(p-q-q)_\alpha \right]$$

sym factor

$$\cdot \frac{-i}{p^2-i\epsilon}\left[g^{\alpha\gamma} + \frac{p^\alpha p^\gamma (1-\xi)}{p^2-i\epsilon} \right] \cdot \frac{-i}{(q-p)^2-i\epsilon}\left[g^{\beta\delta} + \frac{(q-p)^\beta (q-p)^\delta (1-\xi)}{(q-p)^2-i\epsilon} \right]$$

$$\cdot f_{bcd}\left[g_{\nu\gamma}(-q-p)_\delta + g_{\gamma\delta}(p-q+p)_\nu + g_{\delta\nu}(q-p+q)_\gamma \right]$$

Set $\xi = 1$

$$= \frac{g^2}{2} C_2(8)\, \delta_{ab}\left[g_{\mu\alpha}(q+p)_\beta + g_{\alpha\beta}(q-2p)_\mu + g_{\beta\mu}(p-2q)_\alpha \right]$$

$$\times \frac{-i}{p^2} g^{\alpha\gamma} \frac{-i}{(q-p)^2} g^{\beta\delta}\left[g_{\nu\gamma}(-p-q)_\delta + g_{\gamma\delta}(-q+2p)_\nu - g_{\delta\nu}(p-2q)_\gamma \right]$$

$$= -g^2 C_2(8)\, \delta_{ab}\left[g_{\mu\alpha}(q+p)_\beta + g_{\alpha\beta}(q-2p)_\mu + g_{\beta\mu}(p-2q)_\alpha \right] \frac{1}{p^2 (q-p)^2}$$

$$\times \left[g_\nu{}^\alpha (p+q)^\beta + g^{\alpha\beta}(-q+2p)_\nu - g^\beta{}_\nu (p-2q)^\alpha \right]$$

$$= -\frac{g^2}{2} C_2(8)\,\delta_{ab}\Big[-g_{\mu\nu}(p+q)^2 - (q-2p)_\mu (p+q)_\nu$$

$$-(p+q)_\mu (p-2q)_\nu + (2p-q)_\nu (q+p)_\mu$$

$$+\cancel{4}^{\,d}(q-2p)_\mu (2p-q)_\nu + (2p-q)_\nu (p-2q)_\mu$$

$$-(q+p)_\nu (p-2q)_\mu - (p-2q)_\nu (q-2p)_\mu - g_{\mu\nu}(p-2q)^2\Big]$$

$$= -\frac{g^2}{2} C_2(8)\,\delta_{ab}\Big[-g_{\mu\nu}\big((p+q)^2 + (p-2q)^2\big)$$

$$+(2p-q)_\mu (p+q)_\nu + (2p-q)_\nu (p+q)_\mu$$

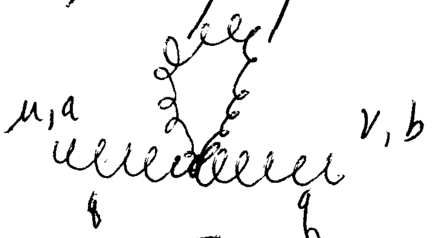
$$-(p-2q)_\nu (p+q)_\mu - (p-2q)_\mu (p+q)_\nu$$

$$+(2p-q)_\mu (p-2q)_\nu + (2p-q)_\nu (p-2q)_\mu$$

$$-\cancel{4}^{\,d}(2p-q)_\mu (2p-q)_\nu\Big]$$

d = no. of ST dimensions

Before proceeding with this, look at the other two graphs that contribute for $\Pi_{ab}^{\mu\nu}$:



$$= -\frac{ig^2}{2}\Big[f_{abe}f_{cce}(g_{\mu\nu}g_{\lambda\lambda'} - g_{\mu\lambda}g_{\nu\lambda'})$$

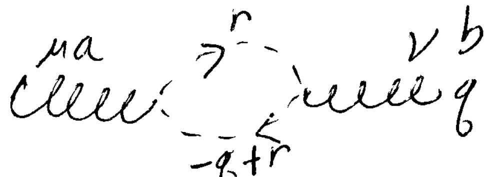
sym → 2

$$- f_{ace}f_{bce}(g_{\mu\lambda}g_{\nu\lambda'} - g_{\mu\nu}g_{\lambda\lambda'})$$

$$+ f_{cbe}f_{ace}(-g_{\lambda\nu}g_{\lambda'\mu} - g_{\lambda\lambda'}g_{\mu\nu})\Big)\Big]\frac{-ig^{\lambda\lambda'}}{p^2}$$

$$= +\frac{ig^2}{2}\, C_2(8)\, \delta_{ab}\, 2\left(g_{\mu\nu} - \overset{d}{4}\, g_{\mu\nu}\right)\frac{-i}{p^2}$$

The third graph contributing to $\Pi^{ab}_{\mu\nu}$ is



$$-\left(g^2\right) f_{ade}\,\frac{r^\mu(-i)}{r^2}\, f_{bed}\,\frac{(r-q)^\nu(-i)}{(r-q)^2}$$

$$= -\,g^2\, C_2(8)\,\delta_{ab}\,\frac{r^\mu (r-q)^\nu}{r^2 (r-q)^2}$$

Combine these, & insert loop mom. integration

$$g^2\, C_2(8)\,\delta_{ab}\int\frac{(dp)}{(2\pi)^4}\,\frac{1}{p^2(p-q)^2}\Big\{ +\frac{g_{\mu\nu}}{2}\left[(p+q)^2+(p-2q)^2\right]$$

$$\tfrac{1}{2}\Big[-(2p-q)_\mu(p+q)_\nu - (2p-q)_\nu(p+q)_\mu + (p-2q)_\mu(p+q)_\nu + (p-2q)_\nu(p+q)_\mu$$

$$-(2p-q)_\mu(p-2q)_\nu - (2p-q)_\nu(p-2q)_\mu + d\,(2p-q)_\mu(2p-q)_\nu\Big]$$

$$+\ (1-d)\, g_{\mu\nu}\ \ \underline{(p-q)^2}\ \ - p_\mu (p-q)_\nu\ \Big\}$$

Now combine the denominators

$$\frac{1}{p^2(p-q)^2} = -\int_0^\infty ds\, s \int_0^1 du\, e^{-is\chi(u)}$$

$$\chi(u) = (1-u)p^2 + u(p-q)^2$$

$$= p^2 - 2pqu + uq^2$$

$$= (p-qu)^2 + q^2u(1-u)$$

So we integrate over $p-qu$:

$$\langle (p-qu)^\mu \rangle = 0$$

$$\langle (p-qu)^\mu (p-qu)^\nu \rangle = A g^{\mu\nu}$$

$$\langle (p-qu)^2 \rangle = dA$$

$$\int \frac{(dp)}{(2\pi)^4} e^{-is(p-qu)^2} = -\frac{i}{16\pi^2 s^2} \quad (4.90a)$$

$$\int \frac{(dp)}{(2\pi)^4} (p-qu)^2 e^{-is(p-qu)^2} = -\frac{2i}{s}\frac{-i}{16\pi^2 s^2} \quad (4.90c)$$

or in d dim

$$\int \frac{d^dp}{(2\pi)^d} e^{-isp^2} = \frac{1}{(2\pi)^d}\left(\frac{\pi}{is}\right)^{d/2} = i^{1-d/2}\left(\frac{1}{4\pi s}\right)^{d/2}$$

$$\int \frac{d^dp}{(2\pi)^d} p^2 e^{-isp^2} = -\frac{1}{i}\frac{d}{ds}\int \frac{d^dp}{(2\pi)^d} e^{-isp^2} = +i^{-d/2}\frac{d}{2}\frac{1}{s}\left(\frac{1}{4\pi s}\right)^{d/2}$$

Return to p. 3:

$$\{\ \} = \frac{g_{\mu\nu}}{2}\left[(p-qu+q(1+u))^2 + (p-qu+q(u-2))^2\right]$$

$$+\frac{1}{2}\Big(-[2(p-qu)+q(2u-1)]_\mu[p-qu+q(1+u)]_\nu + (\mu\leftrightarrow\nu)$$

$$+(p-uq+q(u-2))_\mu(p-qu+q(1+u))_\nu + (\mu\leftrightarrow\nu)$$

$$-(2(p-qu)+q(2u-1))_\mu(p-qu+q(u-2))_\nu + (\mu\leftrightarrow\nu)$$

$$+d[2(p-qu)+q(2u-1)]_\mu[2(p-qu)+q(2u-1)]_\nu\Big)$$

$$+g_{\mu\nu}(1-d)(p-qu+q(u-1))^2 - (p-qu+qu)_\mu(p-qu+q(u-1))_\nu$$

$$\to \frac{g_{\mu\nu}}{2}\left[2(p-qu)^2 + q^2\left((1+u)^2+(u-2)^2\right)\right]$$

$$+\frac{1}{2}\Big(-2(p-qu)_\mu(p-qu)_\nu - q_\mu q_\nu(2u-1)(1+u) - (\mu\leftrightarrow\nu)$$

$$+(p-qu)_\mu(p-qu)_\nu + q_\mu q_\nu(u-2)(1+u) + (\mu\leftrightarrow\nu)$$

$$-2(p-qu)_\mu(p-qu)_\nu - q_\mu q_\nu(2u-1)(u-2) - (\mu\leftrightarrow\nu)$$

$$+d\left[4(p-qu)_\mu(p-qu)_\nu + q_\mu q_\nu(2u-1)^2\right]\Big)$$

$$+g_{\mu\nu}(1-d)\left[(p-qu)^2 + q^2(u-1)^2\right] - (p-qu)_\mu(p-qu)_\nu$$

$$+q_\mu q_\nu u(1-u)$$

Collect quadratically divergent terms: (6)

$$g_{\mu\nu}A\left[d - 2 + 1 - 2 + \frac{4d}{2} + (1-d)d - 1 \right]$$

$$= A g_{\mu\nu}\left[-d^2 + 4d - 4\right] = -A g_{\mu\nu}(d-2)^2$$

Sing. at $d=2$ cancels, $\therefore$ no quad divergence

Therefore: combining all this

$$i\Pi^{\mu\nu}_{ab} = -\int_0^\infty ds\, s \int_0^1 du\, e^{-isu(1-u)q^2} \frac{+i^{1-d/2}}{(4\pi\cdot s)^{d/2}}$$

$$\times\Big\{ + \frac{i}{s} g_{\mu\nu}(d-2)^2 \frac{d}{2}\frac{1}{d}$$

$$+ g_{\mu\nu} q^2 \left[\tfrac{1}{2}(2u^2 - 2u + 5) + (1-d)(u-1)^2\right]$$

$$+ q_\mu q_\nu \Big[-(1+u)(2u-1) + (u-2)(1+u) - (2u-1)(u-2)$$
$$+ \tfrac{1}{2}d(2u-1)^2 + u(1-u)\Big]\Big\}$$

$$= -\frac{i^{1-d/2}}{(4\pi)^{d/2}}\int_0^\infty ds\, s^{1-d/2}\int_0^1 du\, e^{-isu(1-u)q^2}$$

$$\times\Big\{ + \frac{i}{s}\frac{d}{d\,2}(d-2)^2 g_{\mu\nu} + q^2 g_{\mu\nu}\left[\begin{matrix} u^2 - u + \frac{5}{2} \\ +(1-d)(u-1)^2 \end{matrix}\right]$$

$$+ q_\mu q_\nu \left[\begin{matrix} -2u^2 - u + 1 \\ u^2 - u - 2 \\ -2u^2 + 5u - 2 \\ -u^2 + u \end{matrix} \quad + \frac{d}{2}(2u-1)^2\right]$$

!

7

$$= -\frac{i^{1-d/2}}{(4\pi)^{d/2}} \int_0^1 du \Big\{ g_{\mu\nu} \Big[+ i \overbrace{\tfrac{1}{2}(d-2)^2}^{2(\frac{d}{2}-1)(d-2)} \Gamma(1-d/2)\,(i\,u(1-u)q^2)^{\frac{d}{2}-1}$$

$$+ q^2\Big(u^2 - u + \tfrac{5}{2} + (1-d)(u-1)^2\Big)\Gamma(2-d/2) \times (i\,u(1-u)q^2)^{d/2-2}\Big]$$

$$+ q_\mu q_\nu \Big[-4u^2 + 4u - 3 + \tfrac{d}{2}(2u-1)^2\Big) \times \Gamma(2-d/2)\,(i\,u(1-u)q^2)^{d/2-2}$$

$$= -\frac{i^{1-d/2}}{(4\pi)^{d/2}} \int_0^1 du \left[i\,u(1-u)q^2\right]^{d/2-2}\Gamma(2-\tfrac{d}{2}) \Big\{\Big[-i\,(d-2)\,i\,u(1-u)q^2$$

$$+ q^2\Big(u^2 - u + \tfrac{5}{2} + (1-d)(u-1)^2\Big)\Big] g_{\mu\nu}$$

$$+ q_\mu q_\nu \Big[-4u^2 + 4u - 3 + \tfrac{d}{2}(2u-1)^2\Big]\Big\}$$

$$= \frac{-i^{1-d/2}}{(4\pi)^{d/2}} \int_0^1 du \left[i\,u(1-u)q^2\right]^{\frac{d}{2}-2}\Gamma(2-\tfrac{d}{2}) \Big[g_{\mu\nu} q^2 \Big(-d(1-u)(u+(1-u)) + 2u^2 - 2u + u^2 - u + \tfrac{5}{2} + u^2 - 2u + 1\Big)$$

$$+ q_\mu q_\nu \Big[\tfrac{d}{2}(2u-1)^2 - 4u^2 + 4u - 3\Big]\Big]$$

$$= \frac{-i^{1-d/2}}{(4\pi)^{d/2}} \int_0^1 du \left(i\,u(1-u)q^2\right)^{\frac{d}{2}-2}\Gamma(2-\tfrac{d}{2}) \Big\{ q^2 g_{\mu\nu}\Big[-d\overbrace{(1-u)}^{-u+\frac12+\frac12}_{\times(1-2u)} + \overbrace{4u^2 - 5u + \tfrac{7}{2}}^{(2u-1)^2 - u + \frac12 + 2}\Big]$$

$$+ q_\mu q_\nu \Big[\tfrac{d}{2}(2u-1)^2 \underbrace{- 4u^2 + 4u - 3}_{-(2u-1)^2 - 2}\Big]\Big\}$$

Note terms odd in $u-\frac{1}{2}$ integrate to zero
($u \to 1-u$ sub)

$$\therefore\; i\Pi_{\mu\nu}^{ab} = \frac{-i^{1-d/2}}{(4\pi)^{d/2}}\, g^2 C_2(8)\delta_{ab}\, \Gamma\!\left(2-\frac{d}{2}\right)\int_0^1 du\, \left[iu(1-u)q^2\right]^{d/2-2}$$ ⑧

$$\left\{ q^2 g_{\mu\nu}\left[-\frac{1}{2}d(1-2u)^2 + (2u-1)^2 + 2\right] + q_\mu q_\nu\left[\frac{1}{2}d(1-2u)^2 - (2u-1)^2 - 2\right]\right\}$$

$$= -\frac{i^{1-d/2}}{(4\pi)^{d/2}}\,\Gamma(2-d/2)\,(q^2 g_{\mu\nu} - q_\mu q_\nu)\, g^2 C_2(8)\delta_{ab}$$

$$\otimes \int_0^1 du\,\left[iu(1-u)q^2\right]^{d/2-2}\left[(2u-1)^2 + 2 - \frac{d}{2}(1-2u)^2\right]$$

$$d = 4-\epsilon \qquad \epsilon \to 0$$

$$\to -\frac{i^{-1}}{(4\pi)^2}\,\Gamma\!\left(\frac{\epsilon}{2}\right) g^2 C_2(8)\delta_{ab}\,(q^2 g_{\mu\nu} - q_\mu q_\nu)$$

$$\times \left(\frac{q^2}{\mu^2}\right)^{-\epsilon/2} \int_0^1 du\, \underbrace{\underbrace{\left[(2u-1)^2 + 2 - 2(1-2u)^2\right]}_{-(1-4u+4u^2)+2}}_{1+\frac{4}{2}-\frac{4}{3} \;=\; \frac{5}{3}}$$

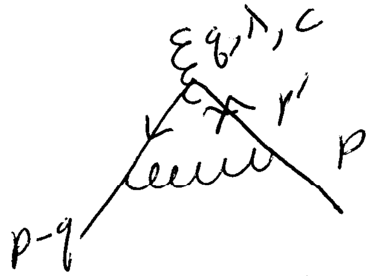
insert arb. mass scale

$$= \frac{i}{(4\pi)^2}\frac{2}{\epsilon}\, g^2 C_2(8)\,\delta_{ab}\,\frac{5}{3}\left(\frac{q^2}{\mu^2}\right)^{-\epsilon/2}(q^2 g_{\mu\nu} - q_\mu q_\nu)$$

$$= \frac{ig^2}{8\pi^2}\frac{1}{\epsilon}\, C_2(8)\,\delta_{ab}\,\frac{5}{3}\left(\frac{q^2}{\mu^2}\right)^{-\epsilon/2}(q^2 g_{\mu\nu} - q_\mu q_\nu)$$

as stated!

2.



$\xi = 1$

$$= -ig\,\gamma^\mu \frac{\lambda^a}{2}\,\frac{-i}{m+\gamma p'}\left(-ig\gamma^\lambda \frac{\lambda^c}{2}\right)\frac{-i}{m+\gamma(p'-q)}\left(-ig\gamma_\mu \frac{\lambda^a}{2}\right)$$

$$\times \frac{-i}{(p-p')^2}$$

$$= -g^3 \frac{\lambda^a}{2}\frac{\lambda^c}{2}\frac{\lambda^a}{2}\,\gamma^\mu \frac{m-\gamma p'}{m^2+p'^2}\,\gamma^\lambda \frac{(m-\gamma(p'-q))}{m^2+(p'-q)^2}\,\gamma_\mu \frac{1}{(p-p')^2}$$

This leads to a logarithmically divergent integral — we are interested in the divergence so we can set $m \to 0$

$$\to -g^3 \frac{\lambda^a\lambda^c\lambda^a}{8}\int \frac{d^dp'}{(2\pi)^d}\,\frac{\gamma^\mu\,\gamma p'\,\gamma^\lambda\,\gamma(p'-q)\,\gamma_\mu}{p'^2(p'-q)^2(p'-p)^2}$$

Note $\frac{\lambda^a}{2}\frac{\lambda^c}{2}\frac{\lambda^a}{2} = \frac{\lambda^a}{2}\frac{\lambda^a}{2}\frac{\lambda^c}{2} + \frac{\lambda^a}{2}\left[\frac{\lambda^c}{2},\frac{\lambda^a}{2}\right]$

$$= C_2(3)\frac{\lambda^c}{2} + \frac{\lambda^a}{2}\, i f^{cab}\frac{\lambda^b}{2}$$

$$= C_2(3)\frac{\lambda^c}{2} + \underbrace{\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right]}_{if^{abd}\frac{\lambda^d}{2}}\frac{1}{2}\, i f^{cab}$$

$\leftarrow$ (6.32)

$$= C_2(3)\frac{\lambda^c}{2} + \frac{1}{2}\, i^2 \frac{\lambda^d}{2}\, C_2(8)\,\delta_{cd}$$

$$= \left(C_2(3) - \frac{1}{2}C_2(8)\right)\frac{\lambda^c}{2}$$

Since we're only picking out the divergent part, we can drastically simplify this

$$-g^3\left(C_2(3) - \tfrac{1}{2}C_2(8)\right)\frac{\lambda^c}{2}\int\frac{d^dp'}{(2\pi)^d}\,\frac{\gamma^\mu\gamma p'\gamma^\lambda\gamma p'\gamma_\mu}{(p')^6}$$

$$p'^\mu p'^\nu \to A p^2 g^{\mu\nu}$$

$$p^2 \to A p^2 d$$

Integral above $= \dfrac{1}{d}\displaystyle\int\frac{d^dp'}{(2\pi)^d}\,\frac{\gamma^\mu\gamma^{\cancel{\lambda}}\gamma^\lambda\gamma_\nu\gamma_\mu}{p'^4} \equiv I$

$$\gamma^\nu\gamma^\lambda\gamma_\nu = -2g^{\lambda\nu}\gamma_\nu - \gamma^\lambda\underbrace{\gamma^\nu\gamma_\nu}_{-d}$$

$$= -2\gamma^\lambda + d\gamma^\lambda$$

$$= (d-2)\gamma^\lambda$$

$$I = \frac{(d-2)^2}{d}\gamma^\lambda\int\frac{d^dp'}{(2\pi)^d}\,\frac{1}{(p'^2)^2} = \frac{(d-2)^2}{d}\int\frac{d^dp'}{(2\pi)^d}\int_0^\infty ds\,s\int_0^1 du\,e^{-is\chi}$$

$$\chi = u(p'^2) + (1-u)p'^2 = p'^2$$

$$= \frac{(d-2)^2}{d}\gamma^\lambda\int_0^\infty ds\,s\,\underbrace{\int\frac{d^dp}{(2\pi)^d}\,e^{-isp^2}}_{i^{1-d/2}\left(\frac{1}{4\pi s}\right)^{d/2}}$$

$$= \frac{(d-2)^2\gamma^\lambda\, i^{1-d/2}}{d\,(4\pi)^{d/2}}\underbrace{\int_0^\infty ds\,s^{1-d/2}}_{\sim\Gamma(2-d/2)}$$

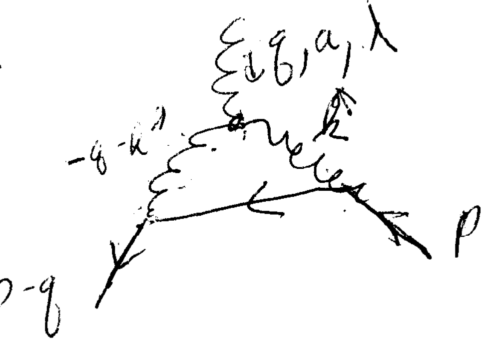
← exponential convergence factor missing because dropped all other momenta + masses

$$\simeq \frac{-i}{(4\pi)^2}\frac{2}{\epsilon}\gamma^\lambda$$

Therefore, this vertex term is

$$-g^3\left(C_2(3)-\tfrac{1}{2}C_2(8)\right)\frac{\lambda^c}{2}\,\frac{-i}{(4\pi)^2}\,\frac{2}{\epsilon}\,\gamma^\lambda$$

Next consider



$$= -ig\gamma^\mu\frac{\lambda^b}{2}\,\frac{-i}{m+\gamma(p-l)}\,(-ig\gamma^\nu)\,\frac{-i}{k^2}\,\frac{-i}{(q-l)^2}$$

$$\times(-g)f_{abc}\left[g_{\lambda\mu}(q-l)_\nu + g_{\mu\nu}(k+q+l)_\lambda + g_{\nu\lambda}(-q-k+q)_\mu\right]\lambda^c$$

$$= +ig^3 f_{abc}\frac{\lambda^b}{2}\frac{\lambda^c}{2}\,\gamma^\mu\,\frac{m-\gamma(p-k)}{m^2+(p-k)^2}\,\gamma^\nu\,\frac{1}{k^2}\,\frac{1}{(q-k)^2}$$

$$\times\left(g_{\lambda\mu}(q-k)_\nu + g_{\mu\nu}(2k+q)_\lambda + g_{\nu\lambda}(-2q-k)_\mu\right.$$

Here $f_{abc}\frac{\lambda^b}{2}\frac{\lambda^c}{2} = \frac{1}{2}f_{abc}\left[\frac{\lambda^b}{2},\frac{\lambda^c}{2}\right] = \frac{1}{2}f_{abc}\,i f_{bcd}\,\frac{\lambda^d}{2}$

$$= \frac{i}{2}C_2(8)\,\delta_{ad}\,\frac{\lambda^d}{2} = \frac{i}{2}C_2(8)\,\frac{\lambda^a}{2}.$$

Again pick out divergent terms by dropping mass + external momenta

$$\to -\frac{1}{2}g^3 C_2(8)\frac{\lambda^a}{2}\int\frac{d^dk}{(2\pi)^d}\,\gamma^\mu\gamma k\gamma^\nu\,\frac{1}{(k^2)^3}\left(-g_{\lambda\mu}k_\nu + 2k_\lambda g_{\mu\nu} - g_{\nu\lambda}k_\mu\right)$$

$$= -\frac{1}{2} g^3 C_2(8) \frac{\lambda^a}{2} \frac{1}{d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(p^2)^2} \left[-\gamma^\lambda \gamma_\nu \gamma^\nu + 2\gamma^\mu \gamma^\lambda \gamma_\mu - \gamma^\nu \gamma_\nu \gamma^\lambda \right]$$

$$= -\frac{1}{2} g^3 C_2(8) \frac{\lambda^a}{2} \frac{i}{d} \left(-\frac{i}{(4\pi)^2} \frac{2}{\epsilon} \right) \underbrace{\left[+4\gamma^\lambda + 4\gamma^\lambda + 4\gamma^\lambda \right]}_{12\gamma^\lambda}$$

$d \approx 4$

$$= + \frac{g^3 i}{(4\pi)^2} \frac{\lambda^a}{2} C_2(8) \frac{2}{\epsilon} \frac{3}{2} \gamma^\lambda$$

Combine this with term on top of p. 11:

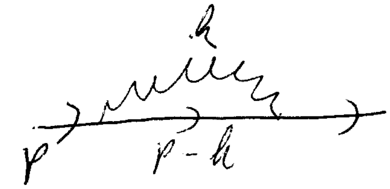
$$\frac{i g^3}{(4\pi)^2} \frac{\lambda^a}{2} \gamma^\lambda \frac{2}{\epsilon} \left[C_2(3) - \frac{1}{2} C_2(8) + \frac{3}{2} C_2(8) \right]$$

$$= \frac{i g^3}{(4\pi)^2} \frac{\lambda^a}{2} \gamma^\lambda \frac{2}{\epsilon} \left[C_2(3) + C_2(8) \right]$$

Since this is a modification to the tree-level quark-gluon vertex $-ig\gamma^\lambda \frac{\lambda^a}{2}$, we identify

$$Z_1 = 1 - \frac{g^2}{(4\pi)^2} \frac{2}{\epsilon} \left(C_2(3) + C_2(8) \right)$$

(the factor $(\mu/\mu_R)^\epsilon$ appears when finite terms included)

3. 

$$\Sigma = -ig\gamma^\mu \frac{\lambda^a}{2} \frac{-i}{k^2} \frac{-i}{m+\gamma(p-k)} \left(-ig\gamma_\mu \frac{\lambda^a}{2}\right)$$

$$= g^2 \frac{\lambda^a}{2}\frac{\lambda^a}{2} \gamma^\mu \frac{1}{k^2} \frac{m-\gamma(p-k)}{m^2+(p-k)^2}\gamma_\mu$$

Again, to estimate divergence, drop external mass ~~+ mom.~~

$\frac{\lambda^a}{2}\frac{\lambda^a}{2} = C_2(3)$ $\qquad\qquad$ $\gamma^\mu\gamma k\gamma_\mu$

so above

$$\Sigma = g^2 C_2(3)\int \frac{d^dk}{(2\pi)^d} \frac{\gamma^\mu\gamma(k-p)\gamma_\mu}{(k^2)(k-p)^2}$$

$$\gamma^\mu\gamma(k-p)\gamma_\mu = (d-2)\gamma(k-p)$$

$$\frac{1}{k^2(k-p)^2} = -\int_0^\infty ds\, s\int_0^1 du\, e^{-is[u(k-p)^2+(1-u)k^2]}$$

$$= -\int_0^\infty ds\, s\int_0^1 du\, e^{-is\underbrace{[k^2-2up+up^2]}_{(k-up)^2+p^2u(1-u)}}$$

$\gamma(k-up)$ integrates to zero

so $\gamma(k-p) = \gamma(k-up+p(u-1)) \rightarrow (u-1)\gamma p$

So

$$\Sigma = -g^2C_2(3)\gamma p\int ds\, s\int_0^1 du\, e^{-isu(1-u)p^2}(u-1)(d-2)\frac{i^{1-d/2}}{(4\pi)^{d/2}s^{d/2}}$$

$$= -g^2C_2(3)\gamma p\frac{i^{1-d/2}}{(4\pi)^{d/2}}\Gamma(2-d/2)(d-2)\int_0^1 du\,[isu(1-u)p^2]^{\frac{d}{2}-2}(u-1)$$

$\underline{d \to 4}$

$$\Sigma = -g^2 C_2(3)\, \gamma p \frac{(-i)}{(4\pi)^2}\, 2\Gamma\left(\tfrac{\epsilon}{2}\right)\left(-\tfrac{1}{2}\right)\left(\frac{p^2}{\mu^2}\right)^{-\epsilon/2}$$

$$= -i g^2 C_2(3) \frac{1}{(4\pi)^2} \frac{2}{\epsilon} \gamma p \left(\frac{p^2}{\mu^2}\right)^{-\epsilon/2}$$

This is interpreted as a modification of the free prop

$$\frac{-i}{m+\gamma p} \to \frac{-i}{\gamma p} \to \frac{-i}{\gamma p} - \frac{1}{\gamma p}\Sigma\frac{-i}{\gamma p}$$

$$= -\frac{i}{\gamma p + i\Sigma} = Z_2 \frac{-i}{\gamma p}$$

$$Z_2 = 1 - \frac{g^2}{(4\pi)^2} C_2(3) \frac{2}{\epsilon}\left(\frac{\mu}{\mu_R}\right)^{\epsilon}$$