Lecture 1. Supersymmetry: Introduction

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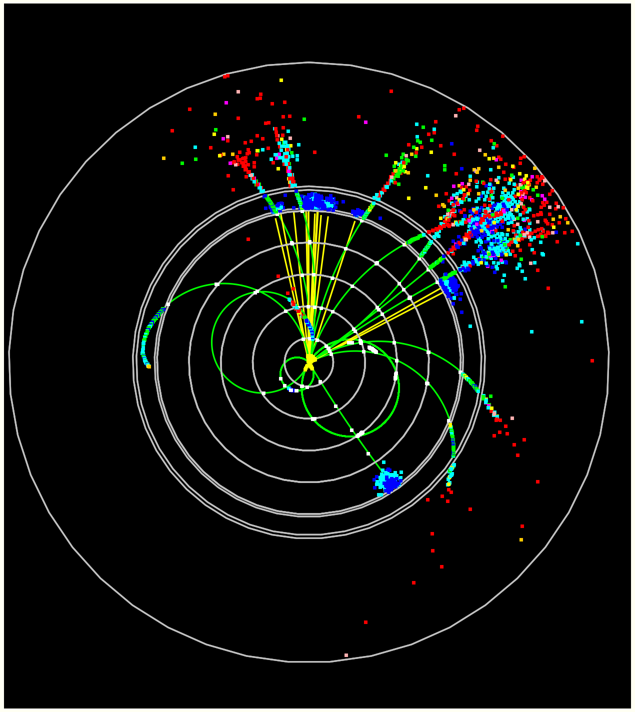
⋆ Outline

- Briefly, the Standard Model
- What is supersymmetry (SUSY)?
- Motivations for SUSY
- The Wess-Zumino model
- SUSY gauge theories
- SUSY breaking



The Standard Model of Particle Physics

- ⋆ gauge symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow g_{\mu A},\ W_{\mu i},\ B_\mu$
- ⋆ matter content: 3 generations quarks and leptons

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \ u_R,\ d_R;\ \begin{pmatrix} \nu \\ e \end{pmatrix}_L,\ e_R \tag{1}$$

- ⋆ Higgs sector $\Rightarrow$ spontaneous electroweak symmetry breaking:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \tag{2}$$

- ⋆ $\Rightarrow$ massive $W^\pm$, Z^0, massless γ, massive quarks and leptons
- ⋆ $\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_{Yuk.} + \mathcal{L}_{Higgs}$: 19 parameters
- ⋆ good-to-excellent description of (almost) *all* accelerator data!

Shortcomings of SM

Data

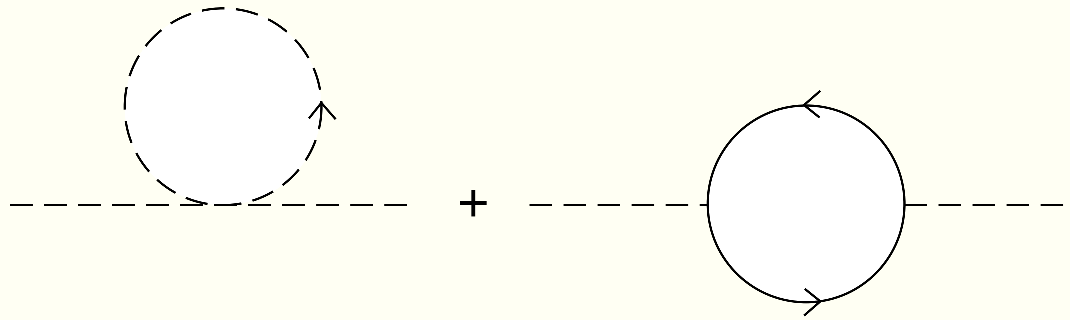
- ⋆ neutrino masses and mixing
- ⋆ baryogenesis (matter anti-matter asymmetry)
- ⋆ cold dark matter
- ⋆ dark energy

Theory

- ⋆ quadratic divergences in scalar sector $\Rightarrow$ fine-tuning
- ⋆ origin of generations
- ⋆ explanation of masses
- ⋆ origin of gauge symmetry/ quantum numbers
- ⋆ unification with gravity

Problem of scalar quadratic divergences

⋆ Structure of scalar self energies



- $\delta m^2_{H_{SM}} = \langle H_{SM} | \frac{g^2 m^2_{H_{SM}}}{32 M_W^2} H^4_{SM} | H_{SM} \rangle = 12 \frac{g^2 m^2_{H_{SM}}}{32 M_W^2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2_{H_{SM}}}$
  $= \frac{12 g^2 m^2_{H_{SM}}}{32 M_W^2} \frac{1}{16\pi^2} \left(\Lambda^2 - m^2_{H_{SM}} \ln \frac{\Lambda^2}{m^2_{H_{SM}}} + \mathcal{O}(\frac{1}{\Lambda^2}) \right)$
- $m^2_{H_{SM}}(\mathrm{phys}) \simeq m^2_{H_{SM}} + \frac{c}{16\pi^2}\Lambda^2$
- If $\Lambda \sim 10^{16}$ GeV, then fine-tuning to 1 part in 10^{26} needed
- Alternatively, if $\Lambda \sim 1$ TeV, then no fine-tuning, but then SM is valid only below 1 TeV scale

Supersymmetry

⋆ Supersymmetry (SUSY) is a spacetime symmetry

- most general extension of Poincare group
- a quantum symmetry- no classical analogue
- introduce spinorial generators Q
- relates fermions to bosons: $Q|B\rangle = |F\rangle$
- SUSY is a “square-root” of a translation

⋆ discovered gradually from ~ 1968 to 1974

- introduce fermions into string theory
- non-linear realizations
- Wess-Zumino 1974: first 4-d SUSY QFT

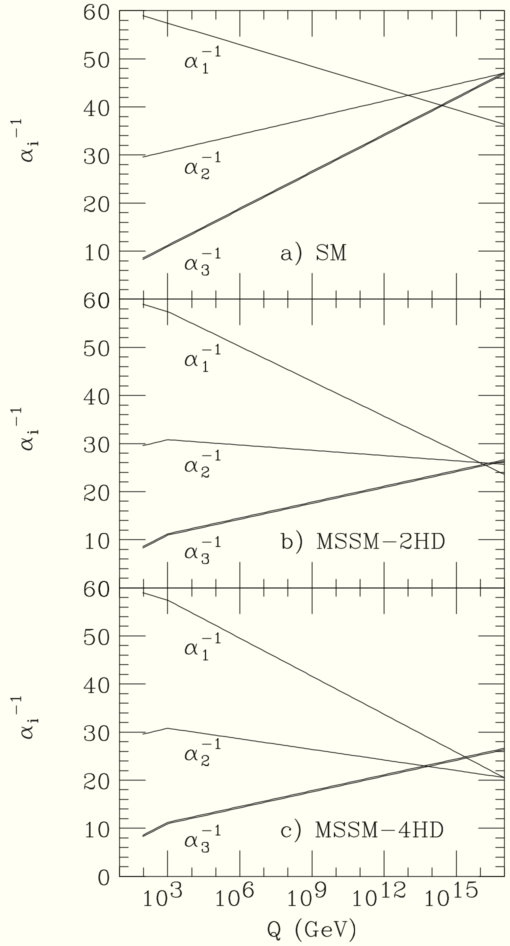
Supersymmetry motivations (theory)

- Aesthetics
  - nature likely to use most general extension of Poincaré group at some level
- ultra-violet behavior and fine-tuning: $m_{SUSY} \sim 1$ TeV!
- connection to gravity: local SUSY $=$ supergravity
- ultraviolet completeness: desert all the way to M_{GUT}?
- connection to strings: world-sheet SUSY vs. space-time SUSY

Supersymmetry motivations (experiment)

- gauge coupling unification: connection to GUTs

Gauge coupling evolution



Supersymmetry motivations (experiment)

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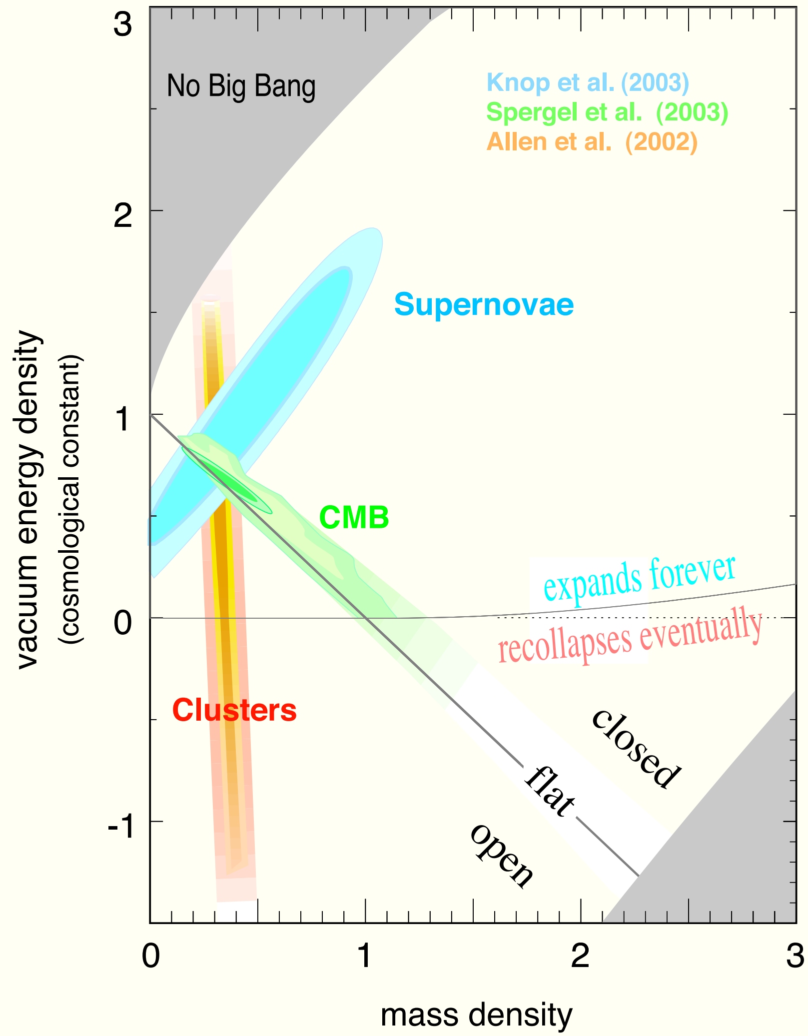
- cold dark matter

Dark matter versus dark energy



Supersymmetry motivations (experiment)

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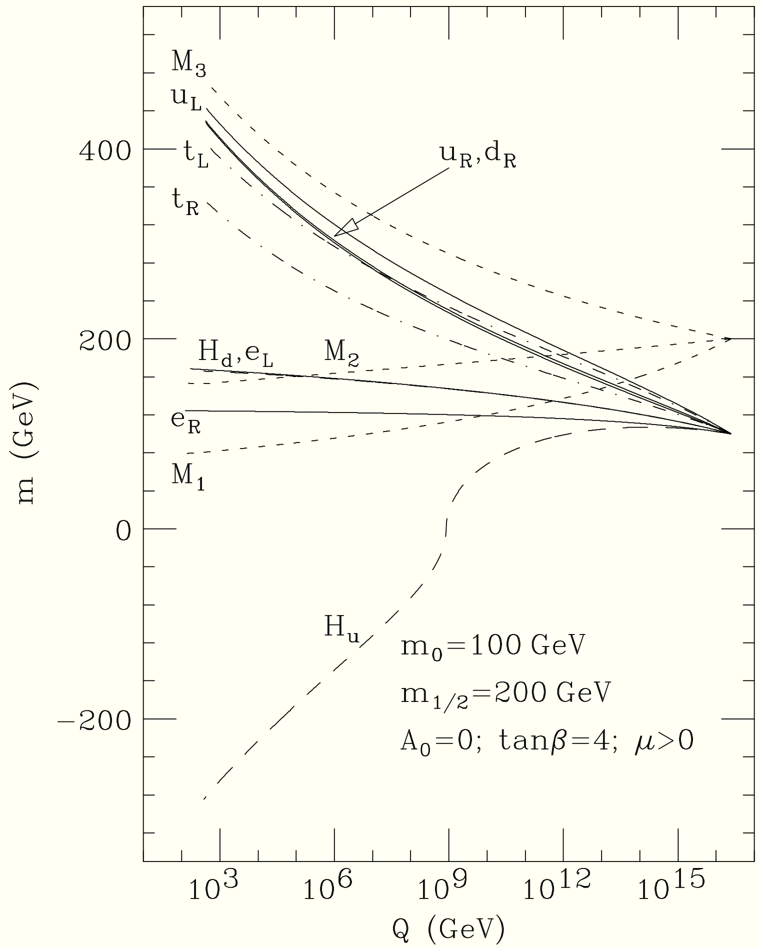
- radiative breakdown of EW symmetry (if $m_t \sim 100 - 200$ GeV)

Soft term evolution and radiative EWSB



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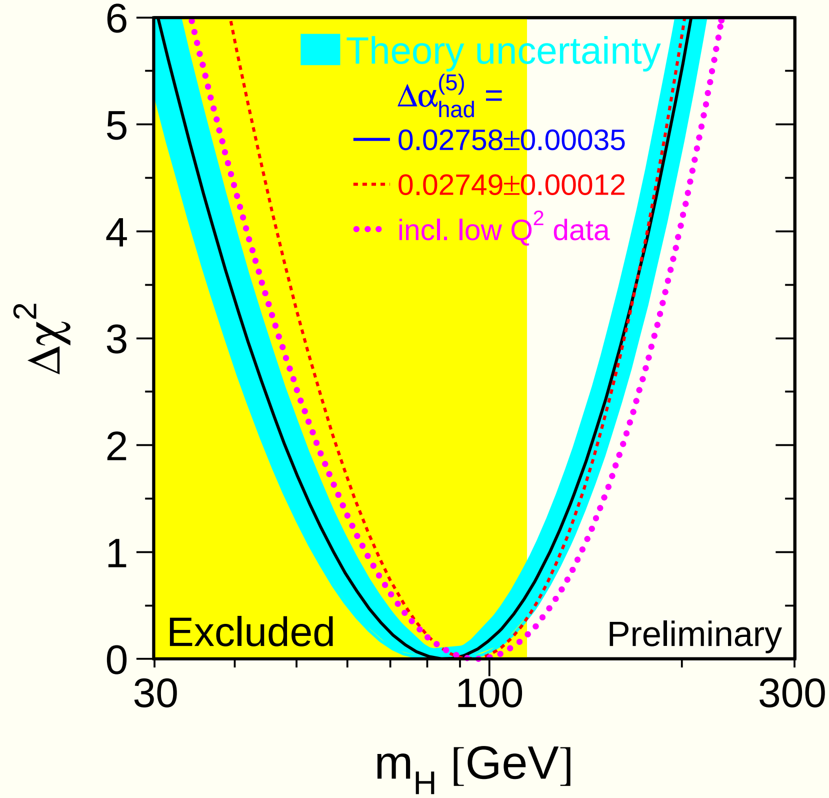
- decoupling in SUSY theories: tiny radiative corrections if $m_{SUSY} \gtrsim 100$ GeV
- mass of the Higgs boson: MSSM $\Rightarrow m_h \lesssim 135$ GeV

Likelihood distribution of Higgs mass



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- mechanisms for baryogenesis: EW, leptogenesis, Affleck-Dine
- neutrino mass: stabilize see-saw scale; GUTs

Focus of these lectures is on *supersymmetry*

"if we consider the main classes of new physics that are currently being contemplated$\cdots$, it is clear that (supersymmetry) is the most directly related to GUTs. SUSY offers a well defined model computable up to the GUT scale and is actually supported by the quantitative success of coupling unification in SUSY GUTs.For the other examples$\cdots$, all contact with GUTs is lost or at least is much more remote. $\cdots$ the SUSY picture$\cdots$ remains the standard way beyond the Standard Model"

G. Altarelli and F. Feruglio, hep-ph/0306265

Wess-Zumino toy SUSY model: 1974

- $\mathcal{L} = \mathcal{L}_{\rm kin.} + \mathcal{L}_{\rm mass}$
  - $\mathcal{L}_{\rm kin.} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\overline{\psi}\,\partial\!\!\!/\psi + \frac{1}{2}(F^2 + G^2)$
  - $\mathcal{L}_{\rm mass} = -m[\frac{1}{2}\bar{\psi}\psi - GA - FB]$
- A and B and real scalar fields with $[A] = [B] = 1$
- ψ is a *Majorana* spinor with $\psi = C\bar{\psi}^T$ and $[\psi] = \frac{3}{2}$
- F and G are *auxiliary* (non-propagating) fields with $[F] = [G] = 2$
  - can be eliminated by E-L equations: $F = -mB$, $G = -mA$

SUSY transformation in WZ model

- $A \to A + \delta A$ with $\delta A = i\bar{\alpha}\gamma_5\psi$
- $\delta B = -\bar{\alpha}\psi,$
- $\delta\psi = -F\alpha + iG\gamma_5\alpha + \not\partial\gamma_5 A\alpha + i\not\partial B\alpha,$
- $\delta F = i\bar{\alpha}\not\partial\psi,$
- $G = \bar{\alpha}\gamma_5\not\partial\psi$

Using Majorana bilinear re-arrangements (e.g. $\bar{\psi}\chi = -\bar{\chi}\psi$) and product rule $\partial_\mu(f\cdot g) = \partial_\mu f\cdot g + f\cdot\partial_\mu g$ and algebra, can show that $\mathcal{L} \to \mathcal{L} + \delta\mathcal{L}$ with

- $\delta\mathcal{L}_{\rm kin} = \partial^\mu(-\frac{1}{2}\bar{\alpha}\gamma_\mu\not\partial B\psi + \frac{i}{2}\bar{\alpha}\gamma_5\gamma_\mu\not\partial A\psi + \frac{i}{2}F\bar{\alpha}\gamma_\mu\psi + \frac{1}{2}G\bar{\alpha}\gamma_5\gamma_\mu\psi),$
- $\delta\mathcal{L}_{\rm mass} = \partial^\mu(mA\bar{\alpha}\gamma_5\gamma_\mu\psi + imB\bar{\alpha}\gamma_\mu\psi)$

Since Lagrangian changes by a total derivation, the *action* $S = \int\mathcal{L}d^4x$ is invariant! (owing to Gauss' law in 4-d) $\int_V d^4x\partial_\mu\Lambda^\mu = \int_{\partial V} d\sigma\Lambda^\mu n_\mu$ Thus, WZ transformation is a *symmetry* of the action!

Aspects of the WZ model:

- Can add interactions:
- $\mathcal{L}_{\text{int}} = -\frac{g}{\sqrt{2}} A\bar{\psi}\psi + \frac{ig}{\sqrt{2}} B\bar{\psi}\gamma_5\psi + \frac{g}{\sqrt{2}}(A^2 - B^2)G + g\sqrt{2}ABF$
- Difficult calculation, but can show $\delta\mathcal{L} \to$ total derivative
- Also, can show: quadratic divergences all *cancel*!
- If SUSY transformations expressed as $\mathcal{S} \to \mathcal{S}' = e^{i\bar{\alpha}Q}\mathcal{S}e^{-i\bar{\alpha}Q} \approx \mathcal{S} + [i\bar{\alpha}Q, \mathcal{S}] = \mathcal{S} + \delta\mathcal{S} \equiv (1 - i\bar{\alpha}Q)\mathcal{S}$, then can show that the generator Q obeys
    - $\{Q_a, \bar{Q}_b\} = 2(\gamma_\mu)_{ab}P_\mu$
    - SUSY is spacetime symmetry! (super-Poincaré algebra)

Weak Scale Supersymmetry

HB and X. Tata

Spring, 2006; Cambridge University Press

★ Part 1: superfields/Lagrangians

– 4-component spinor notation for exp'ts

– master Lagrangian for SUSY gauge theories

★ Part 2: models/implications

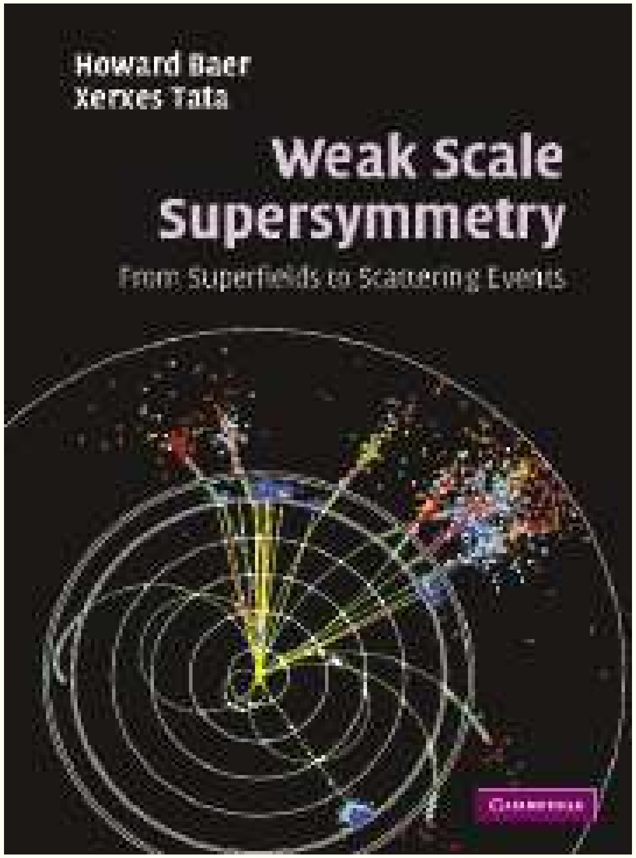
– MSSM, SUGRA, GMSB, AMSB, $\cdots$

★ Part 3: SUSY at colliders

– production/decay/event generation

– collider signatures

– R-parity violation



Constructing supersymmetric models

- While the WZ model is interesting, it was essentially "taken out of a hat", without rules of how to construct SUSY models in general
- Shortly after WZ model appeared in 1974, Salam and Strathdee developed *superfield formalism*, which *does* allow one to construct SUSY models in general.
- How does one combine scalar and spinor fields into a single entity?
- introduce *superspace* $x^\mu \to (x^\mu, \theta_a)$ where θ_a $(a = 1 - 4)$ are four *anti*-commuting dimensions arranged as a Majorana spinor

Some types of superfields

- general superfield:
$\hat{\Phi}(x,\theta) = \mathcal{S} - i\sqrt{2}\bar{\theta}\gamma_5\psi - \frac{i}{2}(\bar{\theta}\gamma_5\theta)\mathcal{M} + \frac{1}{2}(\bar{\theta}\theta)\mathcal{N} + \frac{1}{2}(\bar{\theta}\gamma_5\gamma_\mu\theta)V^\mu$
$+ i(\bar{\theta}\gamma_5\theta)[\bar{\theta}(\lambda + \frac{i}{\sqrt{2}}\not\partial\psi)] - \frac{1}{4}(\bar{\theta}\gamma_5\theta)^2[\mathcal{D} - \frac{1}{2}\Box\mathcal{S}]$
- left chiral scalar superfield: $\hat{\mathcal{S}}_L(x,\theta) = \mathcal{S}(\hat{x}) + i\sqrt{2}\bar{\theta}\psi_L(\hat{x}) + i\bar{\theta}\theta_L\mathcal{F}(\hat{x})$ where $\hat{x}_\mu = x_\mu + \frac{i}{2}\bar{\theta}\gamma_5\gamma_\mu\theta$
- right chiral scalar superfield: $\hat{\mathcal{S}}_R(x,\theta) = \mathcal{S}(\hat{x}^\dagger) - i\sqrt{2}\bar{\theta}\psi_R(\hat{x}^\dagger) - i\bar{\theta}\theta_R\mathcal{F}(\hat{x}^\dagger)$
- multiplication rules
  - LCSSF$\times$ LCSSF$=$ LCSSF
  - RCSSF$\times$ RCSSF $=$ RCSSF
  - LCSSF$\times$ RCSSF $=$ general superfield
- D-term of general SF transforms to total derivative
- F-term of LCSSF or RCSSF transforms into total derivative

Supersymmetric Lagrangians

- Since D and F terms transform into total derivatives, they are candidates for SUSY Lagrangians!
- The *superpotential* $\hat{f}$ is a function of LCSSFs only. Hence it is itself a LCSSF, and its F term is a candidate Lagrangian
- The *Kähler potential* K is a function of LCSSFs times RCSSFs. Hence it is a general superfield and its D term is a candidate Lagrangian
- Augmenting the superfields with *gauge* superfields $\hat{\Phi}_A = \frac{1}{2}(\bar{\theta}\gamma_5\gamma_\mu\theta)V_A^\mu + i\bar{\theta}\gamma_5\theta\cdot\bar{\theta}\lambda_A - \frac{1}{4}(\bar{\theta}\gamma_5\theta)^2\mathcal{D}_A$ (in WZ gauge) or $\hat{W}_A(\hat{x},\theta) = \lambda_{LA}(\hat{x}) + \frac{1}{2}\gamma^\mu\gamma^\nu F_{\mu\nu A}(\hat{x})\theta_L - i\bar{\theta}\theta_L(\not{D}\lambda_R)_A - i\mathcal{D}_A(\hat{x})\theta_L$ allows one to write a *Master formula* for supersymmetric gauge theories!

Master formula for SUSY gauge theories

$$
\begin{aligned}
\mathcal{L} &= \sum_i (D_\mu \mathcal{S}_i)^\dagger (D^\mu \mathcal{S}_i) + \frac{i}{2}\sum_i \bar{\psi}_i \not{D}\psi_i + \sum_{\alpha,A}\left[\frac{i}{2}\bar{\lambda}_{\alpha A}(\not{D}\lambda)_{\alpha A} - \frac{1}{4}F_{\mu\nu\alpha A}F^{\mu\nu}_{\alpha A}\right] \\
&- \sqrt{2}\sum_{i,\alpha,A}\left(\mathcal{S}_i^\dagger g_\alpha t_{\alpha A}\bar{\lambda}_{\alpha A}\frac{1-\gamma_5}{2}\psi_i + \text{h.c.}\right) \\
&- \frac{1}{2}\sum_{\alpha,A}\left[\sum_i \mathcal{S}_i^\dagger g_\alpha t_{\alpha A}\mathcal{S}_i + \xi_{\alpha A}\right]^2 - \sum_i \left|\frac{\partial \hat{f}}{\partial \hat{\mathcal{S}}_i}\right|^2_{\hat{\mathcal{S}}=\mathcal{S}} \\
&- \frac{1}{2}\sum_{i,j}\bar{\psi}_i\left[\left(\frac{\partial^2 \hat{f}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}}\frac{1-\gamma_5}{2} + \left(\frac{\partial^2 \hat{f}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j}\right)^\dagger_{\hat{\mathcal{S}}=\mathcal{S}}\frac{1+\gamma_5}{2}\right]\psi_j ,
\end{aligned}
$$

where the covariant derivatives are given by,

$$
D_\mu \mathcal{S} = \partial_\mu \mathcal{S} + i\sum_{\alpha,A} g_\alpha t_{\alpha A} V_{\mu\alpha A}\mathcal{S},
$$

$$
\begin{aligned}
D_\mu \psi &= \partial_\mu \psi + i \sum_{\alpha, A} g_\alpha (t_{\alpha A} V_{\mu\alpha A}) \psi_L \\
&\quad - i \sum_{\alpha, A} g_\alpha (t^*_{\alpha A} V_{\mu \alpha A}) \psi_R, \\
(\not{D}\lambda)_{\alpha A} &= \not{\partial}\lambda_{\alpha A} + i g_\alpha \left(t^{adj}_{\alpha B} \not{V}_{\alpha B} \right)_{AC} \lambda_{\alpha C}, \\
F_{\mu\nu\alpha A} &= \partial_\mu V_{\nu\alpha A} - \partial_\nu V_{\mu\alpha A} - g_\alpha f_{\alpha ABC} V_{\mu\alpha B} V_{\nu\alpha C}.
\end{aligned}
$$

Supersymmetry breaking

- Spontaneous breaking of global SUSY is possible: $\langle 0|\mathcal{F}_i|0\rangle \neq 0$ or $\langle 0|\mathcal{D}_A|0\rangle \neq 0$ (F or D type breaking)
- May also explicitly break SUSY by adding *soft* SUSY breaking terms to $\mathcal{L}$:
  - linear terms in the scalar field $\mathcal{S}_i$ (relevant only for singlets of all symmetries),
  - scalar masses,
  - and bilinear or trilinear operators of the form $\mathcal{S}_i\mathcal{S}_j$ or $\mathcal{S}_i\mathcal{S}_j\mathcal{S}_k$ (where $\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j$ and $\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j\hat{\mathcal{S}}_k$ occur in the superpotential),
  - and finally, in gauge theories, gaugino masses, one for each factor of the gauge group,

Recipe for SUSY model building

- Choose the gauge symmetry (adopting appropriate gauge superfields for each gauge symmetry)
- Choose matter and Higgs representations included as LCSSFs
- Choose the superpotential $\hat{f}$ as a gauge invariant *analytic function* of LCSSFs; degree is ≤ 3 for renormalizable theory
- Adopt all allowed gauge invariant soft SUSY breaking terms; these are generally chosen to *parametrize our ignorance* of the mechanism of SUSY breaking
- The Master formula, augmented by the soft SUSY breaking terms, gives the final Lagrangian of the theory.

Next lecture:

- Using the $Master\ formula$, we may write down the MSSM
- MSSM includes the SM as a sub-theory, but much more
- Since quadratic divergences cancel, MSSM may be a theory valid $e.g.$ from M_{weak} to M_{GUT}
- RGEs, gauge coupling evolution, REWSB, spectrum calculation
- supergravity
- Grand unified theories:
  - $SU(5)$
  - $SO(10)$