Collider signals II: $\not\!\!E_T$ signatures
including SUSY, Tp, KKp and dark matter connection

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★ Lecture 1: some SUSY basics

- SUSY spectra demos

★ Lecture 2: Sparticle production, decay, event generation

- SUSY LHC events demo

★ Lecture 3: SUSY models and DM connection

★ Lecture 4: SUSY, UED and LHT at LHC

First: why have a special set of 4 lectures on $\not{E}_T$ at LHC?

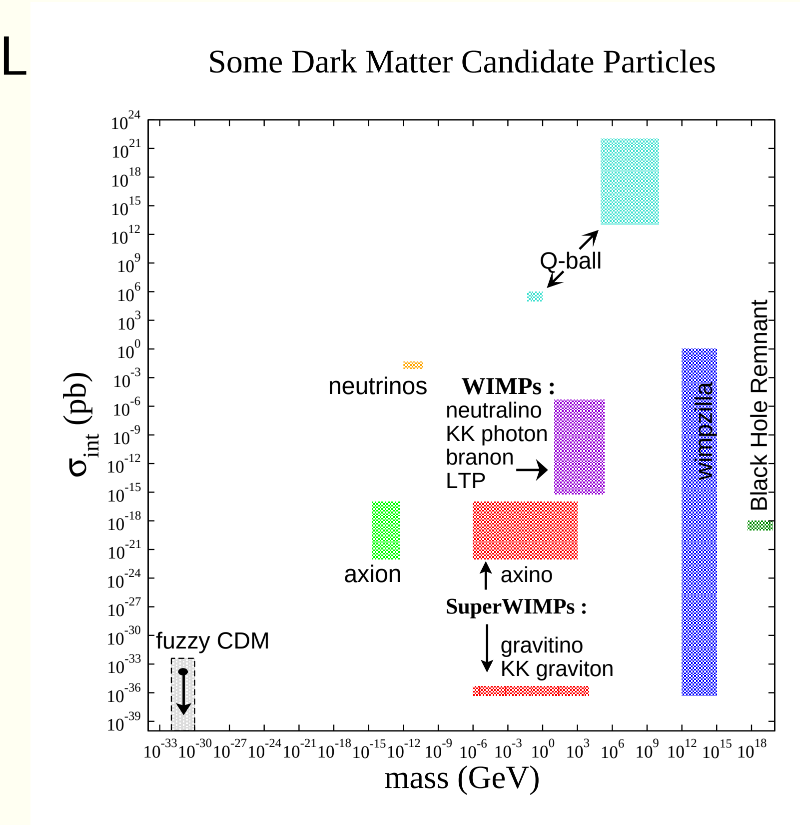
★ $\not{E}_T$ is one of the main signals for new physics to be searched for at the LHC

★ Main motivation nowadays: dark matter

- vast array of astrophysical data show we most likely live in a ΛCDM universe!
  - baryons: $\sim 4\%$
  - dark matter $\sim 25\%$
  - dark energy $\sim 70\%$
  - νs, γs: tiny fraction
- properties of DM
  - massive
  - electric (and likely color) neutral
  - non-relativistic, to seed structure formation
  - one form of DM, νs, are relativistic
- what is the DM? some form of elementary particle not included in the SM

Candidates for Dark Matter

- ⋆ unseen baryons, e.g. BHs, brown dwarves, stellar remnants
  - – inconsistent with BBN element abundance calc'n
  - – limits from MACHO, EROS, OGL



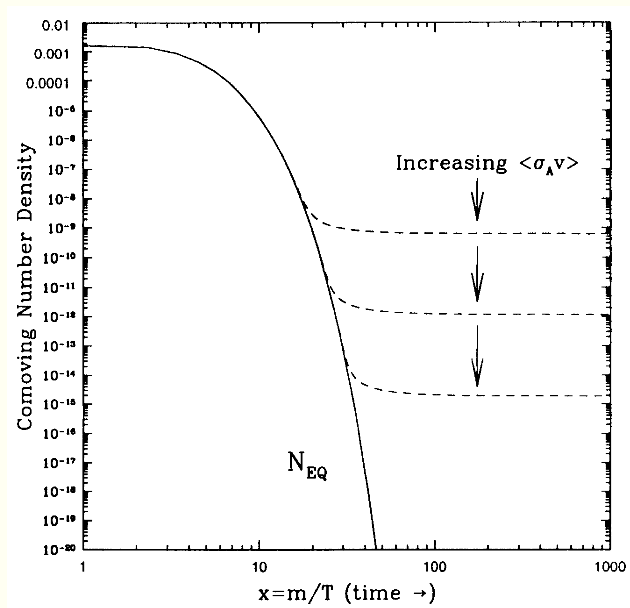
- ⋆ light neutrinos ($= HDM$)
- ⋆ axions/axinos
- ⋆ WIMPS
- ⋆ superWIMPS
- ⋆ Q-balls
- ⋆ primordial BHs

WIMPs: the WIMP miracle!

- Weakly Interacting Massive Particles
- assume in thermal equil'n in early universe
- Boltzman eq'n:

$$-\,dn/dt = -3Hn - \langle \sigma v_{rel} \rangle (n^2 - n_0^2)$$

- $\Omega h^2 = \frac{s_0}{\rho_c/h^2} \left(\frac{45}{\pi g_*}\right)^{1/2} \frac{x_f}{M_{Pl}} \frac{1}{\langle \sigma v \rangle}$
- $\sim \frac{0.1\ pb}{\langle \sigma v \rangle} \sim 0.1 \left(\frac{m_{wimp}}{100\ GeV}\right)^2$
- thermal relic $\Rightarrow$ new physics at M_{weak}!



Some WIMP candidates

- 4th gen. Dirac ν (excluded)
- SUSY neutralino (χ or $\widetilde{Z}_1$)
- UED excited photon B_μ^1
- little Higgs photon B_H
- little Higgs (theory space) N_1 (scalar)
- warped GUTS: LZP KK fermion
- $\cdots$

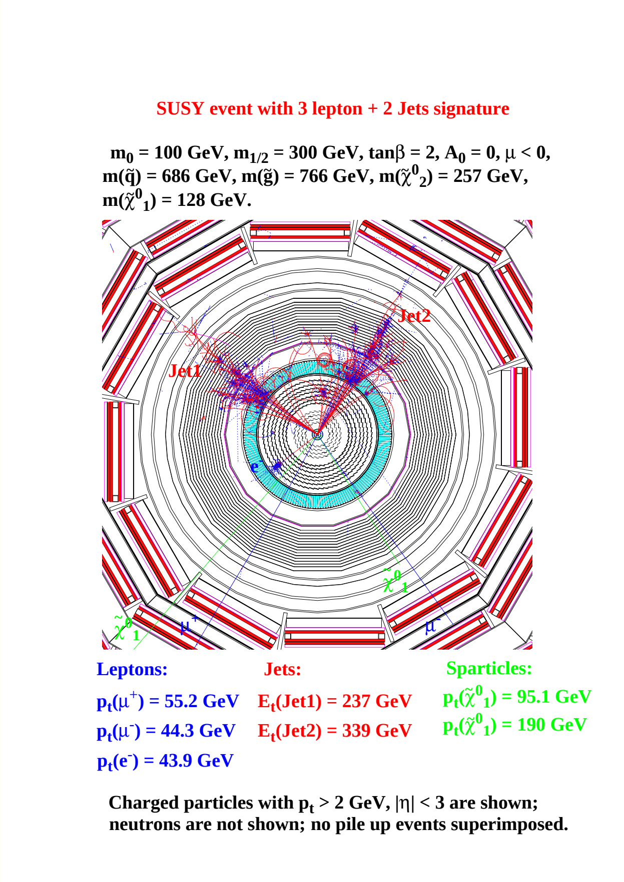
⋆ If DM is a WIMP particle, then LHC may be a DM factory!

⋆ May be able to study properties of DM in a laboratory environment!

⋆ WIMPs, since they are electric and color neutral, will give rise to **missing energy** at LHC!

Lecture 1. SUSY basics, models and spectra generation

- ⋆ Outline
- ⋆ SUSY basics
  - WZ model
  - SUSY master Lagrangian
  - MSSM: construction
  - RGEs, soft term evolution and spectra
  - SUGRA, GMSB, AMSB
  - SUSY spectra demo

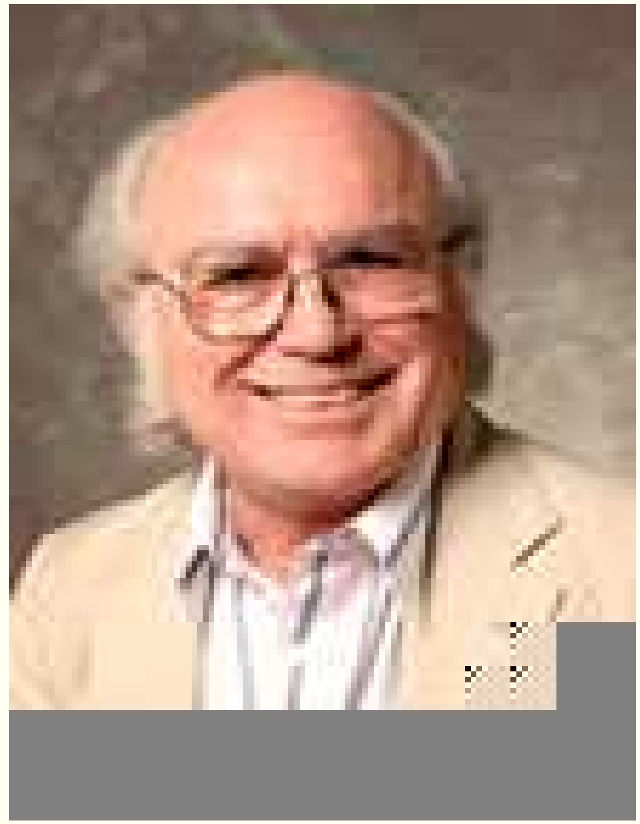


Wess-Zumino toy SUSY model: 1974

- $\mathcal{L} = \mathcal{L}_{\text{kin.}} + \mathcal{L}_{\text{mass}}$
  - $\mathcal{L}_{\text{kin.}} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\overline{\psi}\,\partial\!\!\!/\psi + \frac{1}{2}(F^2 + G^2)$
  - $\mathcal{L}_{\text{mass}} = -m[\frac{1}{2}\bar{\psi}\psi - GA - FB]$
- A and B are real scalar fields with $[A] = [B] = 1$
- ψ is a *Majorana* spinor with $\psi = \psi^c = C\bar{\psi}^T$ and $[\psi] = \frac{3}{2}$
  - $\psi_D(x) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2E_{\mathbf{k}}}\sum_s [c_{\mathbf{k},s}u_{\mathbf{k},s}e^{-ikx} + d^\dagger_{\mathbf{k},s}v_{\mathbf{k},s}e^{ikx}]$
  - $\psi^c_D(x) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2E_{\mathbf{k}}}\sum_s [c^\dagger_{\mathbf{k},s}v_{\mathbf{k},s}e^{ikx} + d_{\mathbf{k},s}u_{\mathbf{k},s}e^{-ikx}]$
  - $\psi_M(x) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2E_{\mathbf{k}}}\sum_s [c_{\mathbf{k},s}u_{\mathbf{k},s}e^{-ikx} + c^\dagger_{\mathbf{k},s}v_{\mathbf{k},s}e^{ikx}]$
- F and G are *auxiliary* (non-propagating) fields with $[F] = [G] = 2$
  - can be eliminated by E-L equations: $F = -mB$, $G = -mA$

Julius Wess (1934-2007) and Bruno Zumino

Julius Wess lecturing at the SUSY07 conference on July 25th, 2007 in Karlsruhe



SUSY transformation in WZ model

- $A \to A + \delta A$ with $\delta A = i\bar{\alpha}\gamma_5\psi$
- $\delta B = -\bar{\alpha}\psi$,
- $\delta\psi = -F\alpha + iG\gamma_5\alpha + \not{\partial}\gamma_5 A\alpha + i\,\not{\partial}B\alpha$,
- $\delta F = i\bar{\alpha}\,\not{\partial}\psi$,
- $G = \bar{\alpha}\gamma_5\,\not{\partial}\psi$

Using Majorana bilinear re-arrangements (e.g. $\bar{\psi}\chi = -\bar{\chi}\psi$) and product rule $\partial_\mu(f\cdot g) = \partial_\mu f\cdot g + f\cdot\partial_\mu g$ and algebra, can show that $\mathcal{L} \to \mathcal{L} + \delta\mathcal{L}$ with

- $\delta\mathcal{L}_{\rm kin} = \partial^\mu(-\frac{1}{2}\bar{\alpha}\gamma_\mu\,\not{\partial}B\psi + \frac{i}{2}\bar{\alpha}\gamma_5\gamma_\mu\,\not{\partial}A\psi + \frac{i}{2}F\bar{\alpha}\gamma_\mu\psi + \frac{1}{2}G\bar{\alpha}\gamma_5\gamma_\mu\psi)$,
- $\delta\mathcal{L}_{\rm mass} = \partial^\mu(mA\bar{\alpha}\gamma_5\gamma_\mu\psi + imB\bar{\alpha}\gamma_\mu\psi)$

Since Lagrangian changes by a total derivation, the *action* $S = \int \mathcal{L}d^4x$ is invariant! (owing to Gauss' law in 4-d) $\int_V d^4x\partial_\mu\Lambda^\mu = \int_{\partial V} d\sigma\Lambda^\mu n_\mu$ Thus, WZ transformation is a *symmetry* of the action!

Aspects of the WZ model:

- Can add interactions:
- $\mathcal{L}_{\rm int} = -\frac{g}{\sqrt{2}}A\bar{\psi}\psi + \frac{ig}{\sqrt{2}}B\bar{\psi}\gamma_5\psi + \frac{g}{\sqrt{2}}(A^2-B^2)G + g\sqrt{2}ABF$
- Difficult calculation, but can show $\delta\mathcal{L} \to$ total derivative
- Also, can show: quadratic divergences all *cancel*!
- If SUSY transformations expressed as $\mathcal{S} \to \mathcal{S}' = e^{i\bar{\alpha}Q}\mathcal{S}e^{-i\bar{\alpha}Q} \approx \mathcal{S} + [i\bar{\alpha}Q, \mathcal{S}] = \mathcal{S} + \delta\mathcal{S} \equiv (1 - i\bar{\alpha}Q)\mathcal{S}$, then can show that the generator Q obeys
  - $\{Q_a, \bar{Q}_b\} = 2(\gamma_\mu)_{ab}P_\mu$
  - SUSY is spacetime symmetry! (super-Poincaré algebra)

Constructing supersymmetric models

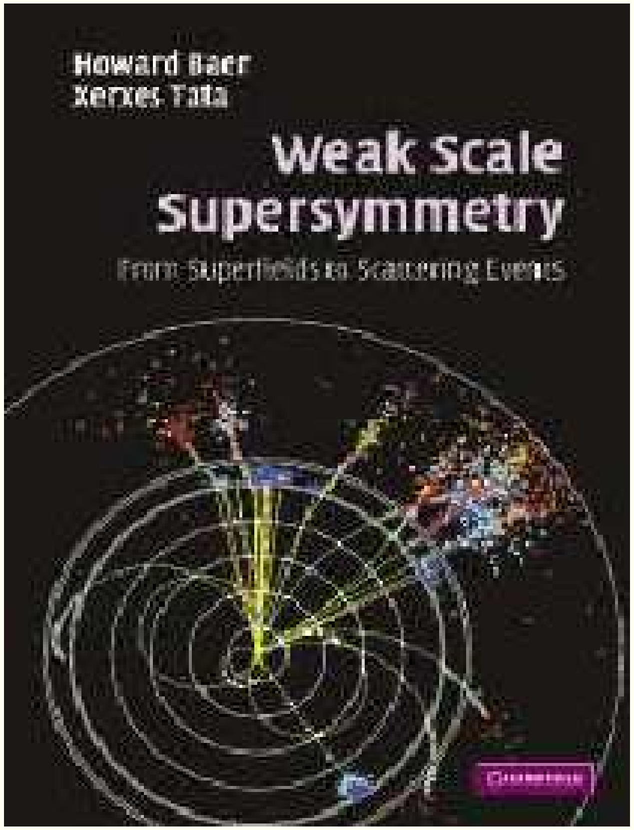
- While the WZ model is interesting, it was essentially "taken out of a hat", without rules of how to construct SUSY models in general
- Shortly after WZ model appeared in 1974, Salam and Strathdee developed *superfield formalism*, which *does* allow one to construct SUSY models in general.
- How does one combine scalar and spinor fields into a single entity?
- introduce *superspace* $x^\mu \to (x^\mu, \theta_a)$ where θ_a $(a = 1 - 4)$ are four *anti*-commuting dimensions arranged as a Majorana spinor

Weak Scale Supersymmetry

HB and X. Tata

Spring, 2006; Cambridge University Press

- ⋆ Part 1: superfields/Lagrangians
  - 4-component spinor notation for exp'ts
  - master Lagrangian for SUSY gauge theories
- ⋆ Part 2: models/implications
  - MSSM, SUGRA, GMSB, AMSB, $\cdots$
- ⋆ Part 3: SUSY at colliders
  - production/decay/event generation
  - collider signatures
  - R-parity violation



Some types of superfields

- general superfield:
$\hat{\Phi}(x,\theta) = \mathcal{S} - i\sqrt{2}\bar{\theta}\gamma_5\psi - \frac{i}{2}(\bar{\theta}\gamma_5\theta)\mathcal{M} + \frac{1}{2}(\bar{\theta}\theta)\mathcal{N} + \frac{1}{2}(\bar{\theta}\gamma_5\gamma_\mu\theta)V^\mu$
$+ i(\bar{\theta}\gamma_5\theta)[\bar{\theta}(\lambda + \frac{i}{\sqrt{2}}\not{\partial}\psi)] - \frac{1}{4}(\bar{\theta}\gamma_5\theta)^2[\mathcal{D} - \frac{1}{2}\Box\mathcal{S}]$
- left chiral scalar superfield: $\hat{\mathcal{S}}_L(x,\theta) = \mathcal{S}(\hat{x}) + i\sqrt{2}\bar{\theta}\psi_L(\hat{x}) + i\bar{\theta}\theta_L\mathcal{F}(\hat{x})$ where $\hat{x}_\mu = x_\mu + \frac{i}{2}\bar{\theta}\gamma_5\gamma_\mu\theta$
- right chiral scalar superfield: $\hat{\mathcal{S}}_R(x,\theta) = \mathcal{S}(\hat{x}^\dagger) - i\sqrt{2}\bar{\theta}\psi_R(\hat{x}^\dagger) - i\bar{\theta}\theta_R\mathcal{F}(\hat{x}^\dagger)$
- multiplication rules
  - LCSSF$\times$ LCSSF$=$ LCSSF
  - RCSSF$\times$ RCSSF $=$ RCSSF
  - LCSSF$\times$ RCSSF $=$ general superfield
- D-term of general SF transforms to total derivative
- F-term of LCSSF or RCSSF transforms into total derivative

Supersymmetric Lagrangians

- Since D and F terms transform into total derivatives, they are candidates for SUSY Lagrangians!
- The *superpotential* $\hat{f}$ is a function of LCSSFs only. Hence it is itself a LCSSF, and its F term is a candidate Lagrangian
- The *Kähler potential* K is a function of LCSSFs times RCSSFs. Hence it is a general superfield and its D term is a candidate Lagrangian
- Augmenting the superfields with *gauge* superfields $\hat{\Phi}_A = \frac{1}{2}(\bar{\theta}\gamma_5\gamma_\mu\theta)V_A^\mu + i\bar{\theta}\gamma_5\theta \cdot \bar{\theta}\lambda_A - \frac{1}{4}(\bar{\theta}\gamma_5\theta)^2\mathcal{D}_A$ (in WZ gauge) or $\hat{W}_A(\hat{x},\theta) = \lambda_{LA}(\hat{x}) + \frac{1}{2}\gamma^\mu\gamma^\nu F_{\mu\nu A}(\hat{x})\theta_L - i\bar{\theta}\theta_L(\not{D}\lambda_R)_A - i\mathcal{D}_A(\hat{x})\theta_L$ allows one to write a *Master formula* for supersymmetric gauge theories!

Master formula for SUSY gauge theories

$$\begin{aligned}
\mathcal{L} &= \sum_i (D_\mu \mathcal{S}_i)^\dagger (D^\mu \mathcal{S}_i) + \frac{i}{2}\sum_i \bar{\psi}_i \not{D}\psi_i + \sum_{\alpha,A}\left[\frac{i}{2}\bar{\lambda}_{\alpha A}(\not{D}\lambda)_{\alpha A} - \frac{1}{4}F_{\mu\nu\alpha A}F^{\mu\nu}_{\alpha A}\right] \\
&- \sqrt{2}\sum_{i,\alpha,A}\left(\mathcal{S}_i^\dagger g_\alpha t_{\alpha A}\bar{\lambda}_{\alpha A}\frac{1-\gamma_5}{2}\psi_i + \text{h.c.}\right) \\
&- \frac{1}{2}\sum_{\alpha,A}\left[\sum_i \mathcal{S}_i^\dagger g_\alpha t_{\alpha A}\mathcal{S}_i + \xi_{\alpha A}\right]^2 - \sum_i \left|\frac{\partial \hat{f}}{\partial \hat{\mathcal{S}}_i}\right|^2_{\hat{\mathcal{S}}=\mathcal{S}} \\
&- \frac{1}{2}\sum_{i,j}\bar{\psi}_i\left[\left(\frac{\partial^2 \hat{f}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}}\frac{1-\gamma_5}{2} + \left(\frac{\partial^2 \hat{f}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j}\right)^\dagger_{\hat{\mathcal{S}}=\mathcal{S}}\frac{1+\gamma_5}{2}\right]\psi_j,
\end{aligned}$$

where the covariant derivatives are given by,

$$D_\mu \mathcal{S} = \partial_\mu \mathcal{S} + i\sum_{\alpha,A} g_\alpha t_{\alpha A} V_{\mu\alpha A}\mathcal{S},$$

$$
\begin{aligned}
D_\mu\psi &= \partial_\mu\psi + i\sum_{\alpha,A} g_\alpha (t_{\alpha A} V_{\mu\alpha A})\psi_L \\
&\quad -i\sum_{\alpha,A} g_\alpha (t^*_{\alpha A} V_{\mu\alpha A})\psi_R, \\
(\not{D}\lambda)_{\alpha A} &= \not{\partial}\lambda_{\alpha A} + i g_\alpha \left(t^{adj}_{\alpha B}\ \not{V}_{\alpha B}\right)_{AC}\lambda_{\alpha C}, \\
F_{\mu\nu\alpha A} &= \partial_\mu V_{\nu\alpha A} - \partial_\nu V_{\mu\alpha A} - g_\alpha f_{\alpha ABC} V_{\mu\alpha B} V_{\nu\alpha C}.
\end{aligned}
$$

Supersymmetry breaking

- Spontaneous breaking of global SUSY is possible:
  $\langle 0|\mathcal{F}_i|0\rangle \neq 0$ or $\langle 0|\mathcal{D}_A|0\rangle \neq 0$ (F or D type breaking)
- May also explicitly break SUSY by adding *soft* SUSY breaking terms to $\mathcal{L}$:
  - linear terms in the scalar field $\mathcal{S}_i$ (relevant only for singlets of all symmetries),
  - scalar masses,
  - and bilinear or trilinear operators of the form $\mathcal{S}_i\mathcal{S}_j$ or $\mathcal{S}_i\mathcal{S}_j\mathcal{S}_k$ (where $\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j$ and $\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j\hat{\mathcal{S}}_k$ occur in the superpotential),
  - and finally, in gauge theories, gaugino masses, one for each factor of the gauge group,

Recipe for SUSY model building

- Choose the gauge symmetry (adopting appropriate gauge superfields for each gauge symmetry)
- Choose matter and Higgs representations included as LCSSFs
- Choose the superpotential $\hat{f}$ as a gauge invariant *analytic function* of LCSSFs; degree is ≤ 3 for renormalizable theory
- Adopt all allowed gauge invariant soft SUSY breaking terms; these are generally chosen to *parametrize our ignorance* of the mechanism of SUSY breaking
- The Master formula, augmented by the soft SUSY breaking terms, gives the final Lagrangian of the theory.

The Minimal Supersymmetric Standard Model (MSSM)

Construction

★ gauge symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$
\begin{aligned}
B_\mu &\rightarrow \hat{B} \ni (\lambda_0, B_\mu, \mathcal{D}_B), \\
W_{A\mu} &\rightarrow \hat{W}_A \ni (\lambda_A, W_{A\mu}, \mathcal{D}_{WA}),\ A = 1,2,3,\ \text{and} \\
g_{A\mu} &\rightarrow \hat{g}_A \ni (\tilde{g}_A, G_{A\mu}, \mathcal{D}_{gA}),\ A = 1,\cdots,8.
\end{aligned}
$$

★ matter content: 3 generations quarks and leptons

$$
\begin{aligned}
\begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix} &\rightarrow \hat{L}_i \equiv \begin{pmatrix} \hat{\nu}_i \\ \hat{e}_i \end{pmatrix}, \\
(e_R)^c &\rightarrow \hat{E}_i^c, \\
\begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} &\rightarrow \hat{Q}_i \equiv \begin{pmatrix} \hat{u}_i \\ \hat{d}_i \end{pmatrix},
\end{aligned}
$$

$$
\begin{aligned}
(u_R)^c &\rightarrow \hat{U}_i^c, \\
(d_R)^c &\rightarrow \hat{D}_i^c,
\end{aligned}
$$

where *e.g.*

$$\hat{e} = \tilde{e}_L(\hat{x}) + i\sqrt{2}\bar{\theta}\psi_{eL}(\hat{x}) + i\bar{\theta}\theta_L\mathcal{F}_e(\hat{x}) \tag{1}$$

while

$$\hat{E}^c = \tilde{e}_R^\dagger(\hat{x}) + i\sqrt{2}\bar{\theta}\psi_{E^cL}(\hat{x}) + i\bar{\theta}\theta_L\mathcal{F}_{E^c}(\hat{x}). \tag{2}$$

SM Dirac fermions are constructed out of Majorana fermions via

$$e = P_L\psi_e + P_R\psi_{E^c}. \tag{3}$$

where in chiral rep. of γ matrices

$$\psi_e = \begin{pmatrix} e_1 \\ e_2 \\ -e_2^* \\ e_1^* \end{pmatrix} \text{ and } \quad \psi_{E^c} = \begin{pmatrix} e_4^* \\ -e_3^* \\ e_3 \\ e_4 \end{pmatrix}.$$

The MSSM (part 2)

Construction

★ Higgs multiplets:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to \hat{H}_u = \begin{pmatrix} \hat{h}_u^+ \\ \hat{h}_u^0 \end{pmatrix}. \quad (4)$$

Now spin $\frac{1}{2}$ higgsinos with $Y = 1$ can circulate in triangle anomalies; cancel with additional $Y = -1$ doublet:

$$\hat{H}_d = \begin{pmatrix} \hat{h}_d^- \\ \hat{h}_d^0 \end{pmatrix}, \quad (5)$$

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\hat{L} = \begin{pmatrix} \hat{\nu}_{eL} \\ \hat{e}_L \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	-1
$\hat{E}^c$	$\mathbf{1}$	$\mathbf{1}$	2
$\hat{Q} = \begin{pmatrix} \hat{u}_L \\ \hat{d}_L \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{3}$
$\hat{U}^c$	$\mathbf{3}^*$	$\mathbf{1}$	$-\frac{4}{3}$
$\hat{D}^c$	$\mathbf{3}^*$	$\mathbf{1}$	$\frac{2}{3}$
$\hat{H}_u = \begin{pmatrix} \hat{h}_u^+ \\ \hat{h}_u^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
$\hat{H}_d = \begin{pmatrix} \hat{h}_d^- \\ \hat{h}_d^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}^*$	**-1**

The MSSM (part 3)

Construction

★ superpotential

$$\hat{f} = \mu \hat{H}_u^a \hat{H}_{da} + \sum_{i,j=1,3} \left[(\mathbf{f}_u)_{ij} \epsilon_{ab} \hat{Q}_i^a \hat{H}_u^b \hat{U}_j^c + (\mathbf{f}_d)_{ij} \hat{Q}_i^a \hat{H}_{da} \hat{D}_j^c + (\mathbf{f}_e)_{ij} \hat{L}_i^a \hat{H}_{da} \hat{E}_j^c \right]. \tag{6}$$

The following terms are gauge invariant and renormalizable, but violate baryon and lepton number. They are excluded if one requires R-parity conservation $R = (-1)^{3(B-L)+2s}$:

$$\hat{f}_{\not{L}} = \sum_{i,j,k} \left[\lambda_{ijk} \epsilon_{ab} \hat{L}_i^a \hat{L}_j^b \hat{E}_k^c + \lambda'_{ijk} \epsilon_{ab} \hat{L}_i^a \hat{Q}_j^b \hat{D}_k^c \right] + \sum_i \mu'_i \epsilon_{ab} \hat{L}_i^a \hat{H}_u^b, \tag{7}$$

and

$$\hat{f}_{\not{B}} = \sum_{i,j,k} \lambda''_{ijk} \hat{U}_i^c \hat{D}_j^c \hat{D}_k^c, \tag{8}$$

The MSSM (part 4)

⋆ soft SUSY breaking terms

$$
\begin{aligned}
\mathcal{L}_{\rm soft} &= -\left[\tilde{Q}_i^\dagger \mathbf{m^2_Q}_{ij}\tilde{Q}_j + \tilde{d}^\dagger_{Ri}\mathbf{m^2_D}_{ij}\tilde{d}_{Rj} + \tilde{u}^\dagger_{Ri}\mathbf{m^2_U}_{ij}\tilde{u}_{Rj}\right.\\
&+ \left.\tilde{L}_i^\dagger \mathbf{m^2_L}_{ij}\tilde{L}_j + \tilde{e}^\dagger_{Ri}\mathbf{m^2_E}_{ij}\tilde{e}_{Rj} + m^2_{H_u}|H_u|^2 + m^2_{H_d}|H_d|^2\right]\\
&- \frac{1}{2}\left[M_1\bar{\lambda}_0\lambda_0 + M_2\bar{\lambda}_A\lambda_A + M_3\bar{\tilde{g}}_B\tilde{g}_B\right]\\
&- \frac{i}{2}\left[M_1'\bar{\lambda}_0\gamma_5\lambda_0 + M_2'\bar{\lambda}_A\gamma_5\lambda_A + M_3'\bar{\tilde{g}}_B\gamma_5\tilde{g}_B\right]\\
&+ \left[(\mathbf{a_u})_{ij}\epsilon_{ab}\tilde{Q}_i^a H_u^b\tilde{u}^\dagger_{Rj} + (\mathbf{a_d})_{ij}\tilde{Q}_i^a H_{da}\tilde{d}^\dagger_{Rj} + (\mathbf{a_e})_{ij}\tilde{L}_i^a H_{da}\tilde{e}^\dagger_{Rj} + \text{h.c.}\right]\\
&+ \left[bH_u^a H_{da} + \text{h.c.}\right],
\end{aligned}
$$

The MSSM (part 5)

- count parameters
  - g_1, g_2, g_3, θ_{QCD}
  - gaugino masses M_1, M_1', M_2, M_2', M_3 (M_3' absorbed into $\tilde{g}$)
  - $m^2_{H_u}$, $m^2_{H_d}$, μ, b (phase of b absorbed)
  - $5 \times (6+3) = 45$ in sfermion mass matrices
  - $3 \times (3 \times 3 \times 2) = 54$ in Yukawa matrices
  - $3 \times (3 \times 3 \times 2) = 54$ in a-term matrices
  - a global $U(3)^5$ transformation in matter allows $45 - 2 = 43$ phases absorbed into matter sfermions
  - total parameters $= 9 + 5 + 45 + 54 + 54 - 43 = 124$
- most choices are excluded: lead to FCNC or CP violating effects
  - solutions: universality, decoupling, alignment

The MSSM (part 6): electroweak breaking

- ⋆ construct scalar potential of MSSM: $V = V_F + V_D + V_{soft}$
- ⋆ minimization conditions: $\partial V/\partial h_u^0 = \partial V \partial h_d^0 = 0$ has solution so $\langle h_u^0 \rangle = v_u$, $\langle h_d^0 \rangle = v_d$ with $\tan\beta \equiv v_u/v_d$
  - $W^\pm$, Z_0 become massive as in SM
  - SM fermions all gain mass *e.g.* $m_e = f_e v_d$
- ⋆ states with same spin/charge can mix
  - predict many new states to exist!

The MSSM (part 7): new matter states

- ⋆ spin $\frac{1}{2}$ massive color octet: gluino $\tilde{g}$
- ⋆ spin $\frac{1}{2}$ bino, wino, neutral higgsinos $\Rightarrow$ neutralinos $\widetilde{Z}_1,\ \widetilde{Z}_2,\ \widetilde{Z}_3,\ \widetilde{Z}_4$
- ⋆ spin $\frac{1}{2}$ charged wino, higgsinos $\Rightarrow$ charginos $\widetilde{W}_1^{\pm},\ \widetilde{W}_2^{\pm}$
- ⋆ spin-0 squarks: $\tilde{u}_L,\ \tilde{u}_R,\ \tilde{d}_L,\ \tilde{d}_R, \tilde{s}_L,\ \tilde{s}_R,\ \tilde{c}_L,\ \tilde{c}_R,\ \tilde{b}_1,\ \tilde{b}_2,\ \tilde{t}_1,\ \tilde{t}_2$
- ⋆ spin-0 sleptons: $\tilde{e}_L,\ \tilde{e}_R,\ \tilde{\nu}_e,\ \tilde{\mu}_L,\ \tilde{\mu}_R,\ \tilde{\nu}_\mu,\ \tilde{\tau}_1,\ \tilde{\tau}_2, \tilde{\nu}_\tau$
- ⋆ spin-0 higgs bosons: $h,\ H,\ A,\ H^{\pm}$ (h usually SM-like)

The MSSM: summary

- ⋆ The MSSM includes the SM as a sub-theory, but also includes many new states of matter
- ⋆ Unlike the SM, the MSSM is free of quadratic divergences in the scalar sector
- ⋆ Thus, the MSSM can accommodate vastly different mass scales, $e.g.$ M_{weak} and M_{GUT} or M_{string}
- ⋆ The 124 parameter MSSM is likely to be the low energy effective theory of some more fundamental theory, perhaps one linked to GUTs or strings
- ⋆ The MSSM provides for us the possible physical states and Feynman rules needed for making predictions of physical phenomena
- ⋆ The MSSM parameters are highly constrained by bounds from FCNCs, CP-violation, etc.

The MSSM: RGEs

- ⋆ If the MSSM is to be valid between vastly different mass scales, then it is important to relate parameters between these scales.
- ⋆ The gauge couplings, Yukawa couplings, μ term and soft breaking parameter evolution is governed by *renormalization group equations*, or RGEs
- ⋆ For gauge couplings, these have the form

$$\frac{dg_i}{dt} = \beta(g_i) \quad with \quad t = \log Q \tag{9}$$

- ⋆ In SM,

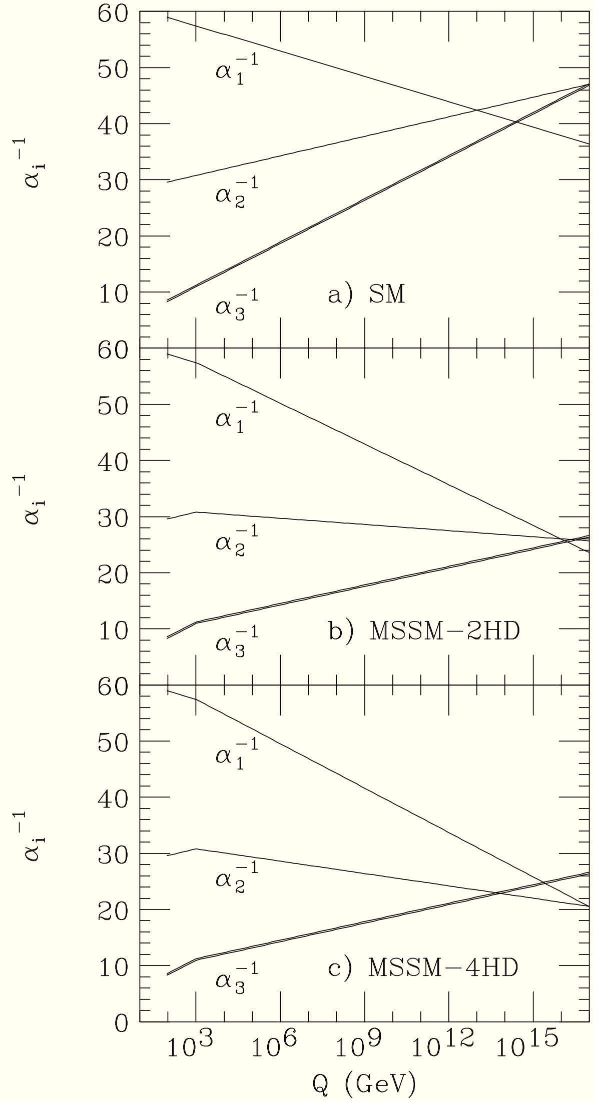
$$\beta(g) = -\frac{g^3}{16\pi^2}\left[\frac{11}{3}C(G) - \frac{2}{3}n_F S(R_F) - \frac{1}{3}n_H S(R_H)\right]. \tag{10}$$

- ⋆ In MSSM, the gauginos, matter and Higgs scalars also contribute:

$$\beta(g) = -\frac{g^3}{16\pi^2}\left[3C(G) - S(R)\right], \tag{11}$$

★ Can use the precision values of g_1, g_2 and g_3 measured at $Q = M_Z$ at LEP2 as boundary conditions, and extrapolate to high energy

Gauge coupling evolution



The MSSM: RGEs continued

$$
\begin{aligned}
\frac{dM_i}{dt} &= \frac{2}{16\pi^2} b_i g_i^2 M_i, \\
\frac{dA_t}{dt} &= \frac{2}{16\pi^2}\left(-\sum_i c_i g_i^2 M_i + 6f_t^2 A_t + f_b^2 A_b\right), \\
\frac{dA_b}{dt} &= \frac{2}{16\pi^2}\left(-\sum_i c_i' g_i^2 M_i + 6f_b^2 A_b + f_t^2 A_t + f_\tau^2 A_\tau\right), \\
\frac{dA_\tau}{dt} &= \frac{2}{16\pi^2}\left(-\sum_i c_i'' g_i^2 M_i + 3f_b^2 A_b + 4f_\tau^2 A_\tau\right), \\
\frac{dB}{dt} &= \frac{2}{16\pi^2}\left(-\frac{3}{5} g_1^2 M_1 - 3g_2^2 M_2 + 3f_b^2 A_b + 3f_t^2 A_t + f_\tau^2 A_\tau\right), \\
\frac{d\mu}{dt} &= \frac{\mu}{16\pi^2}\left(-\frac{3}{5} g_1^2 - 3g_2^2 + 3f_t^2 + 3f_b^2 + f_\tau^2\right),
\end{aligned}
$$

$$\frac{dm_{Q_3}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{1}{15}g_1^2M_1^2 - 3g_2^2M_2^2 - \frac{16}{3}g_3^2M_3^2 + \frac{1}{10}g_1^2S + f_t^2X_t + f_b^2X_b\right),$$

$$\frac{dm_{\tilde{t}_R}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{16}{15}g_1^2M_1^2 - \frac{16}{3}g_3^2M_3^2 - \frac{2}{5}g_1^2S + 2f_t^2X_t\right),$$

$$\frac{dm_{\tilde{b}_R}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{4}{15}g_1^2M_1^2 - \frac{16}{3}g_3^2M_3^2 + \frac{1}{5}g_1^2S + 2f_b^2X_b\right),$$

$$\frac{dm_{L_3}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 - \frac{3}{10}g_1^2S + f_\tau^2X_\tau\right),$$

$$\frac{dm_{\tilde{\tau}_R}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{12}{5}g_1^2M_1^2 + \frac{3}{5}g_1^2S + 2f_\tau^2X_\tau\right),$$

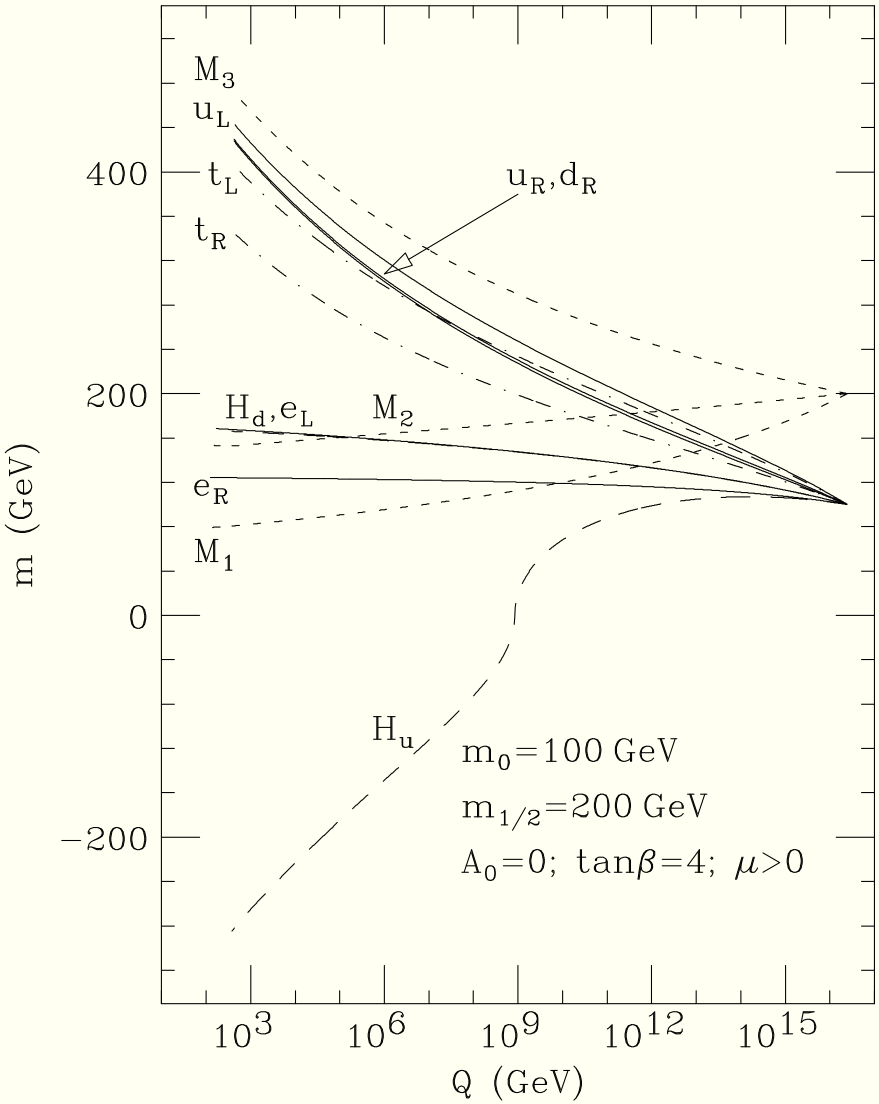
$$\frac{dm_{H_d}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 - \frac{3}{10}g_1^2S + 3f_b^2X_b + f_\tau^2X_\tau\right),$$

$$\frac{dm_{H_u}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 + \frac{3}{10}g_1^2S + 3f_t^2X_t\right),$$

where m_{Q_3} and m_{L_3} denote the mass term for the third generation $SU(2)$ squark and slepton doublet respectively, and

$$\begin{aligned} X_t &= m_{Q_3}^2 + m_{\tilde{t}_R}^2 + m_{H_u}^2 + A_t^2, \\ X_b &= m_{Q_3}^2 + m_{\tilde{b}_R}^2 + m_{H_d}^2 + A_b^2, \\ X_\tau &= m_{L_3}^2 + m_{\tilde{\tau}_R}^2 + m_{H_d}^2 + A_\tau^2, \text{ and} \\ S &= m_{H_u}^2 - m_{H_d}^2 + Tr\left[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_U^2 + \mathbf{m}_D^2 + \mathbf{m}_E^2\right]. \end{aligned}$$

Soft term evolution and radiative EWSB for $m_t \sim 175$ GeV



Supergravity

- ⋆ In SUSY transformation operator $e^{-i\bar{\alpha}Q}$ let $\alpha = \alpha(x)$ so we have a *local* SUSY transformation
- ⋆ Just as for gauge theories, will need to introduce a gauge field to maintain covariance: $\psi_\mu(x)$, a spin $\frac{3}{2}$ vector-spinor (Rarita-Schwinger) field
- ⋆ To maintain local SUSY, will have to introduce bosonic partner: a spin 2 field $g_{\mu\nu}(x)$
  - $g_{\mu\nu}$ is massless, and in classical limit obeys Einstein GR eq'ns of motion: it is the graviton field
  - usually, $g_{\mu\nu}(x)$ is traded for the equivalent vierbein field $e^a_\mu(x)$, where $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$, where η_{ab} is the Minkowski metric
- ⋆ Can derive a Master formula for *supergravity* (SUGRA) gauge theories

Supergravity

- SUGRA is inherently non-renormalizable
- SUGRA theories specified by Kähler function

$$G(\hat{\mathcal{S}}^\dagger, \hat{\mathcal{S}}) = K(\hat{\mathcal{S}}^\dagger, \hat{\mathcal{S}}) + \log |\hat{f}(\hat{\mathcal{S}})|^2, \tag{12}$$

  and gauge kinetic function

$$f_{AB}(\hat{S}). \tag{13}$$

- SUGRA can be spontaneously broken just as SUSY can
- Since SUGRA is local SUSY theory, have a super-Higgs mechanism, wherein the gravitino field ψ_μ gains a mass $m_{3/2}$ while graviton remains massless
- Can embed MSSM in a SUGRA theory along with gauge singlet field(s) $\hat{h}_m$ with superpotential such that SUGRA is spontaneously broken (hidden sector)
- SUGRA breaking communicated from hidden sector to visible sector via gravity: induces soft SUSY breaking terms of order $\sim m_{3/2}$!

Minimal Supergravity model (mSUGRA)

- Assume MSSM embedded in a SUGRA theory
- SUSY broken in hidden sector with $m_{3/2} \sim M_{weak} \sim 1$ TeV
- For simple choice of Kähler function and gauge kinetic function, will induce $universal$ scalar masses m_0, gaugino masses $m_{1/2}$ and trilinears A_0
- Inspired by gauge coupling unification, these universal choices usually taken at $Q = M_{GUT} \simeq 2 \times 10^{16}$ GeV
- Evolve couplings and soft parameters from M_{GUT} to M_{weak}; $m_{H_u}^2 \rightarrow$ negative, breaking EW symmetry.
- All sparticle masses, mixings at $Q = M_{weak}$ calculated in terms of small parameter set:

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ sign(\mu) \tag{14}$$

- The mSUGRA model is paradigm SUSY model for phenomenological analysis, but is not likely to be the complete story.

Precision calculation of SUSY spectrum:

- need full 2-loop RGE running: gauge, Yukawa, SSB terms
- RG-improved 1-loop effective potential evaluated at optimized scale
- t, b, τ threshold effects
- full set of 1-loop sparticle/Higgs mass corrections
- several public codes are available

Sparticle mass spectra

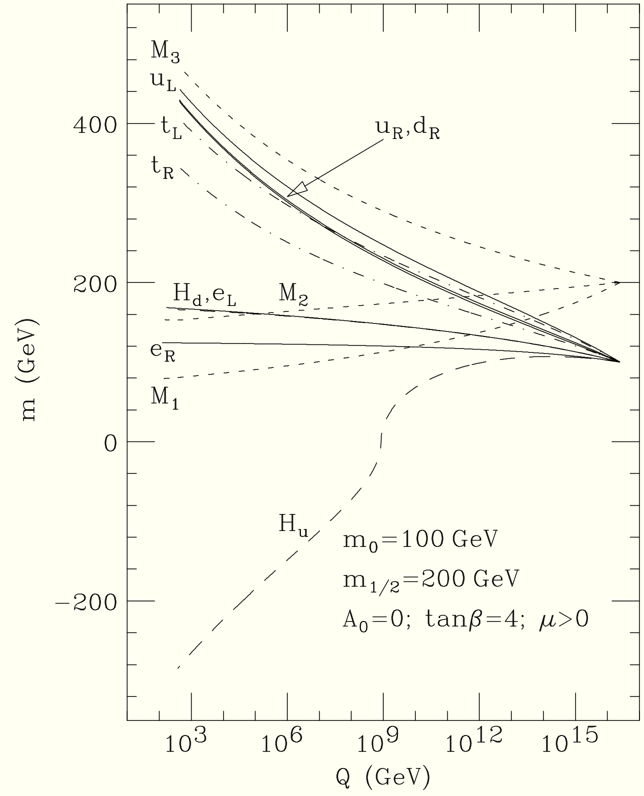
★ Mass spectra codes

★ RGE running: $M_{GUT} \to M_{weak}$

- Isajet (HB, Paige, Protopopescu, Tata)
  - ∗ $\geq$7.72: Isatools
- SuSpect (Djouadi, Kneur, Moultaka)
- SoftSUSY (Allanach)
- Spheno (Porod)

★ Comparison (Belanger, Kraml, Pukhov)

★ Website: `http://kraml.home.cern.ch/kraml/comparison/`



SUSY model #1: minimal supergravity (mSUGRA or CMSSM)

- ⋆ Assume nature described by $N = 1$ supergravity gauge theory Lagrangian:
- ⋆ To accomodate SUSY breaking, must introduce a "hidden sector", consisting of a field or fields which are SM singlets (hence hidden)
- ⋆ Arrange superpotential of hidden sector such that supergravity breaks at mass scale $m \sim 10^{11}$ GeV via *superHiggs* mechanism
- ⋆ Gravitational interactions *induce* exactly the right form of soft SUSY breaking masses, with $m_{SUSY} \sim m_{3/2} \sim m^2/M_P \sim (10^{11}\ GeV)^2/10^{19}\ GeV \sim 10^3\ GeV$
- • gravitino decouples? $\widetilde{Z}_1 = LSP$ or $\tilde{G}$ (see papers by Feng/Ellis)
- ⋆ simplest models ($e.g.$ Polonyi superpotential) give:
  - single scalar mass m_0,
  - gaugino mass $m_{1/2}$,
  - trilinear term A_0, bilinear term B

- ⋆ EWSB radiatively due to large m_t
- ⋆ EWSB condition: $B \to \tan\beta$; μ^2 fixed by M_Z
- ⋆ parameter space: $m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ sign(\mu)$
- ⋆ this is simplest choice and a baseline model, but **many** other possibilities depending on high scale physics
  - non-universal matter scalars: $m^2_{Q_i}$, $m^2_{U_i}$, $m^2_{D_i}$, $m^2_{L_i}$, $M^2_{E_i}$
  - non-universal Higgs scalars: $m^2_{H_u}$, $M^2_{H_d}$
  - non-universal gaugino masses: M_1, M_2, M_3
  - non-universal A terms: A_t, A_b, A_τ
  - FC soft SUSY breaking terms
  - large CP violating phases
  - additional fields beyond MSSM below M_{GUT}?
  - R-parity violating couplings
  - $\cdots$

SUSY model #2: gauge-mediated SUSY breaking (GMSB)

- ⋆ Assume 3 sectors: MSSM, messenger sector, hidden sector
- ⋆ SUSY breaking in HS
- ⋆ SUSY breaking communicated to MSSM via gauge interactions from messenger sector
- ⋆ $m_{SUSY} \sim \frac{g_i^2}{16\pi^2}\frac{\langle F_S\rangle}{M} \sim 1$ TeV, where $M =$messenger mass and $\langle F_S\rangle$ is SUSY breaking scale
- ⋆ gravitino $m_{\tilde{G}} = \frac{\langle F\rangle}{\sqrt{3}M_P}$ can be very light $\sim keV$ so $\tilde{G} = LSP$ and $e.g.$ $\widetilde{Z}_1 \to \gamma\tilde{G}$
- ⋆ EWSB radiatively due to large m_t as usual

GMSB parameter space

⋆ parameter space:

- $\Lambda,\ M,\ n_5,\ \tan\beta,\ sign(\mu), C_{grav}$
- $\Lambda \sim 10-150$ TeV sets sparticle mass scale $m_{SUSY} = \frac{\alpha_i}{4\pi} n_5 \Lambda$
- M =messenger scale $> \Lambda$
- $n_5 = \#$ of messenger fields
- C_{grav} just affects how long lived the NLSP is
- at colliders: get isolated photons from $\widetilde{Z}_1 \to \gamma\tilde{G}$ or long-lived charged tracks if $\tilde{\tau}_1 \to \tau\tilde{G}$ is NLSP

⋆ model solves SUSY flavor problem at price of introducing non-minimal messenger sector

SUSY model #3: anomaly-mediated SUSY breaking (AMSB)

- ⋆ supergravity theories always have 1-loop contributions to soft breaking terms of order $m_{SUSY} \sim m_{3/2}/16\pi^2$ coming from superconformal anomaly: usually suppressed compared to tree level SUGRA contribution
- ⋆ suppose hidden sector is "sequestered" in extra dimensions
- ⋆ then if $m_{3/2} \sim 10 - 100$ TeV, AMSB contribution to sparticle masses is dominant
- ⋆ gauginos: $M_i = \frac{\beta_i}{g_i} m_{3/2}$
- ⋆ scalars: $m^2_{\tilde{f}} = -\frac{1}{4}\left\{\frac{d\gamma}{dg}\beta_g + \frac{d\gamma}{df}\beta_f\right\} m^2_{3/2}$
- ⋆ EWSB radiatively due to large m_t
- ⋆ slepton masses tachyonic $m^2_{\tilde{\ell}} < 0$ so add by hand universal contribution m_0^2 (or other solutions)

AMSB parameter space

- ⋆ parameter space:
  - m_0, $m_{3/2}$ $\tan\beta$, $sign(\mu)$
- ⋆ LSP =lightest $\widetilde{Z}_1$ which is *wino-like*
- ⋆ $m_{\widetilde{W}_1} - m_{\widetilde{Z}_1} \sim 200$ MeV so $\widetilde{W}_1 \to \widetilde{Z}_1\pi^+$ and may give an observable track of few cm length: possibly observable
- ⋆ wino-like $\widetilde{Z}_1$ gives very low relic density: hard to explain dark matter
- ⋆ solves SUSY flavor problem but tachyonic masses...

Conclusions

- ⋆ General formulae for constructing softly broken SUSY gauge theories
- ⋆ most important example: MSSM
  - MSSM $\Rightarrow$ stable hierarchy $M_{weak} - M_{Pl.}$
  - RGEs: can connect weak scale to GUT/string scale physics: desert hypothesis
- ⋆ spectra generation
  - mSUGRA
  - NUSUGRA
  - GMSB
  - AMSB
  - ...
- ⋆ demos: Isasugra, comparison page