Statistical Treatment of Experimental Data

AN INTRODUCTION TO STATISTICAL METHODS

Treating experimental errors

Analyzing experimental data

Concept of probability

Distribution of errors

Propagation of errors

Correlations

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DEDICATION

To Alice,
who shares her
Wonderland
with me.

PREFACE

Every scientist and engineer needs some elementary knowledge of statistical methods for treating experimental errors and analyzing experimental observations. The basic concepts of probability, distribution of errors, propagation of errors, and correlations are an indispensable part of the knowledge of anyone who has contact with numbers related to experimental observations.

Many undergraduate engineering and science students, however, find little or no time in their curricula for an introduction to even the most elementary statistical methods. It is the author's firm belief that some of these techniques should be introduced early in the undergraduate curriculum in science or engineering, so that they may be used in later courses which incorporate laboratory work.

Accordingly, this book has been written with considerable missionary zeal in an attempt to present some of these techniques in a form which is understandable, palatable, and even enjoyable for sophomore science or engineering students with little mathematical sophistication and no previous exposure to the subject of this book. The only mathematical background assumed is a year of elementary calculus. A year of general college physics is helpful in understanding some of the illustrative examples, but is not essential.

Many of the mathematical developments are given a somewhat intuitive rather than a completely rigorous presentation. It is to be expected that this practice will be condemned by specialists in mathematical statistics, but it is followed here deliberately and without apology. The author feels strongly that the student should encounter this material at an early stage in his education rather than waiting until a more rigorous treatment is feasible. The practice of presenting useful formulas with *no* derivation at all has, however, been studiously avoided.

The author's experience in teaching this material to several generations of sophomores majoring in physics at Carnegie Institute of Technology has shown that mastery of it is not beyond the ability of students at this level. It has been incorporated into the first part of a course given students majoring in physics in the first semester of their sophomore year. Most of the material can be covered quite thoroughly in four to six weeks, with three class hours per week and homework assignments for each hour. This material is then followed by laboratory work in which the statistical methods are put to work.

Such a subcourse can be fitted into any course in the sophomore, junior, or senior year in which quantitative laboratory work plays an important part. The book is also sufficiently self-contained so that it may be used for individual study. In either case, exercise in applying the principles is essential. In addition to many illustrative examples in the text, a collection of problems has been included at the end of each chapter. A summary of important formulas is included in Appendix A.

The book is intended primarily to be read from beginning to end. Several chapters may be omitted, however, without too much loss of continuity. The reader who is interested mostly in the Gauss distribution and its consequences may omit Secs. 7, 8, and 11. Sections 10 and 16 also may be omitted if desired.

In conclusion, a statement is necessary about what the book is *not*. It is *not* a treatise of mathematical statistics. Neither is it a comprehensive discussion of all aspects of treatment of experimental data. Several excellent books in these areas already exist. Our aim has been to make some of the most important techniques accessible and useful to those who are just beginning their preparation for the scientific and engineering professions.

Acknowledgments

A preliminary version of the text has been read by sophomore physics students at Carnegie Institute of Technology under its original title, "Thirteen Short Mysteries." (The number of mysteries has since grown to sixteen.) The critical comments and suggestions of these students have helped considerably to clear up sections which were obscurely written. I am indebted to Professor Sir Ronald A. Fisher, Dr. Frank Yates, and to Messrs. Oliver & Boyd, Ltd., Edinburgh, for permission to reprint Tables IV and VI from their book "Statistical Tables for Biological, Agricultural and Medical Research." The very willing and cooperative assistance of Mrs. L. Horton, who typed several versions of the manuscript, is gratefully acknowledged. Particular thanks are due to Dr. Yardley Beers of the National Bureau of Standards, Prof. E. T. Jaynes of Washington University, Prof. Henry Margenau of Yale University, and Prof. Jay Orear of Cornell University. All these people have read the entire manuscript critically, and their comments and suggestions have resulted in substantial improvements in the text. Not all the suggestions have been followed, however; and responsibility for whatever shortcomings remain in the book rests entirely upon the author. The most important acknowledgment of all is on page v.

Hugh D. Young

CONTENTS

LIST OF SYMBOLS

The meanings of symbols which appear several times in the text are listed here for convenient reference.

A	normalization constant in a distribution function
$C(N, n)$	number of combinations of N things taken n at a time
D_μ	deviation of mean $\bar{x}_\mu$ from mean of means $\bar{X}$
M	number of sets of observations
N	number of observations in a set; number of independent events; parameter in binomial distribution
P	probability
P_i	probability of event i; probability for an observation to fall in interval i
Q	quantity calculated from observed quantities $a, b, \ldots$
T	multiple of standard deviation
$\bar{X}$	mean of means $\bar{x}_\mu$
Z	number of sets of observations or trials
a	parameter in Poisson distribution
$a, b, \ldots$	observed quantities
$a_i, b_i, \ldots$	typical observations of quantities $a, b, \ldots$
$\bar{a}, \bar{b}, \ldots$	mean values of quantities $a, b, \ldots$
b	y intercept in linear equation
b'	x intercept in linear equation

d_i	deviation of observation i
$d_{\mu i}$	deviation of observation i in set μ
$f(n)$	function of n; probability distribution for discrete variable n
$f(x)$	function of x; probability distribution for continuous variable x
$f_a(n)$	Poisson distribution function
$f_{N,p}(n)$	binomial distribution function
$f'(x)$	derivative of $f(x)$
h	measure of precision in normal distribution
i	index to denote one of a set of observations
m	slope of line of regression of y on x
m'	slope of line of regression of x on y
p	parameter in binomial distribution
q	parameter in binomial distribution; $q = 1 - p$
r	linear correlation coefficient
w_i	weight of observation i
x	an observed quantity
x_i	typical observation of quantity x
$x_{\mu i}$	observation i in set μ of quantity x
$\bar{x}$	mean value of x; mean of observations x_i
$\bar{x}_\mu$	mean of set μ of observations
y	dependent variable in linear equation
$\Delta a, \Delta b, \ldots$	change in quantity $a, b, \ldots$
α	mean deviation, average deviation, average absolute deviation

ϵ	elemental error used to derive normal distribution
μ	index to denote a set of observations
ν	number of degrees of freedom
σ	standard deviation
σ^2	variance
σ_m	standard deviation of mean
σ_{ma}	standard deviation of mean of a
χ^2	measure of goodness of fit
$\cong$	approximately equal to
$\overset{\circ}{=}$	observed to be equal to